B Decay and the Υ Mass

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(Received 18 September 1998)

Theoretical predictions for inclusive semileptonic B decay rates are rewritten in terms of the $\Upsilon(1S)$ meson mass instead of the b quark mass, using a modified perturbation expansion. This method gives theoretically consistent and phenomenologically useful results. Perturbation theory is well behaved, and the largest theoretical error in the predictions coming from the uncertainty in the quark mass is eliminated. The results are applied to the determination of $|V_{cb}|$, $|V_{ub}|$, and λ_1. [S0031-9007(98)08167-8]

PACS numbers: 13.20.He, 12.15.Hh, 12.38.Bx, 14.20.Mr

Inclusive decay rates of hadrons containing a heavy quark can be systematically expanded in powers of $\alpha_s(m_Q)$ and $\Lambda_{\rm QCD}/m_Q$, where m_Q is the mass of the heavy quark and $\Lambda_{\rm QCD}$ is the nonperturbative scale parameter of the strong interactions. In the $m_Q \to \infty$ limit, inclusive decay rates are given by free quark decay and the order $\Lambda_{\rm QCD}/m_Q$ corrections vanish [1]. The leading nonperturbative corrections of order $\Lambda_{\rm QCD}^2/m_Q^2$ are parametrized by two hadronic matrix elements [2–4]. These results are now used to determine the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements $|V_{cb}|$ and $|V_{ub}|$, using experimental data on inclusive semileptonic B meson decays.

At present, the largest theoretical uncertainties in the $B \to X_c e \bar\nu$ and $B \to X_u e \bar\nu$ decay rates arise from poor knowledge of the b quark mass. The b quark pole mass is an infrared sensitive quantity which is not well defined beyond perturbation theory [5]. This is related to the bad behavior of perturbative corrections to the inclusive decay rate when it is written in terms of the pole mass [6,7]. The decay rate has been rewritten, with the hope of reducing the theoretical uncertainties, in terms of other quantities such as the B meson mass and the $\bar\Lambda$ parameter of HQET, or in terms of the infrared safe modified minimal subtraction ($\overline{\rm MS}$) mass of the b quark. Nonetheless, the uncertainties remain sizable and are a significant part of the present theoretical errors on $|V_{cb}|$ and $|V_{ub}|$.

In this Letter the theoretical predictions for semileptonic B decay rates are rewritten in terms of the $\Upsilon(1S)$ meson mass (which is known to better than 1 MeV) rather than the b quark mass, using a modified perturbation expansion explained below. This eliminates the uncertainty due to the m_b^5 factor in the decay rates, and at the same time improves the behavior of the perturbation series both at low and high orders. Our formulas relate measurable quantities to one another and the resulting perturbation series is free of renormalon ambiguities.

The inclusive decay rate $B \to X_u e \bar\nu$ is [6,7]

$$\Gamma(B \to X_u e \bar\nu) = \frac{G_F^2 |V_{ub}|^2}{192\pi^3}\, m_b^5 \left[1 - 2.41\, \frac{\alpha_s}{\pi}\, \epsilon - 3.22\, \frac{\alpha_s^2}{\pi^2}\, \beta_0 \epsilon^2 - 5.18\, \frac{\alpha_s^3}{\pi^3}\, \beta_0^2 \epsilon^3 - \cdots - \frac{9\lambda_2 - \lambda_1}{2m_b^2} + \ldots \right]. \qquad (1)$$

Here m_b is the b quark pole mass, $\beta_0 = 11 - 2n_f/3$ is the first coefficient of the QCD β function, and α_s is the running coupling constant in the $\overline{\rm MS}$ scheme at the scale $\mu = m_b$. The variable $\epsilon = 1$ denotes the order in our modified expansion. There is a subtlety in the power counting for the Υ mass, for which the difference between powers of α_s and ϵ will be important. Only the part of the $\alpha_s^{2,3}$ corrections proportional to $\beta_0^{1,2}$ [the Brodsky-Lepage-Mackenzie (BLM) piece [8]] is known. It is the dominant part of the two-loop correction in examples where the entire two-loop result is known [see, e.g., Eq. (2)]. The $1/m_b^2$ terms are a few percent, so the α_s/m_b^2 and $1/m_b^3$ corrections are negligible. With $\alpha_s(m_b) = 0.22$ and $n_f = 4$, the perturbative series in Eq. (1) is $1 - 0.17\epsilon - 0.13_{\rm BLM}\epsilon^2 - 0.12_{\rm BLM}\epsilon^3 - \ldots$, where the subscript BLM indicates that only the BLM piece of the $\alpha_s^{2,3}$ terms has been computed. It is difficult to estimate $\Gamma(B \to X_u e \bar\nu)$ reliably, since uncertainties in m_b^5 and in the perturbative expansion seem large. Moreover, the perturbation series at large orders contains a not Borel summable contribution of order $\alpha_s^n \beta_0^{n-1} n!$, leading to a renormalon ambiguity.

