Generation of Spin-Wave Envelope Dark Solitons

Andrei N. Slavin,[1,2] Yuri S. Kivshar,[2,3] Elena A. Ostrovskaya,[3] and Hartmut Benner[2]
[1]*Department of Physics, Oakland University, Rochester, Michigan 48309*
[2]*Institut für Festkörperphysik and SFB 185, Technische Universität Darmstadt, D-64289 Darmstadt, Germany*
[3]*Optical Sciences Centre, Australian National University, Canberra ACT 0200, Australia*
(Received 2 June 1998)

We demonstrate that in nonlinear systems with small group velocity any (odd or even) number of dark solitons can be generated by an input pulse without initially introduced phase modulation. We propose a theoretical explanation of the earlier reported experimental results on the generation of magnetic envelope dark solitons. [S0031-9007(99)08768-2]

PACS numbers: 75.30.Ds, 76.50.+g, 85.70.Ge

Experimental study of nonlinear waves in different physical systems is an attractive area of research, not only from the fundamental point of view, but also for future applications of nonlinear properties of solids. So far the most impressive results have been demonstrated in nonlinear guided-wave optics (see, e.g., Refs. [1,2]). In a sharp contrast, the experimental study of nonlinear waves in solid-state systems has demonstrated much slower progress, in particular due to dissipative losses that make the observation of large nonlinear effects in real systems difficult. Nevertheless, *spin-wave bright envelope solitons* have been observed in magnetic films, for different orientations of the magnetic field and propagation direction [3,4]. Similar results have been reported for other types of nonlinear waves in solids, e.g., *acoustic envelope solitons* generated in a quartz crystal [5].

The first observation of *microwave magnetic-envelope dark solitons* was reported by Chen *et al.* [6], who generated spin waves propagating perpendicularly to the direction of a bias magnetic field in a tangentially magnetized single-crystal yttrium iron garnet (YIG) film. For such a geometry, the dispersion and nonlinear coefficients have the same sign [7] and, therefore, dark solitons can be generated. The experimental results reported in Ref. [6] revealed unusual features of the soliton generation when the number of generated solitons was changing with the input power from even to odd.

The purpose of this Letter is twofold. First, we demonstrate that in the case when an input pulse without any phase modulation enters a nonlinear dispersive medium at a certain point, the generated localized wave acquires *an induced spatial phase shift* accumulated during its generation, the phase shift being inversely proportional to the wave group velocity. Such a phase shift is *negligible* for large group velocities, e.g., for optical solitons in fibers. However, for wave propagation in solids, the induced phase is no longer small, and its effect becomes important, as in the case of spin waves. Second, based on this general concept, we shed light on the experimental results reported in Ref. [6]. We show that an arbitrary small phase shift across the initial pulse can change the character of the soliton generation, and both an odd and even number of dark solitons can emerge. For the same shape and duration of the input pulse, this effect is determined only by the pulse amplitude as observed in [6].

First, we discuss the phenomenon of the phase shift accumulated during the pulse generation. We consider the evolution of a slowly varying wave envelope $A(z,t)$ described by the nonlinear Schrödinger (NLS) equation,

$$i\left(\frac{\partial A}{\partial t} + v_g \frac{\partial A}{\partial z}\right) + \frac{1}{2} D \frac{\partial^2 A}{\partial z^2} - N|A|^2 A = 0\,, \qquad (1)$$

where v_g is the wave group velocity calculated at the carrier wave number $k = k_0$, and the relative signs of the dispersion, D, and nonlinearity, N, coefficients correspond to two different types of solitary waves, bright ($ND < 0$) or dark ($ND > 0$) solitons.

Let us discuss the case when a localized wave is excited at a certain point $z = z_0$ by a finite-duration input signal with no intentional phase modulation. When such a wave of amplitude A_m propagates for the time t_0 in a nonlinear dispersive medium described by the NLS equation (1), the phase of its envelope changes by the amount

$$2\tilde{\phi} = \left(v_g K + \frac{1}{2} D K^2 + N|A_m|^2\right) t_0\,, \qquad (2)$$

where $K = (k - k_0) \ll k_0$ is the characteristic wave number of the wave envelope. By the moment when the whole input signal of the full length $l_0 = v_g t_0$ excited at $z = z_0$ entered the medium, its leading edge had spent time t_0 longer in the medium than its trailing edge. Thus, the pulse accumulates *an induced spatial phase shift* which can be estimated with the help of Eq. (2).

