Measurement of the de Broglie Wavelength of a Multiphoton Wave Packet

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A fourth-order Young interference experiment was done to demonstrate a practical way to measure the de Broglie wavelength of a two-photon wave packet. A two-photon collinear beam is generated by type-II spontaneous parametric down-conversion. By modifying the transverse field profile of the pump laser beam that generates the two-photon beam we demonstrate that it is possible to measure the de Broglie wavelength of the single-photon constituents of the two-photon wave packet, the de Broglie wavelength of the two-photon wave packet as a whole and an ill defined intermediate de Broglie wavelength between the two cases. [S0031-9007(99)08826-2]

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In a recent article, Jacobson *et al.* [1] have shown theoretically that the measured de Broglie wavelength of an object is dependent on its internal structure as well as on the detection system. Motivated by a recent experiment [2,3] with molecules they proposed an idealized interferometer that is capable of measuring the de Broglie wavelength of an incident multiphoton wave packet as a whole. For an ensemble of photons with average number n and wavelength λ_0, the de Broglie wavelength is $\frac{\lambda_0}{n}$. Their proposed Mach-Zehnder interferometer has an "effective" beam splitter (BS) that is a function of a parameter χ that varies from 0 (BS does not divide the composite system in constituent quanta) to 1 (BS divides the composite system in its constituent quanta). A calculation done with an incident state $|\psi_0\rangle = |2,0\rangle$ (two photons incident in one of the ports and zero photons in the other) shows that the oscillation period of the interferometer varies from $\lambda_0(\chi = 1)$, corresponding to single-photon interference, to $\frac{\lambda_0}{2}(\chi = 0)$, corresponding to two-photon interference. It is interesting to notice that for intermediate values of χ the de Broglie wavelength is not well defined, even though the state energy is well defined.

In this article, we demonstrate that we can measure the de Broglie wavelength of a two-photon wave packet (*biphoton*) with a Young double-slit experiment. The incident two-photon wave packet is generated collinearly from a nonlinear crystal by the process of spontaneous parametric down-conversion. The photons transmitted by the double slit form a fourth-order pattern which is a superposition of two Young interference patterns with different periodicity. One of them results from the interference of the individual photons ("the parts of the object" [1]) and has an oscillation period of λ_0. The other pattern is due to the interference of the "object as a whole with itself," i.e., the interference of the "*biphoton*" and shows a periodicity of $\frac{\lambda_0}{2}$. Both interference patterns are functions of the transverse spatial profile of the pump laser beam used for the generation of the down-converted photons [4]. By tailoring the spatial profile of the pump beam it is possible to switch from the interference of the constituent quanta ($\chi \approx 1$ in Jacobson *et al.*'s article [1]) to the interference of the two-photon packet as a whole ($\chi \approx 0$). It is also possible to be in a situation where the de Broglie wavelength is not well defined.

In the process of parametric down-conversion, one photon of a pump laser (frequency ω_p and wave vector $\vec{k}_p$) is converted into a photon pair: signal (ω_s, $\vec{k}_s$) and idler (ω_i, $\vec{k}_i$). A complete theory [4] that takes into account the spatial correlation of the generated down-converted parametric photons was used to calculate the number of transmitted *biphotons* through the double slit. The two-photon wave packets are generated collinearly from the crystal (Fig. 1a). The number of transmitted two-photon packets hitting the detection screen is proportional to the fourth-order correlation function calculated at position x,

$$N_c(x) \propto \langle \hat{E}_i^-(x)\hat{E}_s^-(x)\hat{E}_i^+(x)\hat{E}_s^+(x)\rangle, \qquad (1)$$

where $\hat{E}_i^+(x)$ and $\hat{E}_s^+(x)$ are the idler and the signal transmitted electric field operators, respectively. The transmitted electric field operators are obtained by analogy with the classical calculation of the electric field transmitted through an aperture when the angular spectrum of the field before the aperture (double slit) is known [5]. The electric field operator can be written as

$$\hat{E}_j^+(x) \propto e^{ikz} \int dq_j \int dq_j'\, \hat{a}(q_j')T(q_j - q_j') \times \exp i\left[q_j x - \frac{q_j^2(z - z_A)}{2k} - \frac{q_j'^2 z_A}{2k}\right], \qquad (2)$$

where $j = i$(idler), s(signal); $k = k_s = k_i$ is the down-converted field wave vector magnitude; q_j, q_j' are x-transverse components of the wave vectors $\vec{k}$; $\hat{a}(q')$ is the annihilation operator associated with the mode q'; $T(u)$ is the Fourier transform of the double-slit aperture; z_A is the distance from the crystal to the double slit, x is the transverse position of the two-photon detector, and z is the longitudinal coordinate with origin ($z = 0$) at the crystal center (Fig. 1a).