The pole mass m_b is an infrared sensitive quantity. It can be related to an infrared safe mass such as the $\overline{\rm MS}$ mass $\overline{m}_b$ via (for $n_f = 4$) [9]

$$\frac{m_b}{\overline{m}_b(m_b)} = 1 + \frac{4\alpha_s}{3\pi}\, \epsilon + (1.56\beta_0 - 1.07)\, \frac{\alpha_s^2}{\pi^2}\, \epsilon^2 + \ldots. \qquad (2)$$

This relation also has terms of the form $\alpha_s^n \beta_0^{n-1} n!$ at high orders. There is a cancellation between the $\alpha_s^n \beta_0^{n-1} n!$ terms in Eqs. (1) and (2) when the inclusive decay rate is rewritten in terms of the $\overline{\rm MS}$ mass [10]. While this cancellation is present at high orders, the perturbation series in Eq. (1) with $m_b \to \overline{m}_b$ is $1 + 0.30\epsilon + 0.19_{\rm BLM}\epsilon^2 + 0.05_{\rm BLM}\epsilon^3$ [7], so there are still large corrections at low orders. Furthermore, using the $\overline{\rm MS}$ mass does not remove the quark mass uncertainty in the decay rate.

A simple method of avoiding problems with the quark mass is to use instead the hadron mass. Unfortunately, the B meson and b quark masses differ by order $\Lambda_{\rm QCD}$, and so this reintroduces a $\Lambda_{\rm QCD}/m_b$ correction to the inclusive decay rate. A better method is to rewrite expressions like Eq. (1) in terms of the Υ mass to obtain well defined formulas for B decay rates in terms of m_Υ. The resulting expressions are free of renormalon ambiguities, and they express one measurable quantity in terms of another. We will also see numerically that the α_s corrections are small when the B decay rate is written in terms of the Υ mass.

There is an interesting theoretical subtlety in the behavior of the perturbation series for the Υ mass in terms of the b quark pole mass. This is simplest to illustrate in the large β_0 (i.e., bubble summation) approximation. Schematically, the perturbative expansion of the Υ mass in terms of m_b is

$$m_\Upsilon/(2m_b) \sim 1 - [(\alpha_s C_F)^2/8][1 + (\alpha_s\beta_0/\pi)(\ell + 1) + (\alpha_s\beta_0/\pi)^2(\ell^2 + \ell + 1) + \cdots + (\alpha_s\beta_0/\pi)^n(\ell^n + \ell^{n-1} + \cdots + 1) + \ldots], \tag{3}$$

where $\ell = \ln[\mu/(m_b\alpha_s C_F)]$, $C_F = 4/3$, and the precise coefficients are not shown. At low orders this series is of the form $\{\alpha_s^2, \alpha_s^3\beta_0, \alpha_s^4\beta_0^2, \ldots\}$, whereas the corrections in Eqs. (1) and (2) are of order $\{\alpha_s, \alpha_s^2\beta_0, \alpha_s^3\beta_0^2, \ldots\}$. An explicit calculation using the Borel transform of the static quark potential [11] shows that this mismatch disappears at higher orders. The terms in Eq. (3) of the form $(\ell^n + \ell^{n-1} + \cdots + 1)$ exponentiate to give $\exp(\ell) = \mu/(m_b\alpha_s C_F)$ and correct the mismatch between the powers of α_s and β_0. This has to happen since m_Υ is a physical quantity, so the renormalon ambiguities must cancel in Eq. (3) between $2m_b$ and the potential plus kinetic energies [12].

