# Reversing Measurement and Probabilistic Quantum Error Correction

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We consider a probabilistic reversing operation that returns the measured system to its original state by means of a physical process, and derive a trade-off relation between the unsharpness of the measurement and the best efficiency of the reversing operation. Such a reversing operation is shown to serve as a probabilistic quantum error correction, which will be useful when the numbers of qubits and gate operations are limited. [S0031-9007(99)08786-4]

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A striking difference between quantum and classical mechanics is that, in quantum mechanics, one cannot freely measure a system without disturbing its state. As a consequence, one cannot measure the wave function of a single system, or equivalently, an arbitrary unknown quantum state cannot be cloned [1,2]. With the growing interest in the field of quantum information theory, much effort has been devoted to characterizing this restriction in more detail. One approach has been to identify those restrictions on the initial state that allow deterministic cloning [1–5]. Very recently, the probabilistic cloning condition was also discussed, and its best efficiencies were derived [6]. Another approach is to start with an arbitrary initial state, and then to find those restrictions on the measurement that allow the postmeasurement state to be reversed to its original state. For "measurements" that allow deterministic reversal (namely, reversal with unit probability), general conditions have recently been derived by Nielsen and Caves [7]. Other cases have been reported [8,9] in which the postmeasurement state can be reversed only with a nonzero probability of success. Since a sharp measurement (i.e., one with no measurement error) allows no chance of reversal, it is natural to expect a trade-off relation between the unsharpness of a measurement and the degree of physical reversibility—the maximum probability of the premeasurement state being reproduced from the postmeasurement state by means of a physical process.

In this Letter, we generally characterize such *probabilistically reversible measurements* and show that among the many indices characterizing the unsharpness of a measurement, the degree of physical reversibility is determined by one particular index, the fraction of the background, which is defined as the fraction of the outcomes that are independent of the measured state. We also propose that such probabilistic reversal serves as a means of error correction in quantum computation, which would be particularly useful when the numbers of qubits and gate operations are limited.

Suppose that we perform measurement $M=\{\hat{A}_\nu\}$ on a state represented by density operator $\hat{\rho}$ defined on subspace $H$ of the entire Hilbert space and that outcome $\nu$ is obtained. The postmeasurement state $\hat{\rho}_\nu$ is given by

$$\hat{\rho}_\nu = \frac{\hat{A}_\nu \hat{\rho} \hat{A}_\nu^\dagger}{\mathrm{Tr}[\hat{A}_\nu \hat{\rho} \hat{A}_\nu^\dagger]} . \tag{1}$$

On this postmeasurement state, we perform reversing measurement $R^{(\nu)}=\{\hat{R}_\mu^{(\nu)}\}$ such that if a particular outcome, say $\mu=0$, is obtained, the postmeasurement state of $R^{(\nu)}$ is identical to initial state $\hat{\rho}$. We will call $\mu=0$ the "successful outcome." To put it mathematically,

$$\frac{\hat{R}_0^{(\nu)} \hat{A}_\nu \hat{\rho} \hat{A}_\nu^\dagger \hat{R}_0^{(\nu)\dagger}}{\mathrm{Tr}[\hat{R}_0^{(\nu)} \hat{A}_\nu \hat{\rho} \hat{A}_\nu^\dagger \hat{R}_0^{(\nu)\dagger}]} = \hat{\rho} . \tag{2}$$

We impose no constraints on initial state $\hat{\rho}$ other than the restriction that its support is included in $H$. We assume that reversing measurement $R^{(\nu)}$ restores any initial state when the outcomes of successive measurements $M$ and $R^{(\nu)}$ are $\nu$ and 0, respectively. We call a measurement process described by operator $\hat{A}_\nu$ *physically reversible* if it has a reversing measurement, $R^{(\nu)}$.

