

## Shear-Induced Changes in Two-Dimensional Foam

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The effects of shear strain on a two-dimensional foam, comprising a monolayer of bubbles bridging from a soap solution to a cover glass, have been studied. For initially disordered foam,  $\mu_2$ , the second central moment of the distribution of bubble coordination numbers, fell under strain, indicating ordering. The reduction in  $\mu_2$  is proportional to its initial value. Further, the accompanying bubble-level changes occur in clusters, some of which are large. Comparisons are drawn with recent studies. [S0031-9007(99)08528-2]

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Foams are paradigms of disordered cellular structures. They are beautiful and even today remain somewhat mysterious, due largely to the difficulties of direct observation of their three-dimensional structure and behavior. Apart from the intrinsic interest of such disordered systems, they are also widely relevant in science and technology. Many applications involve the complex rheology of foam. However, the fundamental basis of this remains poorly understood [1,2]. In particular, there is little understanding of the microscopic, bubble-level processes underlying the phenomena. It is difficult to visualize these in three-dimensional foam. We have therefore studied two-dimensional (2D) foam subject to shear to illuminate certain aspects of these processes.

Computer simulation has been widely used to study 2D foam, including its response to applied stress [3–6]. In 2D cellular systems order is quantified via the coordination numbers of the cells,  $n$ , order being associated with  $n = 6$ , disorder with a broadening of the distribution  $P(n)$ , usually quantified by its second central moment,  $\mu_2$ . Various predictions of the simulations include the following:

(1) Counterintuitively, initially disordered foam (large  $\mu_2$ ) becomes more ordered ( $\mu_2$  falls) on application of stress [3]. However, ordered foam becomes disordered.

(2) Occurrence of avalanches of change in foam subject to stress [4–6].

The latter aspect is somewhat controversial, depending to some extent on the exact model used. Using a vertex model of dry foam [7] system-wide avalanches at large shear strains were found [4], in particular, long-ranged correlations of bubble displacements and power-law distributions of elastic energy release, etc. In studies of the response to extensional stress Hutzler *et al.* [5] found that with increasing foam wetness (towards the rigidity loss transition at gas fraction  $\phi = 0.84$ ) system-wide avalanches of topological change due to small strain changes became more probable. In contrast, Durian's bubble-level model [6] predicts power-law distributions of the energy release in avalanches but with an exponential cutoff. We use a novel experimental approach to study both predictions.

We have adapted a model 2D foam suggested by Fortes *et al.* [8] and Vaz *et al.* [9] to permit application of stress. A monolayer of foam is made by bubbling  $N_2$  gas through a long hypodermic needle into a soap solution ( $\sim 4$  cm deep) covered with a glass plate to which the bubbles bridge. Loss of gas from the bubbles is inhibited, the foam lasting essentially indefinitely, allowing its behavior to be studied [9–11]. The rate of gas flow determines the bubble size, ordered foam arising if it is kept constant. Varying the gas flow while moving the needle erratically results in disordered foam. The bubbles, which averaged 3.5 mm in diameter in these experiments, resemble sessile drops in shape, large bubbles being significantly flattened. Apart from the very smallest bubbles, such monolayer foams appear to be of rather uniform thickness (average bubble depth  $\sim 2$  mm) and can be viewed as nearly two dimensional. The aspect ratio of the bubbles will vary from 1:1 for the smallest bubbles to  $\geq 4:1$  for the largest, of diameter  $\geq 8$  mm. The foam is rather wet; comparison with images from simulations [2] suggest a gas fraction  $\phi \sim 0.93$ – $0.97$ .

The 2D foam, comprising  $\sim 1500$  cells, is constrained within a square cell (13 cm on a side), defined by an elastic peripheral band stretched around four pegs, which can be affinely deformed into a parallelogram while maintaining constant area. High shear strains cannot be reached, the maximum being  $\gamma = 0.75$ . Temporal evolution of such foams starts some 10–12 hours after formation [11], whereas the data here derive from foams less than 1 hour old. The number of bubbles in the foam is conserved during our experiments. Our results do *not* originate from coarsening of the foam. Further, shear does not cause bubbles to move into the third dimension.