The expression for the Υ mass in terms of m_b is [13]

$$\frac{m_\Upsilon}{2m_b} = 1 - \frac{(\alpha_s C_F)^2}{8}\left\{1\epsilon + \frac{\alpha_s}{\pi}\left[\left(\ell + \frac{11}{6}\right)\beta_0 - 4\right]\epsilon^2 + \left(\frac{\alpha_s\beta_0}{2\pi}\right)^2\left(3\ell^2 + 9\ell + 2\zeta(3) + \frac{\pi^2}{6} + \frac{77}{12}\right)\epsilon^3 + \ldots\right\}. \tag{4}$$

The ellipses denote terms of order α_s^4 with at most one power of β_0 or β_1 (which are known), as well as terms of order α_s^5. The arguments following Eq. (3) show that to ensure the cancellation of renormalon ambiguities when we combine Eqs. (1) and (4), terms of order α_s^n in Eq. (4) should be viewed as if they were only of order α_s^{n-1}. For this reason, the power of ϵ in Eq. (4) is one less than the power of α_s. One should also choose the same renormalization scale, μ, in Eqs. (1) and (4). With this prescription, it is also expected that the infrared sensitivity present separately in Eqs. (1) and (4) will cancel to all orders in perturbation theory in ϵ. For μ of order m_b, Eq. (4) shows no sign of convergence; for $\mu = m_b$ it yields $m_\Upsilon = 2m_b(1 - 0.011\epsilon - 0.016\epsilon^2 - 0.024_{\rm BLM}\epsilon^3 - \ldots)$. The bad behavior of this series is unimportant, since the only physical question is what happens when we use Eq. (4) to predict B decay rates in terms of m_Υ.

An important theoretical uncertainty in applying the above approach is the size of nonperturbative corrections to Eq. (4). The dynamics of the Υ system can be described using NRQCD [14]. The leading nonperturbative corrections to m_Υ arise from matrix elements in the Υ of $H_{\rm light}$, the Hamiltonian of the light degrees of freedom. In B mesons, the leading nonperturbative correction to the B meson mass is due to the matrix element of $H_{\rm light}$, which is the $\bar\Lambda$ parameter of order $\Lambda_{\rm QCD}$. The $\Lambda_{\rm QCD}$ dependence is different for the Υ. $H_{\rm light}$ is the integral of a local Hamiltonian density, $H_{\rm light} = \int d^3x\,\mathcal{H}_{\rm light}(x)$. The radius of the Υ is $a \sim 1/(m_b\alpha_s)$, so the matrix element of $H_{\rm light}$ is of order $a^3\Lambda_{\rm QCD}^4$, by dimensional analysis. (Note that the matrix element of $\mathcal{H}_{\rm light}$ is of order $\Lambda_{\rm QCD}^4$, not m_b^4. Terms that grow with m_b can be treated using NRQCD perturbation theory.) Using $1/a \sim 1$ GeV, and $\Lambda_{\rm QCD} \sim 350$ MeV, of order a constituent quark mass, gives a nonperturbative correction of 15 MeV. Using instead $\Lambda_{\rm QCD} \sim 500$ MeV gives a correction of 60 MeV. We will use 100 MeV as a conservative estimate of the nonperturbative contribution to m_Υ.

Substituting Eq. (4) into Eq. (1) and collecting terms of a given order in ϵ gives the $B \to X_u e\bar\nu$ decay rate in the large β_0 approximation in terms of the Υ mass,

$$\Gamma(B \to X_u e\bar\nu) = \frac{G_F^2|V_{ub}|^2}{192\pi^3}\left(\frac{m_\Upsilon}{2}\right)^5\left[1 - 0.115\epsilon - 0.035_{\rm BLM}\epsilon^2 - 0.005_{\rm BLM}\epsilon^3 - \frac{9\lambda_2 - \lambda_1}{2(m_\Upsilon/2)^2} + \ldots\right], \tag{5}$$

using $\mu = m_b$ and $\alpha_s(m_b) = 0.22$. The non-BLM parts of the $\epsilon^{2,3}$ terms have been neglected. The perturbation series, $1 - 0.115\epsilon - 0.035_{\rm BLM}\epsilon^2 - 0.005_{\rm BLM}\epsilon^3$, is far better behaved than the series in Eq. (1), $1 - 0.17\epsilon - 0.13_{\rm BLM}\epsilon^2 - 0.12_{\rm BLM}\epsilon^3$, or the series expressed in terms of the $\overline{\rm MS}$ mass, $1 + 0.30\epsilon + 0.19_{\rm BLM}\epsilon^2 + 0.05_{\rm BLM}\epsilon^3$. The uncertainty in the B decay rate using Eq. (5) is much smaller than that in Eq. (1), both because the perturbation series is better behaved, and because the Υ mass is better known (and better defined) than the b quark mass.

