Precursor of Chiral Symmetry Restoration in the Nuclear Medium

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Spectral enhancement near the $2m_\pi$ threshold in the $I = J = 0$ channel in nuclei is shown to be a distinct signal of the partial restoration of chiral symmetry. The relevance of this phenomenon with the possible detection of $2\pi^0$ and 2γ in hadron-nucleus and photon-nucleus reactions is discussed. [S0031-9007(99)08890-0]

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One of the intriguing phenomena in the physics of strong interactions is the dynamical breaking of chiral symmetry (DBχ). This explains the existence of the pion and dictates most of the low energy phenomena in hadron physics. In the context of quantum chromodynamics (QCD), the fundamental theory of strong interactions, DBχ is associated with the condensation of quark-antiquark pairs in the vacuum. This is analogous to the condensation of Cooper pairs in the theory of superconductivity [1]. Furthermore, as the baryon density and/or the temperature are raised, the QCD vacuum is supposed to undergo a phase transition to the chirally symmetric phase [2]. In particular, some theoretical analyses suggest that a partial restoration of chiral symmetry occurs even at a relatively low baryon density (ρ) close to the nuclear matter density $\rho_0 = 0.17$ fm^{-3} [2]. Then, it is of fundamental importance to clarify what observables reflect most properly this partial symmetry restoration. The purpose of this Letter is to give a candidate of such observables and to propose experiments to test the idea with nuclear targets.

The general wisdom of many-body physics [3–5] tells us that the fluctuation of the order parameter becomes large as the system approaches the critical point of the phase transition. In QCD, this corresponds to a softening of a collective excitation having the same quantum number as that of the vacuum, namely, the scalar-isoscalar ($I = J = 0$) meson, σ [6]. The softening (the redshift of the spectrum) in turn causes the decrease of the decay width of σ by the phase-space suppression of the reaction $\sigma \to 2\pi$. This leads to a conjecture that σ may appear as a sharp resonance at finite temperature (T) [6,7], although it can be elusive due to the large width in the free space [8,9]. A later analysis at $T \neq 0$ showed that the spectral function in the σ channel has a characteristic enhancement just above the two-pion threshold [10].

In this Letter, we demonstrate, using a toy model, that the spectral enhancement near the threshold is a distinct signal of the partial chiral restoration also at $\rho \neq 0$. As possible experiments to observe this enhancement, we will propose to detect the neutral-dipion ($2\pi^0$) and diphoton (2γ) in reactions with heavy nuclear targets [11]. We will also mention the relevance of the softening to the recent data on the near-threshold $\pi^+\pi^-$ production in π^+-nucleus reactions by the CHAOS Collaboration [14].

Before presenting our explicit model calculation, let us describe the general features of the spectral enhancement near the two-pion threshold. Consider the propagator of the σ meson at rest in the medium: $D_\sigma^{-1}(\omega) = \omega^2 - m_\sigma^2 - \Sigma_\sigma(\omega;\rho)$, where m_σ is the mass of σ in the tree level, and $\Sigma_\sigma(\omega;\rho)$ is the loop corrections in the vacuum as well as in the medium. The corresponding spectral function $\rho_\sigma(\omega) = -\pi^{-1}\,\mathrm{Im}\,D_\sigma(\omega)$ reads

$$\rho_\sigma(\omega) = -\frac{1}{\pi}\,\frac{\mathrm{Im}\,\Sigma_\sigma}{(\omega^2 - m_\sigma^2 - \mathrm{Re}\,\Sigma_\sigma)^2 + (\mathrm{Im}\,\Sigma_\sigma)^2}\,. \tag{1}$$