The foam was strained ( $\Delta\gamma \approx 0.05$ ) and allowed to reach apparent equilibrium then photographed. Unfortunately the optical contrast was low, inhibiting image analysis of video records. The photographs were manually analyzed, limiting some of the studies undertaken.

Strain obviously changes initially disordered foam (Fig. 1), but quantitative analysis of  $P(n)$  is required to determine whether order increases. Figure 2 shows

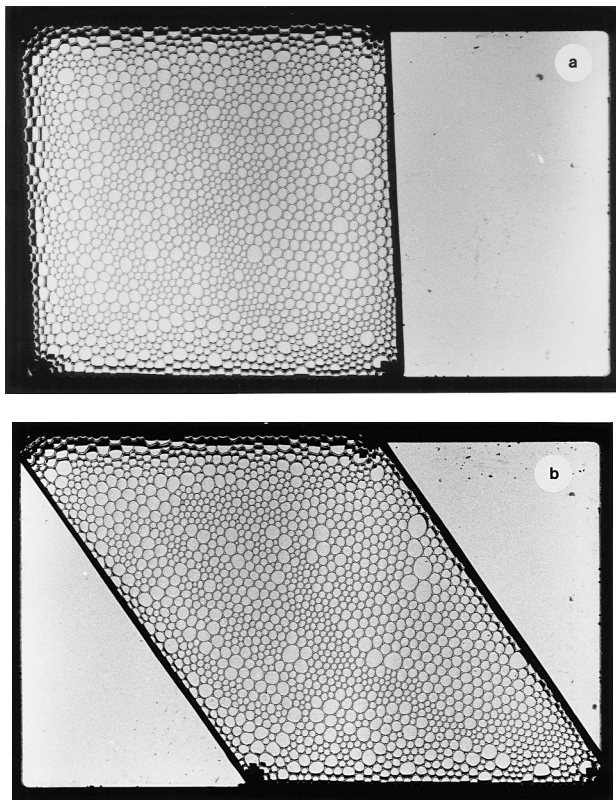


FIG. 1. Images of a rather disordered foam for shear strains: (a)  $\gamma = 0$  and (b) 0.75. Initial  $\mu_2 = 1.06$ .

$P(n)$  for  $\gamma = 0$  and 0.75:  $P(6)$  increases and the whole distribution narrows significantly as  $\gamma$  increases.

We see clear evidence of changes in  $n$  in our foam: under shear large bubbles deform and so lose neighbors, causing adjacent small ones to gain them. The basic path of a collapse of  $P(n)$  towards  $n = 6$  is as anticipated [3]. Relatively ordered areas become less ordered, but this

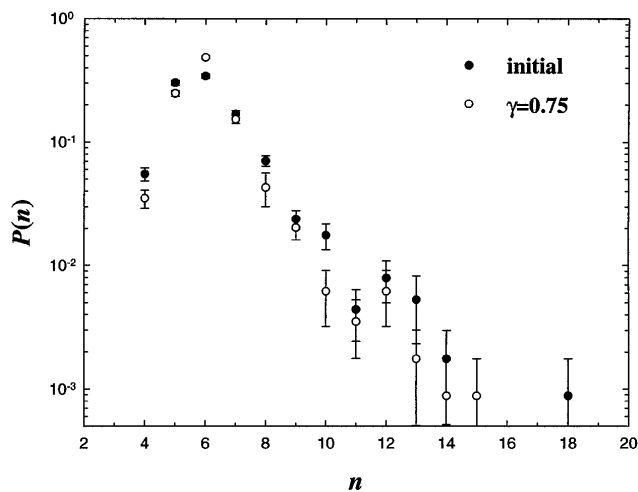


FIG. 2. The distribution of  $n$  in a disordered foam (edge bubbles omitted) in its initial state ( $\mu_2 = 2.32$ ) and at  $\gamma = 0.75$ . Note the logarithmic scale for  $P(n)$ .

is swamped by the decrease in disorder in initially very disordered regions.