The non-BLM order α_s^2 corrections to b decay have been calculated only for $b \to c$ decay, at three values of the invariant mass of the lepton pair [15]. Extrapolating to $m_c \to 0$ gives the estimate that the complete α_s^2 correction to $b \to u$ decay is about $(90 \pm 10)\%$ of the order $\alpha_s^2\beta_0$ result [6]. With this estimate, and including the entire ϵ^2

term in Eq. (4) gives at order ϵ^2

$$\Gamma(B \to X_u e \bar{\nu}) = \frac{G_F^2 |V_{ub}|^2}{192\pi^3} \left(\frac{m_\Upsilon}{2}\right)^5 [1 - 0.115\epsilon - (0.045 \pm 0.013)\epsilon^2 - (0.20\lambda_2 - 0.02\lambda_1)/\text{GeV}^2], \quad (6)$$

where the error on the ϵ^2 term is due to the $\pm 10\%$ uncertainty in the α_s^2 term in $b \to u$ decay. Equation (6) yields a relation between $|V_{ub}|$ and the total semileptonic $B \to X_u e \bar{\nu}$ decay rate with very small uncertainty,

$$|V_{ub}| = (3.06 \pm 0.08 \pm 0.08) \times 10^{-3} \times \left(\frac{\mathcal{B}(B \to X_u e \bar{\nu})}{0.001} \frac{1.6 \text{ ps}}{\tau_B}\right)^{1/2}, \quad (7)$$

where we have used $\lambda_2 = 0.12$ GeV^2 and $\lambda_1 = (-0.25 \pm 0.25)$ GeV^2. The first error is obtained by assigning an uncertainty in Eq. (6) equal to the value of the ϵ^2 term and the second is from assuming a 100 MeV uncertainty in Eq. (4). The scale dependence of $|V_{ub}|$ due to varying μ in the range $m_b/2 < \mu < 2m_b$ is less than 1%. The uncertainty in λ_1 makes a negligible contribution to the total error. It is unlikely that $\mathcal{B}(B \to X_u e \bar{\nu})$ will be measured without significant experimental cuts, for example, on the hadronic invariant mass [16]. Our method should reduce the uncertainties in such analyses as well.

The $B \to X_c e \bar{\nu}$ decay depends on both m_b and m_c. It is convenient to express the decay rate in terms of m_Υ and λ_1 instead of m_b and m_c, using Eq. (4) and

$$m_b - m_c = \overline{m}_B - \overline{m}_D + \left(\frac{\lambda_1}{2\overline{m}_B} - \frac{\lambda_1}{2\overline{m}_D}\right) + \ldots, \quad (8)$$

where $\overline{m}_B = (3m_{B^*} + m_B)/4 = 5.313$ GeV and $\overline{m}_D = (3m_{D^*} + m_D)/4 = 1.973$ GeV. The α_s correction to free quark decay is known analytically [17], and the full order α_s^2 result [15] can be estimated numerically (at the scale $\mu = m_b$) by multiplying the order $\alpha_s^2\beta_0$ correction [6] by 0.9 ± 0.05. We then find

$$\Gamma(B \to X_c e \bar{\nu}) = \frac{G_F^2 |V_{cb}|^2}{192\pi^3} \left(\frac{m_\Upsilon}{2}\right)^5 0.533[1 - 0.096\epsilon - 0.031\epsilon^2 - (0.28\lambda_2 + 0.12\lambda_1)/\text{GeV}^2], \quad (9)$$

where the phase space has also been expanded in ϵ. For comparison, the perturbation series in this relation when written in terms of the pole mass is $1 - 0.12\epsilon - 0.06\epsilon^2 - \ldots$. Equation (9) implies

$$|V_{cb}| = (41.6 \pm 0.8 \pm 0.7 \pm 0.5) \times 10^{-3} \times \eta_{\text{QED}} \left(\frac{\mathcal{B}(B \to X_c e \bar{\nu})}{0.105} \frac{1.6 \text{ ps}}{\tau_B}\right)^{1/2}, \quad (10)$$

where $\eta_{\text{QED}} \sim 1.007$ is the electromagnetic radiative correction. The uncertainties come from assuming an error in Eq. (9) equal to the ϵ^2 term, the 0.25 GeV^2 error in λ_1, and a 100 MeV error in Eq. (4), respectively. The second uncertainty is reduced to ± 0.3 by extracting λ_1 from the electron spectrum in $B \to X_c e \bar{\nu}$; see Eq. (11). The agreement of $|V_{cb}|$ with other determinations (such as exclusive decays) is a check that nonperturbative corrections to Eq. (4) are indeed small.