Near the two-pion threshold, the phase space factor gives $\mathrm{Im}\,\Sigma_\sigma \propto \theta(\omega - 2m_\pi)\sqrt{1 - (4m_\pi^2/\omega^2)}$ in the one-loop order. On the other hand, the partial restoration of chiral symmetry indicates that m_σ^* [the "effective mass" of σ defined through $\mathrm{Re}\,D_\sigma^{-1}(\omega = m_\sigma^*) = 0$] approaches to m_π. Therefore, there exists a density ρ_c at which $\mathrm{Re}\,D_\sigma^{-1}(\omega = 2m_\pi)$ vanishes even before the complete σ-π degeneracy takes place; namely, $\mathrm{Re}\,D_\sigma^{-1}(\omega = 2m_\pi) = [\omega^2 - m_\sigma^2 - \mathrm{Re}\,\Sigma_\sigma]_{\omega=2m_\pi} = 0$. At this point, the spectral function is solely dictated by the imaginary part of the self-energy;

$$\rho_\sigma(\omega \simeq 2m_\pi) = -\frac{1}{\pi\,\mathrm{Im}\,\Sigma_\sigma} \propto \frac{\theta(\omega - 2m_\pi)}{\sqrt{1 - (4m_\pi^2/\omega^2)}}\,. \tag{2}$$

This shows that, even if there is no sharp resonance in the scalar channel in the free space, there arises a mild (integrable) singularity just above the threshold in the medium. We emphasize that this is a general phenomenon correlated with the partial restoration of chiral symmetry.

To make the argument more quantitative, let us evaluate $\rho_\sigma(\omega)$ in a *toy* model, namely, the SU(2) linear σ model:

$$\mathcal{L} = \frac{1}{4}\,\mathrm{tr}\left[\partial M \partial M^\dagger - \mu^2 M M^\dagger - \frac{2\lambda}{4!}(MM^\dagger)^2 - h(M + M^\dagger)\right], \tag{3}$$

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where tr is for the flavor index and $M = \sigma + i\vec{\tau}\cdot\vec{\pi}$. Although the model is not a precise low energy representation of QCD [15], it is known to describe the pion dynamics qualitatively well up to 1 GeV as shown by Chan and Haymaker [16].

The coupling constants μ^2, λ, and h have been determined in the vacuum to reproduce $f_\pi = 93$ MeV, $m_\pi = 140$ MeV, as well as the s-wave π-π scattering phase shift in the one-loop order. Resultant parameters in the modified minimal subtraction ($\overline{\rm MS}$) renormalization scheme are given in [10] and are recapitulated for two characteristic cases in Table I in which $m_\sigma^{\rm peak}$ is defined as a peak position of $\rho_\sigma(\omega)$.

The interaction Lagrangian of M with the nucleon field N with SU(2) chiral symmetry is modeled as

$$\mathcal{L}_I(N,M) = -g\chi\bar{N}U_5N - m_0\bar{N}U_5N\,, \tag{4}$$

where we have used a polar representation $\sigma + i\vec{\tau}\cdot\vec{\pi}\gamma_5 \equiv \chi U_5$ for convenience [17]. The first term in (4) with the coupling constant g is a standard chiral invariant coupling in the linear σ model. The second term with a new parameter m_0, which is usually not taken into account in the literature, is also chiral invariant and nonsingular.

With the dynamical breaking of chiral symmetry ($\langle\sigma\rangle_{\rm vac} \equiv \sigma_0 \neq 0$), Eq. (4) expanded in terms of σ/σ_0 and $\vec{\pi}/\sigma_0$ reads $\mathcal{L}_I(N,M) = -m_N\bar{N}N - \bar{N}(g_s\tilde{\sigma} + ig_p\vec{\tau}\cdot\vec{\pi}\gamma_5)N + \frac{1}{2}(m_0/\sigma_0^2)\bar{N}\vec{\pi}^2N + 0(\tilde{\sigma}^{l\geq 1}\times\pi^{n\geq 1})$, where $\tilde{\sigma} = \sigma - \sigma_0$, $m_N \equiv m_0 + g\sigma_0$, $g_s \equiv g$, and $g_p \equiv g_s + m_0/\sigma_0$. Because of m_0, the standard constraint $g_s = g_p$ can be relaxed without conflicting with chiral symmetry. Also, the term proportional to $m_0\pi^2$ appears to preserve chiral symmetry. g_p is constrained by the Goldberger-Treiman relation; $g_p = m_N/\sigma_0 \simeq m_N/f_\pi = g_{\pi N} = 13.5$. On the other hand, g_s is independent of g_p and can be treated as a free parameter. With this freedom, one can circumvent the well-known problem that $g_s = g_p$ combined with Eq. (3) does not reproduce the known nuclear matter properties in the mean-field level [18]. We remark that the dilated chiral model [19] can also avoid $g_s = g_p$ away from the "dilaton" limit and has been applied to study the scalar meson in nuclear matter [20].