The degree of disorder, assessed via  $\mu_2$ , decreases steadily with applied strain (Fig. 3a). The first strain increment applied causes a fall in  $\mu_2$ , suggesting that the yield strain is  $\leq 0.05$ , consistent with simulations of wet foam [5]. Reversal of the strain increases  $\mu_2$  but hysteresis is evident, reflecting the history dependence of the properties of foam [2]. The reduction  $\Delta\mu_2$  as  $\gamma \rightarrow 0.75$  increases with the initial  $\mu_2$  (Fig. 3b). Values from simulations of simple [3] or extensional [12] shear of 2D foam are in excellent accord with the experimental variation. The trend of the present data suggests that the ordering effects of strain should disappear for initial values of  $\mu_2$  below about 0.5, as expected [3,12].

Indeed, shear strain does disorder initially very ordered 2D foam. The parametrization of disorder via  $\mu_2$  is here not feasible, due to the overwhelming number of six-coordinated bubbles. We thus use the number of dislocations,  $N$ , which increases with strain until  $\gamma \approx 0.4$ , after which it decreases. This decrease seems to be a consequence of our finite sample cell. Many of the

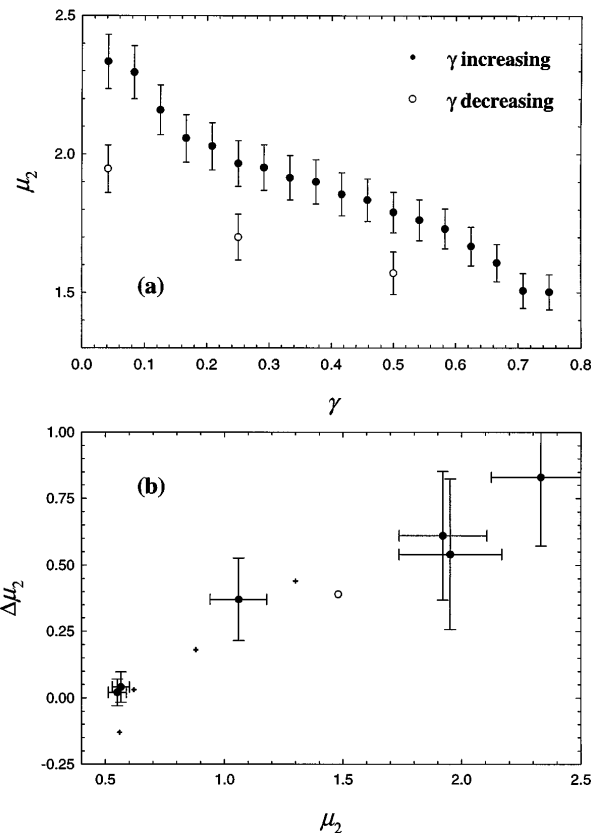


FIG. 3. (a) The variation of  $\mu_2$  with shear strain for a disordered foam. A few points taken as the foam was returned to  $\gamma = 0$  are shown (open points). (b) The reduction  $\Delta\mu_2$  on going from  $\gamma = 0$  to 0.75 for various initial values of  $\mu_2$ . Data from simulations are shown for comparison: + from [12]; o inferred by us from [3].

dislocations generated by strain tend to glide through the foam. At a cell wall they may be reflected or trapped. Reflection may cause one dislocation to meet another: if their Burgers vectors are opposite they annihilate. Trapped dislocations can disappear through slip of one of the constituent lines of bubbles relative to its neighbors. These mechanisms are not intrinsic to 2D foam, and the artifactual decrease of  $N$  at large strain is not in conflict with the simulations [3]. These effects are unlikely to contribute to the ordering of disordered foam, as dislocations will be trapped by the disorder, unable to glide as in a near-perfect lattice.

We have thus verified the prediction that  $\mu_2$  for disordered 2D foam is reduced under strain, validating recent computer simulations [3,12]. The reduction in  $\mu_2$  increases with the initial value of this parameter; the variation accords well with results from simulations. Ordered foam appears to become disordered, as expected.