When $\hat{A}_\nu$ is physically reversible and the reversing operator is given by $\hat{R}_0^{(\nu)}$, Eq. (2) by definition should hold for any pure state $\hat{\rho}=|\Phi\rangle\langle\Phi|$. This means that any state vector $|\Phi\rangle$ in $H$ is an eigenvector of operator $\hat{R}_0^{(\nu)}\hat{A}_\nu$, that is,

$$\hat{R}_0^{(\nu)} \hat{A}_\nu \hat{P}_H = c^{(\nu)} \hat{P}_H , \tag{3}$$

where $\hat{P}_H$ is the projection operator onto $H$, and $c^{(\nu)}$ is a nonzero complex number. This implies that a necessary (but not necessarily sufficient) condition for the physical reversibility of $\hat{A}_\nu$ is that $\hat{A}_\nu$ has left inverse $\hat{A}_\nu^L$ such that

$$\hat{A}_\nu^L \hat{A}_\nu \hat{P}_H = \hat{P}_H . \tag{4}$$

This condition is equivalent to requiring $M$ to be *logically reversible*, that is, initial state $\hat{\rho}$ can be calculated (but need not be restored by means of a physical process) from outcome $\nu$ and from postmeasurement state $\hat{\rho}_\nu$ [9,10].

Using Eq. (3), we obtain

$$|c^{(\nu)}|^2 = \mathrm{Tr}[\hat{R}_0^{(\nu)} \hat{A}_\nu \hat{\rho} \hat{A}_\nu^\dagger \hat{R}_0^{(\nu)\dagger}] \equiv p_{\mathrm{rev}}^{(\nu)} . \tag{5}$$

This means that $c^{(\nu)}$ introduced in Eq. (3) gives joint probability $p_{\mathrm{rev}}^{(\nu)}$ such that the first measurement yields outcome $\nu$ and the subsequent one gives rise to successful

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reversal. Note that this probability is independent of initial state $\hat{\rho}$. We are interested in an upper bound for $p_{\rm rev}^{(\nu)}$, which is obtained as follows. Because set $\{\hat{R}_{\mu}^{(\nu)}\}$ represents a measurement, the following closure relation must hold:

$$\sum_{\mu} \hat{R}_{\mu}^{(\nu)\dagger} \hat{R}_{\mu}^{(\nu)} = \hat{1} \,. \tag{6}$$

This requires that $\hat{1} - \hat{R}_0^{(\nu)\dagger} \hat{R}_0^{(\nu)}$ be a positive semidefinite operator, or equivalently,

$$\sup_{|\Psi\rangle} \frac{\langle \Psi | \hat{R}_0^{(\nu)\dagger} \hat{R}_0^{(\nu)} | \Psi \rangle}{\langle \Psi \mid \Psi \rangle} \leq 1 \,. \tag{7}$$

On the other hand, for arbitrary state vectors $|\Phi\rangle$ in $H$,

$$\begin{aligned} \sup_{|\Psi\rangle} \frac{\langle \Psi | \hat{R}_0^{(\nu)\dagger} \hat{R}_0^{(\nu)} | \Psi \rangle}{\langle \Psi \mid \Psi \rangle} &\geq \sup_{|\Psi\rangle = \hat{A}_\nu |\Phi\rangle} \frac{\langle \Psi | \hat{R}_0^{(\nu)\dagger} \hat{R}_0^{(\nu)} | \Psi \rangle}{\langle \Psi \mid \Psi \rangle} \\ &= \sup_{|\Phi\rangle \in H} \frac{\langle \Phi | \hat{A}_\nu^{\dagger} \hat{R}_0^{(\nu)\dagger} \hat{R}_0^{(\nu)} \hat{A}_\nu | \Phi \rangle}{\langle \Phi | \hat{A}_\nu^{\dagger} \hat{A}_\nu | \Phi \rangle} \\ &= |c^{(\nu)}|^2 \left[ \inf_{|\Phi\rangle \in H} \frac{\langle \Phi | \hat{A}_\nu^{\dagger} \hat{A}_\nu | \Phi \rangle}{\langle \Phi \mid \Phi \rangle} \right]^{-1} , \end{aligned} \tag{8}$$

where we used Eq. (5) to derive the last equality. Combining Eqs. (5), (7), and (8), we find that an upper bound for $p_{\rm rev}^{(\nu)}$ is given by

$$p_{\rm rev}^{(\nu)} \leq \inf_{|\Phi\rangle \in H} \frac{\langle \Phi | \hat{A}_\nu^{\dagger} \hat{A}_\nu | \Phi \rangle}{\langle \Phi \mid \Phi \rangle} \equiv B(\nu) \,. \tag{9}$$