In the following, we treat the effect of the meson loop as well as the baryon density as a perturbation to the vacuum quantities. Therefore, our loop expansion is valid only at relatively low densities. The full self-consistent treatment of the problem requires a systematic resummation of loops similar to what was developed at finite T [10]. Let us first consider the chiral condensate in nuclear matter $\langle\sigma\rangle$ and parametrize it as

$$\langle\sigma\rangle \equiv \sigma_0\Phi(\rho)\,. \tag{5}$$

In the linear density approximation, $\Phi(\rho) = 1 - C\rho/\rho_0$ with $C = (g_s/\sigma_0 m_\sigma^2)\rho_0$. Instead of using g_s, we use Φ as a basic parameter in the following analysis. The plausible value of $\Phi(\rho = \rho_0)$ is $0.7 \sim 0.9$ [2].

The one-loop corrections to the self-energy for σ can be read off from the diagrams in Fig. 1: $\Sigma_\sigma(\omega;\rho) = \Sigma_{\rm vac}^A + \Sigma_{\rm vac}^B + \Sigma_{\rm MF}(\rho) + \Sigma_{\rm ph}(\rho)$. $\Sigma_{\rm vac}^A$ ($\Sigma_{\rm vac}^B$) corresponds to Fig. 1a and Fig. 1b, respectively. Only the latter has ω dependence and the imaginary part. The explicit formula for $\Sigma_{\rm vac}^{A+B}$ renormalized in the $\overline{\rm MS}$ scheme is given in Appendix A of [10].

$\Sigma_{\rm MF}(\rho)$ corresponds to the mean-field correction in the nuclear matter (Fig. 1c). $\Sigma_{\rm ph}(\rho)$ is a correction from the nuclear particle-hole excitation. We take only the density dependent part in these diagrams and neglect the problematic vacuum loops of the nucleon [18].

The leading term in the mean-field part is easily estimated as

$$\Sigma_{\rm MF}(\rho) = \lambda\sigma_0(\langle\sigma\rangle - \sigma_0) = -\lambda\sigma_0^2[1 - \Phi(\rho)]\,. \tag{6}$$

The leading term in the particle-hole part (Fig. 1d) in terms of $k_F = (3\pi^2\rho/2)^{1/3}$ reads $\Sigma_{\rm ph}(\rho) \simeq \frac{2g_s^2}{5\pi^2}\frac{k_F^5}{M^3}$, which starts from $O(\rho^{5/3})$ and is not more than a few percent of $\Sigma_{\rm MF}(\rho)$ at $\rho = \rho_0$. This is in contrast to the case of the pion, where both $\Sigma_{\rm MF}(\rho)$ and $\Sigma_{\rm ph}(\rho)$ are proportional to ρ and cancel with each other due to chiral symmetry. Because of this cancellation, we can neglect the two-loop contribution related to the medium modification of the low-momentum pions near the $2m_\pi$ threshold.

Up to this order, ${\rm Im}\,\Sigma_\sigma$ solely comes from ${\rm Im}\,\Sigma_{\rm vac}^B$, since there is no Landau damping and scalar-vector mixing for the σ meson at rest in nuclear matter:

$${\rm Im}\,\Sigma_\sigma(\omega;\rho) = {\rm Im}\,\Sigma_{\rm vac}^B = -\frac{\lambda^2}{96\pi}\,\sigma_0^2\sqrt{1 - \frac{4m_\pi^2}{\omega^2}}\,, \tag{7}$$

for $2m_\pi \leq \omega \leq 2m_\sigma$.

Now, let us look at the spectral function defined in (1). As we have already discussed, the threshold peak is prominent when ${\rm Re}\,D_\sigma^{-1} = \omega^2 - m_\sigma^2 - {\rm Re}\,\Sigma_\sigma = 0$. In the parametrization given in (5), this condition is rewritten

TABLE I. Parameters for $m_\sigma^{\rm peak} = 550$ and 750 MeV.