We turn to the question of avalanches of change in disordered foam under stress [4–6]. In simulations these are observed as cascades of topological changes (neighborhood switching: the  $T_1$  process [13]) as the foam is reequilibrated following a very small increment of strain. In our study such a definition cannot be used due to the experimental problems mentioned above. From photographs of foam which had apparently reached equilibrium after a strain increment we could identify bubbles whose topological class ( $n$ ) had changed from the previous picture or which had moved so far that it was impossible to identify the corresponding bubble to compare  $n$ . The two definitions are not the same (e.g., a single isolated bubble of changed  $n$  must have involved several  $T_1$  events), but we may gain some insight into the scales of change induced in the foam by application of shear. As we will see, our definition is rather conservative.

On average some 10% ( $\sim 150$ ) of the bubbles satisfy our criteria. They are not randomly distributed but rather form clusters of size  $k$  running from 1 up to  $\geq 20$ . Figure 4a shows a typical example of these changes. The most probable clusters have  $k = 4$ , many clearly being due to a single  $T_1$  process (any other  $k$  implies more than one topological change).

These phenomena are not cell edge effects. Roughly 25% of the clusters at the edges of the cell lay along the stationary edge: the changes are not directly driven by motion of the cell periphery. Many of the larger clusters do not penetrate to within (say) five bubble diameters from the periphery, suggesting that they are not a wall effect.

Figure 4b shows the frequency distribution of the sizes of the clusters of change observed for all  $\gamma$  in one experiment; the number of changed bubbles and of clusters do not seem to vary with  $\gamma$ , so we consider all the data together. The general trend is a quasiexponential decrease with  $k$  ( $\langle k \rangle = 5.0$ ), but there is a tail to large  $k$ . Clusters which extend to the peripheral row of bubbles

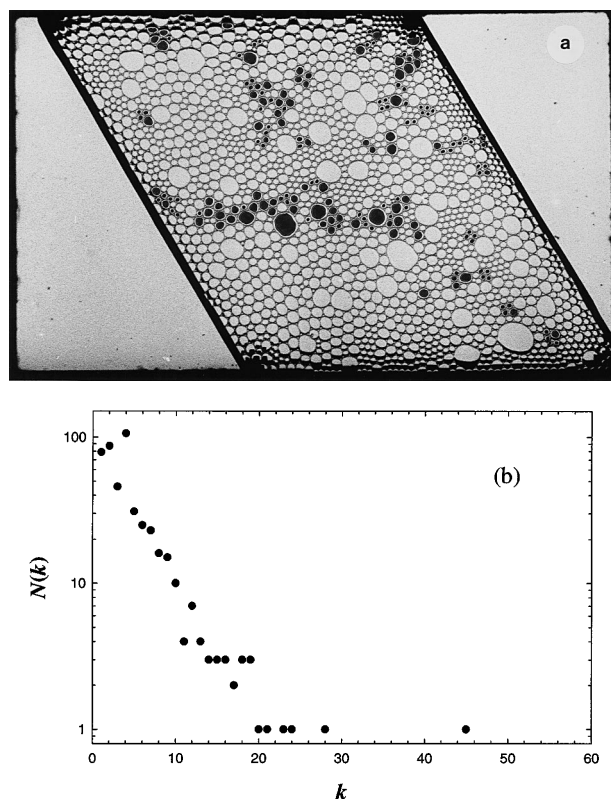


FIG. 4. (a) Bubbles which have changed their  $n$  during a strain increment are marked. (b) The frequency distribution of the size  $k$  of such clusters in one experiment.

are difficult to fully define. However,  $N(k)$  for clusters wholly within the body of the foam and for those reaching the edges of the foam are very similar, so we introduce little bias by including both populations in Fig. 4b. In fact, there are fewer edge clusters of very large  $k$  (and of  $k = 4$ ), so that including this population reduces the apparent significance of the large  $k$  events in Fig. 4b.

We believe that these observations imply the occurrence of very large avalanches of change in response to shear strain. Various arguments support this conclusion:

(1) We see large events; the largest in the experiment represented in Fig. 4b was  $k = 45$ .

(2) In many images there are substantial areas of the foam without any changes: the observed clusters of change are not randomly distributed but tend to be associated. With an average of  $\sim 30$  clusters per image, the probability that 20% of the total area should lack any cluster, as in Fig. 4a, is  $\sim 10^{-3}$ .