For measurement process $\hat{A}_\nu$ to be physically reversible, $p_{\rm rev}^{(\nu)} > 0$ so that $B(\nu) > 0$ is necessary. This is equivalent to requiring $\hat{A}_\nu$ to have a bounded left inverse because the norm of the left inverse is given by $\sqrt{B(\nu)^{-1}}$. When subspace $H$ has a finite dimension, the condition $B(\nu) > 0$ is equivalent to the logical reversibility of $\hat{A}_\nu$ because a linear operator in a Hilbert subspace with a finite dimension is always bounded. When the dimension is infinite, the condition $B(\nu) > 0$ is more stringent than that of logical reversibility.

The quantity $\langle \Phi | \hat{A}_\nu^{\dagger} \hat{A}_\nu | \Phi \rangle / \langle \Phi \mid \Phi \rangle$ gives the probability that measurement $M$ yields outcome $\nu$ for state $|\Phi\rangle / \sqrt{\langle \Phi \mid \Phi \rangle}$. The definition of $B(\nu)$ in Eq. (9) therefore implies that in measurement $M$, outcome $\nu$ is obtained with a probability not less than $B(\nu)$ for any state. We thus find that fraction $B(\nu)$ of the outcome provides no information on the measured system. This is the price we have to pay to make the measurement process of $\hat{A}_\nu$ physically reversible. To gain further insight into quantity $B(\nu)$, suppose that we have a source that produces $N$ identical quantum states represented by the same density operator, $\hat{\rho}$, and that we conduct measurement $M(=\{\hat{A}_\nu\})$ on each state. Histogram $H(\nu)$ for the outcome will, on average, be given by the sum of two nonnegative parts:

$$\begin{aligned} \overline{H(\nu)} &= NB(\nu) + N[\mathrm{Tr}[\hat{A}_\nu \hat{\rho} \hat{A}_\nu^{\dagger}] - B(\nu)] \\ &\equiv NB(\nu) + ND(\nu, \hat{\rho}) \,, \end{aligned} \tag{10}$$

where the first term, $NB(\nu)$, is independent of measured state $\hat{\rho}$. Definition (9) ensures that among the decompositions of $\overline{H(\nu)}$ into $\hat{\rho}$-independent and $\hat{\rho}$-dependent nonnegative terms, i.e., $\overline{H(\nu)} = NB'(\nu) + ND'(\nu, \hat{\rho})$, choice $B'(\nu) = B(\nu)$ gives the largest $B'(\nu)$. This implies that the signal component $ND(\nu, \hat{\rho})$ containing the information on the measured state is formed on top of fixed component $NB(\nu)$. Because of this property, we call $B(\nu)$ the *background* of measurement $M$.

The necessary condition for the physical reversibility of $\hat{A}_\nu$ we have derived so far, i.e., $B(\nu) > 0$, is also a sufficient condition. This can be seen by explicitly constructing reversing measurement $R^{(\nu)}$:

$$\begin{aligned} R^{(\nu)} = &\{\sqrt{B(\nu)}\, \hat{A}_\nu^{L} \hat{P}_\nu (= \hat{R}_0^{(\nu)}), \\ &\sqrt{\hat{P}_\nu - B(\nu) P_{\hat{\nu}} \hat{A}_\nu^{L\dagger} \hat{A}_\nu^{L} \hat{P}_\nu}\,, \hat{1} - \hat{P}_\nu\}, \end{aligned} \tag{11}$$

where $P_{\hat{\nu}}$ is the projection onto the image of $\hat{A}_\nu \hat{P}_H$. Since $p_{\rm rev}^{(\nu)}$ is equal to $B(\nu)$ in this example, $B(\nu)$ is actually the least upper bound for $p_{\rm rev}^{(\nu)}$, namely,

$$\max p_{\rm rev}^{(\nu)} = B(\nu) \,. \tag{12}$$

Let $P_{\mathrm{rev}|\nu}$ be the maximum conditional probability that the second measurement, $R^{(\nu)}$, yields a successful reversal, on condition that the outcome of first measurement $M$ was $\nu$. This probability depends on initial state $\hat{\rho}$ and is written as

$$P_{\mathrm{rev}|\nu} = \frac{\max p_{\rm rev}^{(\nu)}}{\mathrm{Tr}[\hat{A}_\nu \hat{\rho} \hat{A}_\nu^{\dagger}]} = \left[ 1 + \frac{D(\nu, \hat{\rho})}{B(\nu)} \right]^{-1} . \tag{13}$$

We can say that $P_{\mathrm{rev}|\nu}$ decreases as the signal-to-noise ratio $(D/B)$ improves.