	$m_\sigma^{\rm peak}$ (MeV)	$\sqrt{-\mu^2}$ (MeV)	$\lambda/4\pi$	$h^{1/3}$ (MeV)
(I)	550	284	5.81	123
(II)	750	375	9.71	124

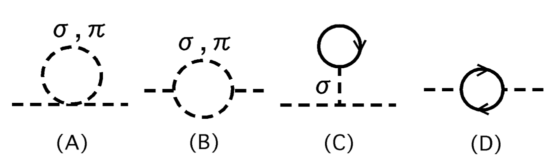

FIG. 1. One-loop self-energy. The dashed line denotes either σ or π. The solid line denotes the nucleon.

as $\Phi(\rho_c) = 0.74$ [case (I)], and $\Phi(\rho_c) = 0.76$ [case (II)]. The numbers on the right hand side are insensitive to the parameters in Eq. (3) as far as the physical quantities in the vacuum such as f_π and m_π are fixed. In the linear density formula $\Phi(\rho) = 1 - C\rho/\rho_0$ with a reasonable value $C \simeq 0.2$, we obtain $\rho_c \simeq 1.25\rho_0$ which is not far from the normal nuclear matter density. This implies that we could see the threshold enhancement in experiments with heavy nuclear targets.

The spectral functions together with $\mathrm{Re}\, D_\sigma^{-1}(\omega)$ for two cases (I) and (II) are shown in Fig. 2 and in Fig. 3, respectively. In both figures, the characteristic enhancement just above the $2m_\pi$ threshold is seen for $\rho \simeq \rho_c$ [21].

We notice that the enhancement is caused by (a) partial restoration of chiral symmetry where m_σ^* approaches toward m_π and (b) the cusp structure of $\mathrm{Re}\, D_\sigma^{-1}(\omega)$ at $\omega = 2m_\pi$; see the lower panels of Figs. 2 and 3. Although the cusp is not prominent at zero density, it eventually hits the real axis at $\rho = \rho_c$ because $\mathrm{Re}\, D_\sigma^{-1}(\omega)$ increases associated with $m_\sigma^* \to 2m_\pi$. This is a general phenomenon for systems where the internal symmetry is partially restored in the medium [22]. Another important observation is that, even at densities well below the point where m_σ^* and m_π are degenerate, one can expect the spectral enhancement near the $2m_\pi$ threshold [23].

To confirm the threshold enhancement associated with the partial chiral restoration, measuring $2\pi^0$ and 2γ in experiments with hadron/photon beams off the heavy nuclear targets should be most appropriate. Measuring $\sigma \to 2\pi^0 \to 4\gamma$ is experimentally feasible [12], and one can avoid the possible $I = J = 1$ background from the ρ meson inherent in the $\pi^+\pi^-$ measurement. Measuring the direct electromagnetic decay $\sigma \to 2\gamma$ is also important because of the small final state interactions. However, the branching ratio is small in this case ($\mathrm{Br} = \Gamma_{\sigma\to 2\gamma}/\Gamma_{\sigma\to 2\pi} = O(10^{-5})$ [8]), and one needs to fight with a large background of photons mainly coming from π^0's. Nevertheless, if the enhancement is prominent and changes rapidly as the mass number of the target nucleus, there is a chance to find the signal. There is also a possibility that one can detect dileptons through the scalar-vector mixing in matter: $\sigma \to \gamma^* \to e^+e^-$ [13]. In this case, the dileptons are produced only when σ has a finite three momentum.

To enhance the production cross section for the critical fluctuation in the $I = J = 0$ channel, a $(d, {}^3\mathrm{He})$ reaction is useful. The incident kinetic energies of the deuteron in the laboratory system E can be estimated to be $1.1 < E < 10$ GeV, to cover the spectral function in the range $2m_\pi < \omega < 750$ MeV [24].