Expression (1) is calculated by using the multimode two-photon wave function [6]. In the monochromatic

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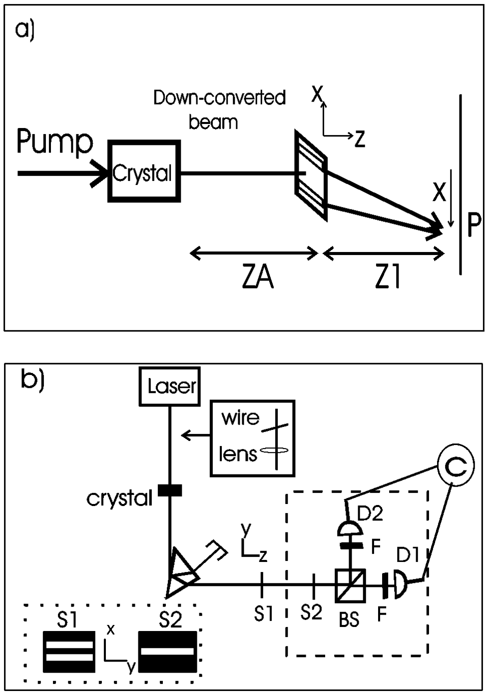

FIG. 1. (a) Schematic drawing of a Young's double-slit experiment where $2a$ is the slit's width; $2d$ is the double slit separation; z_1 is the distance between the double slit and the detector plane; z_A is the distance between the crystal and the double-slit plane; x, y are the transverse coordinates, and z is the longitudinal coordinate. (b) Outline of the experimental setup. BS is the beam-splitter polarizer; M1 is the mirror; F is an interference or color glass cutoff filter; C is the coincidence detection system; S1 is a double slit; S2 is a single-slit; D1 and D2 are detectors. The dashed square shows the two-photon detector. (Inset) In some of the measurements a copper wire with a lens was positioned at the pump laser beam path parallel to the Young's slits.

($\Delta\omega_j \ll \omega_j, j = s, i, p$), paraxial ($|q_j| \ll |k|$), and thin crystal approximation, the state generated by the parametric down-conversion process can be approximated by [7]

$$|\Psi\rangle = |\text{vac}\rangle + \text{const} \int dq_s \int dq_i\, v(q_s + q_i) \times |1, q_s\rangle |1, q_i\rangle\,, \tag{3}$$

where $v(q_s + q_i)$ is the angular spectrum of the pump beam. As it was shown recently [7], the angular spectrum of the pump laser beam is transferred to the fourth-order correlation function of the down-converted two-photon pairs. After some algebra, we obtain the number of *biphotons* as a function of position x in the detection screen from expression (1) (Fig. 1a):

$$\begin{aligned} N_c(x) \propto{}& A(x) + 4B_1(x)B_2(x) \cos\left(\frac{kd^2}{z_A} + \frac{kx2d}{z_1}\right) \\ &+ 4B_2(x)B_4(x) \cos\left(\frac{kd^2}{z_A} - \frac{kx2d}{z_1}\right) \\ &+ 2B_1(x)B_4(x) \cos\left[\frac{2kx(2d)}{z_1}\right], \end{aligned} \tag{4}$$

with

$$A(x) = |B_1(x)|^2 + 4|B_2(x)|^2 + |B_4(x)|^2, \tag{5}$$

$$B_1(x) = 4\sqrt{W(d, z_A)}\, a^2 \operatorname{sinc}^2\left[\left(\frac{k(x - d)a}{z_1}\right)\right], \tag{6}$$

$$\begin{aligned} B_2(x) ={}& 4\sqrt{W(0, z_A)}\, a^2 \operatorname{sinc}^2\left[\left(\frac{kx}{z_1} + \frac{kd}{L}\right)a\right] \\ &\times \operatorname{sinc}^2\left[\left(\frac{kd}{L} - \frac{kx}{z_1}\right)a\right], \end{aligned} \tag{7}$$

$$B_4(x) = 4\sqrt{W(-d, z_A)}\, a^2 \operatorname{sinc}^2\left[\left(\frac{k(x + d)a}{z_1}\right)\right], \tag{8}$$