Taking the summation of $p_{\rm rev}^{(\nu)}$ over $\nu$, we find that the total probability of successful reversal is given by

$$P_{\rm rev} \equiv \sum_{\nu} \max p_{\rm rev}^{(\nu)} = \sum_{\nu} B(\nu) \,. \tag{14}$$

This is the primary result of this Letter. Note that this result still holds even if $B(\nu) = 0$ for some values of $\nu$. The left-hand side of Eq. (14), $P_{\rm rev}$, represents the maximum probability of successfully reversing the postmeasurement state of $M$ back to its initial state, regardless of the outcome of the measurement. We can interpret this quantity as the degree of physical reversibility of measurement $M$. Unlike conditional reversibility $P_{\mathrm{rev}|\nu}$, total reversibility $P_{\rm rev}$ is independent of initial state $\hat{\rho}$. The right-hand side of Eq. (14) represents the total fraction of the background of $M$. We thus conclude that backaction, or disturbance, caused by a measurement on the measured state can be canceled with a probability equal to (and never larger than) the fraction of the background of the measurement. We can say that an unknown initial quantum state can be restored with a probability equal to the degree of our in-principle ignorance of that state.

If $P_{\mathrm{rev}}$ or $P_{\mathrm{rev}|\nu}$ is unity, the postmeasurement state of $M$ can be returned to its original state by means of a unitary process, hence with unit probability. Such a measurement process is called a unitarily reversible quantum operation [7]. Substituting $P_{\mathrm{rev}} = 1$ into Eq. (14) gives an equivalent condition for a measurement to be unitarily reversible in terms of the background, that is, $\sum_\nu B(\nu) = 1$. This means that all outcomes belong to the background, which is independent of the measured state. An alternative but equivalent condition for a measurement process to be unitarily reversible was derived by Nielsen and Caves [7].

As an illustration of our general result, Eq. (14), we now consider a quantum nondemolition (QND) photon-number measurement using the optical Kerr effect [9,11,12]. Here, a signal light (the system to be measured) is coupled to a probe light (the measuring apparatus) via the unitary evolution of the Kerr effect:

$$\hat{U} \equiv e^{i\kappa \hat{n}_s \hat{n}_p}, \tag{15}$$

where $\kappa$ is the strength of the optical Kerr effect, which is proportional to the third nonlinear susceptibility and to the interaction length, $\hat{n}_s$ is the signal photon-number operator, and $\hat{n}_p$ is the probe photon-number operator. After this interaction, a quadrature amplitude of the probe light is measured using homodyne detection. Real-number readout $\nu$ of this detection gives the outcome of the measurement. The probe light is initially prepared in a coherent state with amplitude $\alpha = |\alpha| e^{-i\theta}$, and the set of operators $\{\hat{A}(\nu)\}$ describing this measurement is written as [9]

$$\hat{A}(\nu) = \left(\frac{2}{\pi}\right)^{1/4} \sum_{n=0}^{\infty} \exp\{-[|\alpha| \sin(\kappa n - \theta) - \nu]^2\} \times e^{i\theta_n} |n\rangle\langle n|, \tag{16}$$

where $|n\rangle$ are the Fock states of the signal light and $\theta_n$ are the unimportant phase factors. It is easy to verify the closure relation:

$$\int_{-\infty}^{\infty} \hat{A}^\dagger(\nu)\hat{A}(\nu)\, d\nu = \hat{1}. \tag{17}$$