(3) Two or more clusters often seem spatially correlated. We see cases where the spatial relationship of intervening bubbles has changed but their  $n$  values remain the same, implying several successive topological changes. Such correlated clusters may form the remains of a single larger sequence of topological changes, although we cannot prove such a causal connection *a posteriori*. Our definition leads to minimal clusters.

These observations establish the existence of rather large regions of topological change in 2D foam as a result of strain. While this seems broadly consistent with the avalanches found in some simulations [4,5], further work is required to conclusively establish the connections.

Both conclusions of the present work run counter to certain recent studies. However, there may be no real conflict. The only earlier experimental study of strain of 2D foam [14] involved a foamlike state of a spread molecular monolayer; neither ordering nor large avalanches were observed (the maximum number of simultaneous  $T_1$  processes seen for  $\Delta\gamma = 0.03$  was 4). However, the experimental system may differ from the soap froths studied in simulations. In particular, as noted in [14] there are dissipative processes involved, such as the subphase viscosity, which are absent from froths (and much lower in the present system). This may reduce the probability of a disturbance elsewhere causing change in a given region of the foam, as is implicit in system-wide avalanches. The lack of reduction of  $\mu_2$  may simply be a matter of statistics: the number of cells observed in [14] was significantly less ( $\sim 100$ ) than here, causing  $\mu_2$  to be very uncertain ( $\pm 35\%$ ). Light scattering studies of 3D foam subject to shear [15] do not demonstrate the expected reduction in the rate of coarsening due to the narrowing of  $P(n)$  implied by reduction of  $\mu_2$  [3] and have further been interpreted as implying that rearrangements are purely local. However, these conclusions may not be so definitive: the experiment involved *cyclic* shear [15], and it is well known that such nonmonotonic stress markedly affects the structure and dynamics of foam [2,7,16].

Further work is needed to elucidate differences between different model 2D foams and to remove ambiguities in the studies of the 3D case just mentioned. It would also be of interest to investigate foams of different wetness [5].

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- [1] A. M. Kraynik, *Ann. Rev. Fluid Mech.* **20**, 325 (1988).
  - [2] D. Weaire and M. A. Fortes, *Adv. Phys.* **43**, 685 (1994).
  - [3] D. Weaire, F. Bolton, T. Herdtle, and H. Aref, *Philos. Mag. Lett.* **66**, 293 (1992).
  - [4] T. Okuzono and K. Kawasaki, *Phys. Rev. E* **51**, 1246 (1995).
  - [5] S. Hutzler, D. Weaire, and F. Bolton, *Philos. Mag. B* **71**, 277 (1995).
  - [6] D. J. Durian, *Phys. Rev. E* **55**, 1739 (1997).
  - [7] K. Kawasaki, T. Okuzono, and T. Nagai, *J. Mech. Behav. Mater.* **4**, 51 (1992); T. Okuzono, K. Kawasaki, and T. Nagai, *J. Rheol.* **37**, 571 (1993).
  - [8] M. A. Fortes, D. Groshans, and M. F. Vaz (unpublished).
  - [9] M. F. Vaz and M. A. Fortes, *J. Phys. Condens. Matter* **9**, 8921 (1997).
  - [10] M. E. Rosa and M. A. Fortes, *Philos. Mag. A* **77**, 1423 (1998).
  - [11] A. Abd el Kader and J. C. Earnshaw, *Philos. Mag. A* **76**, 1251 (1997); *Phys. Rev. E* **56**, 3251 (1997); **58**, 760 (1998).
  - [12] S. Hutzler, *The Physics of Foams* (Verlag MIT Tiedeman, Bremen, Germany, 1997), Chap. 5.
  - [13] D. Weaire and N. Rivier, *Contemp. Phys.* **25**, 59 (1984).
  - [14] M. Dennin and C. M. Knobler, *Phys. Rev. Lett.* **78**, 2485 (1997).
  - [15] A. D. Gopal and D. J. Durian, *Phys. Rev. Lett.* **75**, 2610 (1995).
  - [16] D. Weaire and J. P. Kermode, *Philos. Mag. B* **50**, 379 (1984).