where $2d$ is the separation of the double slit, $2a$ is the width of each slit, z_1 is the distance from the double-slit plane to the detector plane, $L = z_1 z_A/z_1 + z_A$, and $\operatorname{sinc}(x) = \sin(x)/x$. $W(x, z_A)$ is the spatial intensity distribution of the pump laser beam at the transverse position x and at the longitudinal distance z_A from the crystal. We notice that the second and third terms in the right hand side of expression (4) show the interference with the periodicity of the down-converted photons and the fourth term express an expected interference pattern from a light beam with the pump wavelength $\frac{\lambda}{2}$. Expressions (6), (7), and (8) show that since $B_1(x)$, $B_2(x)$, and $B_4(x)$ are proportional to the second power of sinc functions, the diffraction terms will be proportional to the fourth power of sinc functions. Note also that $B_1(x)$, $B_2(x)$, and $B_4(x)$ are proportional to the square root of the transverse intensity distribution of the pump laser at the position of the double slit. This is a very interesting and useful result because it can be used to select one of the superposed interference patterns. We can, for example, have a pump transverse intensity distribution at $z = z_A$ that is zero for $x = 0$, i.e., $W(0, z_A) = 0$, and obtain an interference pattern with periodicity $\frac{\lambda}{2}$. We can select, therefore, the Young interference pattern of the "*biphoton* as a whole," i.e., the two-photon wave packet interference with itself. It is also worth mentioning that this Young interference pattern depends on the spatial profile of the pump at the double slit and not at the crystal position as in the Young one-photon interference pattern [8]. Therefore, by preparing the two-photon light state in different ways we are able to measure the de Broglie wavelength of the two-photon packet as a whole or of its constituent quanta.

A 5 mm × 5 mm × 7 mm BBO nonlinear crystal pumped by a 400 mW Argon laser emitting at $\lambda = 351.1$ nm was used to generate down-conversion parametric luminescence. The crystal, whose optical axis lies in a horizontal plane, is cut for collinear type-II phase matching. The down-converted photons have the same wavelength around 702.2 nm. The pump laser beam was blocked by a laser mirror after being transmitted through the crystal. Two 0.9 nm bandwidth Gaussian interference filters F, centered at 702.2 nm were used for alignment of the down-converted beams. A Young double slit (S1) was placed 460 mm far from the crystal (Fig. 1b). The double-slit plane (xy plane) was aligned perpendicular to the pump laser beam direction (z direction) with the small dimension of the slits parallel to the x direction (see Fig. 1b). The width of each slit $2a = 0.1$ mm and the separation between them $2d = 0.3$ mm were measured with a microscope. The two-photon interference pattern was recorded by displacing a "two-photon detector" perpendicular to the pump beam (x direction) [9,10]. The "two-photon detector" consists of a 0.3 mm single slit (S2) oriented parallel to the double slit, a (50/50)% beam-splitter polarizer (BS) and two avalanche photodiodes (D1, D2) detecting the photons in coincidence. Fourth-order interference patterns were obtained from the coincidence counts between D1 and D2 as a function of the two-photon detector transverse position. The distance between the planes of the Young double slit and the two-photon detector slit was 690 mm. The narrow-band interference filters were replaced by color glass cutoff filters when the double-slit interference experiments were performed. The Gaussian transverse field profile of the pump laser was modified in some of the Young's interference experiments by positioning a 0.2 mm diameter copper wire in its path parallel to the Young's slits and 1540 mm before the crystal. Its image was projected 460 mm after the crystal with the help of a $f = 500$ mm lens placed 540 mm before the crystal (Fig. 1b, inset). The transverse intensity profile of the pump laser beam with and without the wire was measured by displacing transversely a powermeter with a 0.025 mm diameter pinhole. The profile was measured at $z_A = 460$ mm from the crystal.

Figure 2a shows the transverse Gaussian profile of the pump laser in the x direction, measured at $z_A = 460$ mm from the crystal (at the double-slit position). For this pump laser Gaussian profile we obtain the fourth-order interference pattern shown in Fig. 2b. This corresponds to a situation where most of the interference occurs between each "individual constituent" of the two-photon wave packets with itself. The wavelength inferred from this pattern corresponds to the wavelength of the downconverted photons $\lambda = 702.2$ nm.