In Ref. [18] $\bar{\Lambda}$ and λ_1 were extracted from the lepton spectrum in $B \to X_c e \bar{\nu}$ decay. With our approach, there is no dependence on $\bar{\Lambda}$, so we can determine λ_1 directly with small uncertainty. Considering the observable $R_1 = \int_{1.5 \text{ GeV}} E_e (d\Gamma/dE_e)\, dE_e / \int_{1.5 \text{ GeV}} (d\Gamma/dE_e)\, dE_e$, a fit to the same data yields

$$\lambda_1 = (-0.27 \pm 0.10 \pm 0.04) \text{ GeV}^2. \quad (11)$$

The central value includes corrections of order $\alpha_s^2\beta_0$ [19]. The first error is dominated by $1/m_b^3$ corrections [20]. We varied the dimension-six matrix elements between $\pm(0.5 \text{ GeV})^3$, and combined their coefficients in quadrature in the error estimate. The second error is from assuming a 100 MeV uncertainty in Eq. (4). The central value of λ_1 at tree level or at order α_s is within 0.03 GeV^2 of the one in Eq. (11).

We can attempt to apply the above results to $D \to X e \nu$ decay, using $\alpha_s(m_c) = 0.35$ and $n_f = 3$. Nonperturbative effects are clearly much larger in the J/ψ than in the Υ, so one might expect the entire analysis to break down completely. It is remarkable that this does not occur. Using $m_{J/\psi} = 2m_c(1 - 0.027\epsilon - 0.059\epsilon^2 - 0.130\epsilon^3 - \ldots)$, neglecting m_s, and following the same procedure as for $b \to u$ decay, we find

$$\Gamma(D \to X e \nu) = \frac{G_F^2(|V_{cs}|^2 + |V_{cd}|^2)}{192\pi^3} \left(\frac{m_{J/\psi}}{2}\right)^5 \times [1 - 0.13\epsilon - 0.03\epsilon^2 - (1.9\lambda_2 - 0.2\lambda_1)/\text{GeV}^2]. \quad (12)$$

The ϵ^3 contribution to Eq. (12) is larger than the order ϵ^2 term. The perturbation series expressed in terms of the pole mass has a much worse behavior, roughly $1 - 0.27\epsilon - 0.32\epsilon^2$. Using $\lambda_2(m_c) = 0.14$ GeV^2 and λ_1 from Eq. (11), we obtain

$$|V_{cs}|^2 + |V_{cd}|^2 = (1.00 \pm 0.06 \pm 0.04) \times \left(\frac{\mathcal{B}(D^\pm \to X e \nu)}{0.17} \frac{1.06 \text{ ps}}{\tau_{D^\pm}}\right), \quad (13)$$

where the uncertainties come from assuming an error in Eq. (12) equal to the ϵ^2 term and the error in λ_1, respectively. We have not included an estimate of nonperturbative corrections to the J/ψ mass, or of scale dependence. The LEP measurements of the hadronic W width yield $|V_{cs}| = 0.99 \pm 0.11$ [21]. The uncertainty in Eq. (13) is comparable to this, since the experimental error of $\mathcal{B}(D^\pm \to X e \nu)$ is about 10%. Equation (13) has theoretical uncertainties which we cannot estimate. The validity of quark-hadron duality may be questionable since the final states are almost saturated by K and K^*. In addition, an estimate similar to that for the Υ suggests that the nonperturbative contribution to the J/ψ mass is of order 500 MeV (using $1/a \sim 0.5$ GeV and $\Lambda_{\text{QCD}} \sim 500$ MeV). This gives an uncertainty of order 100% in $|V_{cs}|^2 + |V_{cd}|^2$. The agreement of Eq. (13) with the experimental results may be a coincidence, or may signal that nonperturbative corrections in the mass relation are much smaller than naive expectations.