Recently the CHAOS Collaboration [14] measured the $\pi^+\pi^\pm$ invariant mass distribution $M^A_{\pi^+\pi^\pm}$ in the reaction $A(\pi^+, \pi^+\pi^\pm)X$ with the mass number A ranging from 2 to 208: They observed that the yield for $M^A_{\pi^+\pi^-}$ near the $2m_\pi$ threshold is close to zero for $A = 2$ but increases dramatically with increasing A. They identified that the $\pi^+\pi^-$ pairs in this range of $M^A_{\pi^+\pi^-}$ is in the $I = J = 0$ state. Attempts so far in hadronic models without considering the partial chiral restoration failed to

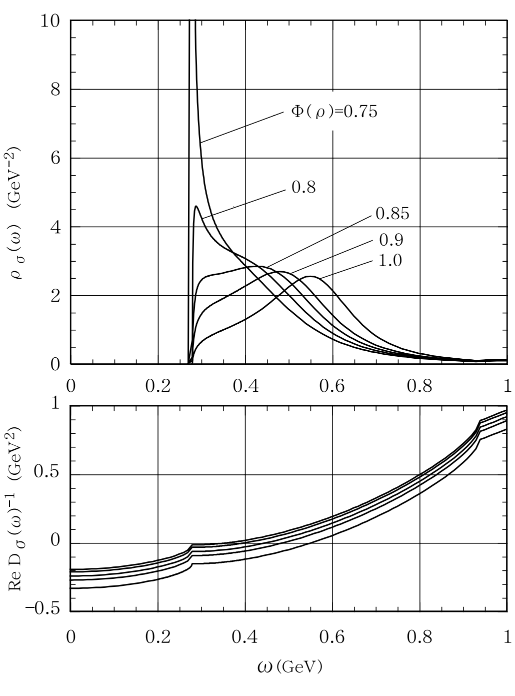

FIG. 2. Spectral function for σ and the real part of the inverse propagator for several values of $\Phi = \langle\sigma\rangle/\sigma_0$ with $m_\sigma^{\mathrm{peak}} = 550$ MeV [case (I) in Table I]. In the lower panel, Φ decreases from bottom to top.

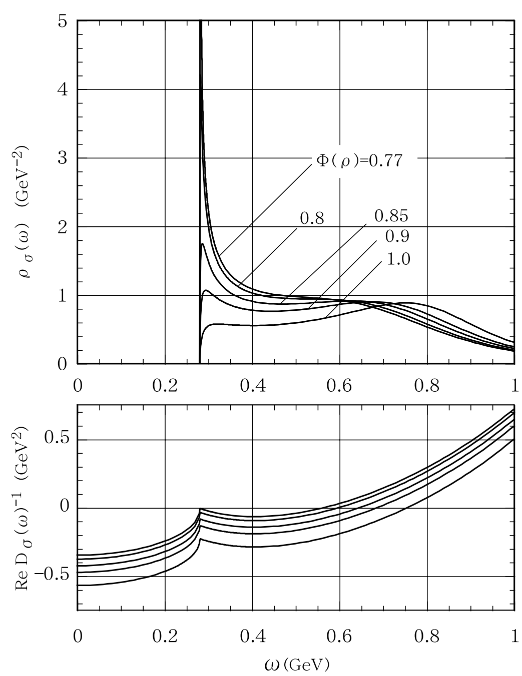

FIG. 3. Same as Fig. 2 for $m_\sigma^{\mathrm{peak}} = 750$ MeV [case (II) in Table I].

reproduce this enhancement [25,26]. On the other hand, the invariant mass distribution presented in [14] near $2m_\pi$ threshold for large A has a close resemblance to our model calculation in Fig. 2, which suggests that this experiment may already provide a hint about how the chiral symmetry is (partially) restored at finite density.

In summary, we have shown that the spectral function in the $I = J = 0$ channel has a large enhancement near the $2m_\pi$ threshold even at nuclear matter density due to the partial chiral restoration. Detection of the dipion and diphoton spectral distribution in the reactions of hadron/photon with heavy nucleus is suitable to confirm the idea of partial chiral restoration in nuclei.

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Note added.—After the completion of this paper, we became aware of a very recent study on the CHAOS data (R. Rapp *et al.* [27]). They do not address the question of the partial chiral restoration, and the underlying medium effect responsible for the near-threshold enhancement is different from ours.

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