For the range of signal photon numbers $n$ satisfying $|\kappa n - \theta| \ll 1$, the QND measurement gives an estimate of $n$ with a resolution (the length of the error bar) of the order of $(\sqrt{|\alpha|\kappa})^{-1}$. The resolution and range of the measurement thus depend on the nonlinearity $\kappa$. Substituting Eq. (16) into Eq. (9), we obtain

$$B(\nu) = \sqrt{\frac{2}{\pi}} \exp[-2(|\alpha| + |\nu|)^2], \tag{18}$$

which shows that the background, $B(\nu)$, for outcome $\nu$ depends on parameter $\alpha$ of the measuring apparatus, and does not depend on the coupling constant, $\kappa$, between the system and the apparatus. Substituting Eq. (18) into Eq. (14), we find that the degree of physical reversibility, $P_{\mathrm{rev}}$, of the measurement is given by

$$\begin{aligned} P_{\mathrm{rev}} &= \int_{-\infty}^{\infty} \sqrt{\frac{2}{\pi}} \exp[-2(|\alpha| + |\nu|)^2]\, d\nu \\ &= \frac{2}{\sqrt{\pi}} \int_{\sqrt{2}|\alpha|}^{\infty} e^{-x^2}\, dx \equiv \mathrm{erfc}(\sqrt{2}|\alpha|). \end{aligned} \tag{19}$$

This is a function of the amplitude of the probe light alone and is independent of the nonlinearity $\kappa$. Provided that amplitude $\alpha$ of the probe light is held constant, changing strength $\kappa$ of the Kerr effect changes the resolution of the measurement, but keeps the degree of physical reversibility unaltered. This is a good example of our general result, that is, the degree of physical reversibility is determined by the fraction of the background alone and is not directly related to the other measurement parameters, such as the resolution of the measurement.

Next, we discuss how our knowledge on the initial state is influenced by the reversing operations. Consider a measured system and a measuring apparatus as a single quantum system, and suppose that a quantum correlation between the measured system and the measuring apparatus has been established, but that the outcome has not yet been read out. At this stage, one can, in principle, construct a unitary evolution operator that brings the state of the measured system and that of the apparatus back to their respective initial states. In this case, the information about the measured state is erased by obliterating (or throwing out the chance to read out) the measurement result itself. On the other hand, once the outcome has been read out, it can no longer be "forgotten." There is, however, still a way to erase the information about the initial state (even from one's mind, presumably): rather than forgetting the results, one can attempt to extract more information about the measured state by performing a second measurement. The trick here is to suitably tailor the second measurement so that one of the outcomes gives an inference about the measured state that is exactly opposite to the inference given by the first measurement. If such a counterbalancing outcome is obtained in the second measurement, the bias formed about the measured state on the basis of the first measurement is neutralized. Reversing measurement $\hat{R}^{(\nu)}$ has this effect, as seen in Eq. (5).

This technique for erasing information utilizes only (classical) statistical inference. There is no need to invoke quantum mechanics. It would be interesting to find out what happens if, instead of applying reversing measurement $\hat{R}^{(\nu)}$, we apply an arbitrary measurement $Q$ that has the information-erasing property after the first measurement, $M_0$. Suppose that the first measurement yielded outcome $\nu_1$, and the second one, $Q$, yielded "successful" outcome $\mu_0$. Consider $M_0$ and $Q$ as one single measurement $M$, and denote the combination of the outcomes $\{\nu_1, \mu_0\}$ by a single index $\nu$. Then, complete erasure of the information means $D(\nu, \hat{\rho}) = \mathrm{Tr}[\hat{A}_\nu \hat{\rho} \hat{A}_\nu^\dagger] - B(\nu) = 0$ for these $M$ and $\nu$. Substituting this into Eq. (13) yields $P_{\mathrm{rev}|\nu} = 1$. This implies that the state after measurement $Q$ can be restored to the initial state by a unitary operation. This result indicates that *any* measurement that has a nonzero probability of erasing the information obtained by the first measurement can work as a reversing measurement.