We have chosen to write our B decay results in terms of the $\Upsilon(1S)$ mass. One could equally well write them in terms of the mass of excited states, such as the $\Upsilon(2S)$. The perturbation series is expected to be worse behaved than for the $\Upsilon(1S)$. The main difference is in the estimate of nonperturbative corrections to the $\Upsilon(2S)$ mass. The radius of the $2S$ state is about 4 times that of the $1S$, so the nonperturbative corrections, which grow as a^3, are approximately 64 times larger. This implies a similar increase in the error on the CKM angles. Ignoring nonperturbative corrections for the moment, the analog of Eq. (4) for the $\Upsilon(2S)$ evaluated at the scale $\mu = m_b$ is $m_{\Upsilon(2S)} = 2m_b(1 - 0.0027\epsilon - 0.0059\epsilon^2 - 0.0117_{\rm BLM}\epsilon^3 - \ldots)$. Numerically, the first few corrections are smaller than for the $\Upsilon(1S)$, but the convergence of the series is worse. The $B \to X_u e\bar{\nu}$ decay rate in the large β_0 approximation in terms of the $\Upsilon(2S)$ mass is then

$$\Gamma(B \to X_u e\bar{\nu}) = \frac{G_F^2|V_{ub}|^2}{192\pi^3}\left(\frac{m_{\Upsilon(2S)}}{2}\right)^5[1 - 0.155\epsilon - 0.098_{\rm BLM}\epsilon^2 - 0.065_{\rm BLM}\epsilon^3 - \ldots]. \quad (14)$$

Compared to Eq. (5), the convergence is worse, as expected. Nevertheless, even this formula gives a reasonable extraction of $|V_{ub}|$. The ratio of $|V_{ub}|^2$ extracted using the $2S$ and $1S$ masses is [Eq. (14)]/[Eq. (5)] $= \{1.34, 1.27, 1.17, 1.08\}$, where the nth number is obtained by truncating both equations at order ϵ^{n-1}, and neglecting the $\lambda_{1,2}$ corrections. The large difference at "tree level," $(m_{\Upsilon(2S)}/m_\Upsilon)^5 = 1.34$ is reduced by the series of perturbative corrections. Expressing the $B \to X_c e\bar{\nu}$ decay rate in terms of the $\Upsilon(2S)$ mass, the perturbative corrections in Eq. (9) become

$$\Gamma(B \to X_c e\bar{\nu}) = \frac{G_F^2|V_{cb}|^2}{192\pi^3}\left(\frac{m_{\Upsilon(2S)}}{2}\right)^5 0.447 \times [1 - 0.107\epsilon - 0.046\epsilon^2 + \ldots]. \quad (15)$$

Again, the convergence of the series becomes worse. However, the ratio of $|V_{cb}|^2$ extracted using the $2S$ and $1S$ masses is consistent with our estimates of the uncertainties, [Eq. (15)]/[Eq. (9)] $= \{1.12, 1.10, 1.08\}$, where the nth number is obtained by truncating both expressions at order ϵ^{n-1}. The difference between the $\Upsilon(2S)$ and $\Upsilon(1S)$ results provides an estimate of nonperturbative contributions to the Υ mass. They suggest that nonperturbative effects are smaller than the conservative estimate we have used; they are certainly much smaller than the naive estimate above of a 64×100 MeV $= 6.4$ GeV nonperturbative contribution to the $\Upsilon(2S)$ mass.

We have shown that inclusive semileptonic B decay rates can be predicted in terms of the $\Upsilon(1S)$ mass instead of the b quark mass. It is crucial to our analysis to use the modified expansion in ϵ rather than the conventional expansion in powers of α_s. Our formulas relate only physical quantities to one another. They result in smaller theoretical uncertainties than existing numerical predictions, and the behavior of the perturbation series is improved. Moreover, the uncertainties can be estimated without resorting to cumbersome arguments, and they can be checked using the experimental data.

Our main results are Eqs. (10) and (7), which relate the total semileptonic $B \to X_{c,u} e\bar{\nu}$ decay rates to $|V_{cb}|$ and $|V_{ub}|$. The uncertainties are below 5% at present, and it may be possible to reduce them further. Our determination of λ_1 is given in Eq. (11). We hope that applications of the method introduced in this paper will prove useful—besides reducing the uncertainties of $|V_{cb}|$ and $|V_{ub}|$—in analyzing a large class of data emerging from present and future B decay experiments. Details of our method, as well as other applications, such as to nonleptonic and exclusive semileptonic B decays, will be discussed elsewhere [22].

We thank Mark Wise for useful discussions. This work was supported in part by DOE Grant No. DOE-FG03-97ER40546 and by NSF Grant No. PHY-9457911.

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