Figure 3 shows experimentally that by modifying the transverse *biphoton* beam profile at the double-slit position ($z = z_A$), via the pump laser beam, we can control the periodicity of the fourth-order Young interference pattern. This control is done by focusing or defocusing the image of the wire projected at the double slit. Figure 3a shows the pump transverse profile where the image of wire is defocused. A Young interference pattern with mixed periodicity is shown in Fig. 3b. The measured de Broglie wavelength from this pattern is not well defined. When the image of the wire is focused on the double slit, we have $W(0, z_A) = 0$ (Fig. 3c). With the pump profile shown in Fig. 3c we obtained a Young interference pattern with the pump laser periodicity (Fig. 3d). The *biphoton* interferes with itself. The measured wavelength is then the de Broglie wavelength of the *biphoton*: $\lambda/2$. The continuous curves in Figs. 2b and 3d are obtained from the theoretical expression (4) slightly changed by including one normalization constant as free parameter and by taking into account the finite size of the "two-photon detector." For generating the curve in Fig. 3b, we added a second free parameter that is able to displace the transverse pump profile $W(x, z_A)$ relatively to the double slit center. The finite size of the detector was taken into account by integrating the *biphotons* number distribution

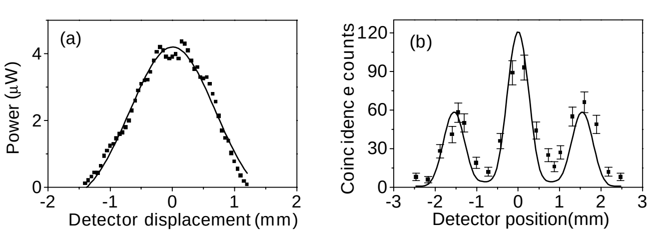

FIG. 2. (a) Transverse profile of the pump laser (x direction), measured at the position of the double slit, i.e., at $z_A = 460$ mm from the crystal. (b) Fourth-order Young's interferogram showing "one-photon" interference for a double slit with separation $2d = 0.3$ mm and slit width $2a = 0.1$ mm. The continuous curves in (a) is experimental fit and (b) is the theoretical curve with one free parameter (see text). Coincidence counts detection time were 600 s. The measured wavelength is $\lambda = 702$ nm.

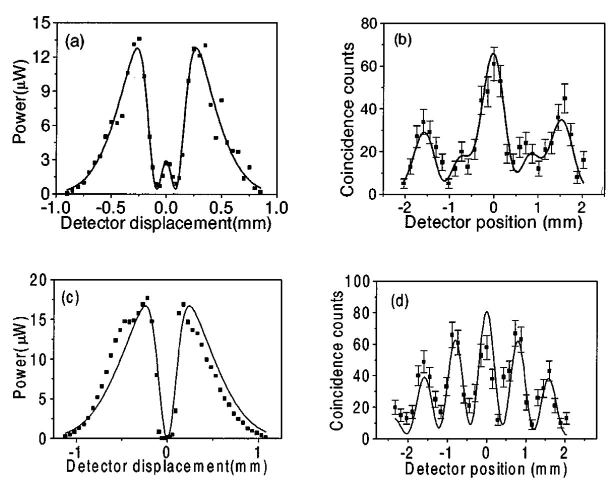

FIG. 3. (a) and (c) are images of a wire out of focus and in focus at the double-slit position, i.e., at $z_A = 460$ mm from the crystal. (b) and (d) are down-converted fourth-order Young's interferograms for the transverse pump beam profiles shown in (a) and (c), respectively. Interferogram in (d) shows "two-photon" interference and the measured wavelength is $\lambda/2 = 351$ nm. The continuous curves in (a) and (c) are experimental fits and in (b) and (d) are the theoretical curves with two and one free parameter, respectively (see text). Coincidence counts detection time were 600 s (b) and 700 s (d).

between $x_o - \frac{b}{2}$ and $x_o + \frac{b}{2}$, where b is the width of the single slit S1 of the two-photon detector and x_o is the two-photon detector transverse position. The theoretical function $W(x, z_A)$ was obtained from the experimental fits shown in Figs. 2a, 3a, and 3c. The interference pattern with two different oscillation periods is a pure two-photon effect, seen only in the fourth-order interferograms. Second-order interference patterns didn't show it.

In conclusion, we have demonstrated that the de Broglie wavelength of two-photon wave packets can be measured with a Young double-slit experiment. Light in the two-photon state was generated by type-II spontaneous parametric down-conversion. By modifying the transverse pump field profile that generates a *biphoton* beam we were able to measure the de Broglie wavelength of the single-photon constituents of the two-photon wave packets. We also measured the de Broglie wavelength of the two-photon wave packet as a whole and an undefined intermediate de Broglie wavelength between the two cases. Theoretical predictions agree very well with experimental data. By modifying the transverse field profile of the pump laser beam in the parametric down-conversion process we demonstrated that a Young interference experiment can be a real alternative to the idealized Mach-Zehnder interferometer proposed by Jacobson *et al.* [1]. The dependence of the periodicity of the fourth-order Young's interferogram on the transverse field profile of the pump laser beam that generates the *biphoton* beam is a pure two-photon effect.

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