The reversibility of a measurement has recently attracted considerable attention in the context of quantum computation. This is because unwanted decoherence caused by the coupling between quantum registers and the environment can be regarded as the backaction of a measurement performed on the Hilbert subspace of the quantum registers. A reversing operation that possibly restores the initial state from the postmeasurement state and from the outcome of the measurement offers a means of error correction, which cancels the effect of the decoherence. An example of such a restoration scheme was proposed by Mabuchi and Zoller [13]. Their scheme complements the error correction schemes that use redundant coding [14–17] in the sense that while the mechanism of decoherence must be known and some information must be collected from the environment, no overhead of qubits is necessary. Their scheme, however, works only if the decoherence process is unitarily reversible, and any failure to obey this stringent requirement leads to an imperfect error correction and hence to an unreliable computation result. In such situations our reversing measurement scheme can be applied as a scheme of *probabilistic quantum error correction*.

To illustrate this, consider the case in which Mabuchi and Zoller's system [13] has an imbalance, $\delta > 0$, in the nominally half beam splitter, which is part of the detection system, with transmissivity $T = (1 + \delta)/2$. The qubit is initially in the state $|\psi_{\rm i}\rangle_{\rm m} = c_0|0\rangle_{\rm m} + c_1|1\rangle_{\rm m}$, and undergoes the decoherence [18]. The original error-correcting procedure leaves the qubit in the state $|\psi_{\rm c}\rangle_{\rm m} = \mathcal{N}(c_0\sqrt{1-\delta}\,|0\rangle_{\rm m} + c_1\sqrt{1+\delta}\,|1\rangle_{\rm m})$, where $\mathcal{N}$ is a normalization constant. This state neither is the same as the initial state, nor can be converted to the initial state by a unitary operation. This is due to the fact that the decoherence process here is not unitarily reversible. It is, however, physically reversible, and a reversing measurement can be constructed. In this example, an optimum reversing measurement can be made by using an additional working qubit prepared in $|0\rangle_{\rm w}$, a few gate operations, and a reading-out of the working qubit. The gate operations consist of three processes: a single-bit rotation on the working qubit, $R$:$\{|0\rangle_{\rm w} \to \cos\theta|0\rangle_{\rm w} + \sin\theta|1\rangle_{\rm w}, |1\rangle_{\rm w} \to \cos\theta|1\rangle_{\rm w} - \sin\theta|0\rangle_{\rm w}\}$, a controlled-NOT operation that exchanges $|1\rangle_{\rm m}|0\rangle_{\rm w}$ and $|1\rangle_{\rm m}|1\rangle_{\rm w}$, and another single-bit rotation, $R^{-1}$. These processes convert the state $|\psi_{\rm c}\rangle_{\rm m}|0\rangle_{\rm w}$ into

$$\mathcal{N}[c_0\sqrt{1-\delta}\,|0\rangle_{\rm m}|0\rangle_{\rm w} + c_1\sqrt{1+\delta}\,|1\rangle_{\rm m}(\sin2\theta|0\rangle_{\rm w} + \cos2\theta|1\rangle_{\rm w})]. \tag{20}$$

By setting $\theta$ to $\sin2\theta = \sqrt{1-\delta}/\sqrt{1+\delta}$, the first qubit precisely returns to its original state, $|\psi_{\rm i}\rangle_{\rm m}$, if the readout of the working qubit is $|0\rangle_{\rm w}$. This error-correcting scheme works probabilistically, and it is not difficult to see that the overall probability of success is $1 - \delta$.

The error-correcting scheme described above does not necessarily increase the probability $p$ of obtaining the correct answer in a single run of computation, in comparison with the imperfect unitary scheme. The distinct advantage of our scheme, however, is that it allows us to be sure whether or not the correction is successful. With our scheme, after repeating the computation $k$ times, the probability of obtaining the correct result is $1 - (1 - p)^k$. If we do not know whether the error correction was successful in each run, we must take a majority vote of all the answers, which results in a smaller probability of obtaining the correct answer. The advantage of our scheme is particularly prominent in decision problems for which the output is "yes" or "no". If we do not know whether the error correction was successful in such problems, increasing the number of runs does not help at all for $p < 1/2$. If we can know for sure, we obtain the right answer in $p^{-1}$ runs on average.

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[18] Here we consider the case in which a particular one (detector **A** in the notation in [13]) of the two detectors registered a photon. The discussion is similar in the other case.