This phase shift can be easily evaluated for a rectangular (or boxlike) dark input pulse of duration T. Such a pulse has the length $L = v_g T$ and the characteristic wave number of its envelope is $K = 2\pi/L$. The full length of the input signal can be estimated as $l_0 = bL = bTv_g$, where $b > 1$ is a phenomenological parameter that takes into account the influence of the cw background on the process of the phase shift generation. The resulting phase shift ϕ is then given by the expression

$$\phi = \tilde{\phi} - 2\pi = \frac{2T}{T_D}\left[1 + \frac{1}{2}\left(\frac{S_0}{\pi}\right)^2\right], \quad (3)$$

where we took $b = 2$, and introduced T_D, the dispersion time defined as

$$T_D = \frac{v_g^2 T^2}{\pi^2 |D|}. \quad (4)$$

In Eq. (3) we subtracted a constant phase 2π resulting from the first term in Eq. (2), and also introduced the dimensionless input pulse area $S_0 = |A_m| L\sqrt{(N/D)}$.

As follows from Eqs. (3) and (4), the induced phase shift ϕ, for a given duration T of the input dark rectangular pulse, is *inversely proportional* to the square of the wave group velocity v_g, and it grows with the input pulse area S_0. For example, the typical duration of the input pulse in optical fibers is much smaller than the dispersion time, $T \ll T_D$, so that in optics the accumulated phase shift is practically zero. To see the importance of this effect in solids, we calculate the phase shift (3) for the parameters of the experiment [6], i.e., $T = 15$ ns, $v_g = 4.5 \times 10^6$ cm s^{-1}, $D = -7.8 \times 10^3$ cm^2 s^{-1}, and $N = -1.0 \times 10^{10}$ s^{-1}. As a result, the accumulated phase varies from 0.16π to 0.4π when the area of the input pulse S_0 increases from 0 to 2π. A qualitative difference between optical and magnetic systems was first pointed out by Boardman *et al.* [8].

Generally speaking, the soliton generation by a localized source is a rather complicated problem, and so far no analytical solution has been found. Application of the inverse scattering transform (IST), based on the idea of the inversion of time and space, requires additional information about the field derivatives at the generation point [9] which are not known *a priori*. The solution of a forced NLS model (similar to that carried for some other models [10]) also meets a lot of difficulties. Here, to estimate qualitatively the effect of the accumulated phase on the process of dark soliton generation, we employ the IST technique, where the phase shift (3) is introduced into the boxlike input. This procedure seems to be only qualitatively correct, but it captures the basic physics of the underlying problem. Moreover, many examples of dark soliton generation [2] demonstrate the crucial importance of the phase rather than amplitude variation of the input pulse. Thus, we expect other effects (e.g., pulse amplitude distortions) to be small or/and less important for the process of dark soliton generation.

Below we consider the normalized form of the NLS equation (1) for $ND > 0$ in the form

$$i\frac{\partial u}{\partial \tau} - \frac{\partial^2 u}{\partial x^2} + 2|u|^2 u = 0, \quad (5)$$

where u is the normalized complex envelope, x is a normalized coordinate in the reference frame moving with group velocity, and τ is the normalized time calculated from the moment of the pulse generation, $t = T$. The continuous wave (cw) solution $|u(x,t)| = u_0$ of Eq. (5) is modulationally stable, and localized waves exist on a stable cw background of the amplitude u_0 in the form of dark solitons [2].

According to Zakharov and Shabat [11], in order to investigate which type of initial input pulse $u(x,0)$ generates solitons and to find their parameters, we have to analyze the eigenvalue problem for the auxiliary two-component eigenfunction $\psi = (\psi_1, \psi_2)^T$,

$$\frac{\partial \psi_1}{\partial x} = i\lambda\psi_1 - iu(x,0)\psi_2, \qquad \frac{\partial \psi_2}{\partial x} = -i\lambda\psi_2 + iu^*(x,0)\psi_1, \quad (6)$$

where the asterisk stands for the complex conjugate, and nonvanishing boundary conditions, $|u(x,0)| \to u_0$, are assumed.

The spectral problem (6) may possess a discrete spectrum with real eigenvalues $|\lambda_n| < u_0$; the discrete spectrum is invariant in τ when the time evolution is included. Each real eigenvalue λ_n corresponds to a dark soliton with amplitude $\kappa_n = (u_0^2 - \lambda_n^2)^{1/2}$ moving with the velocity $2\lambda_n$. Thus, the asymptotic evolution of the input pulse is described by the discrete eigenvalues of the scattering problem (6).

To analyze the dark-soliton generation by an input pulse with an induced phase shift, we consider a symmetric boxlike initial condition with the total phase shift 2ϕ in the form $u(x,0) \equiv u_0(x) = u_0 \exp[i\,\mathrm{sgn}\,(x)\phi]$, for $|x| > L/2$, and $u(x,0) = u_1 e^{i\alpha x}$, for $|x| \le L/2$, with a nonzero minimum at the center, $u_1 < u_0$, and $\alpha = 2\phi/L$ for the phase continuity. To solve the eigenvalue equations for ψ_1 and ψ_2, we notice that from Eqs. (6) we can find the equivalent equation for ψ_1,

$$\frac{\partial^2 \psi_1}{\partial x^2} - \frac{u'}{u}\frac{\partial \psi_1}{\partial x} + \left(\lambda^2 - |u|^2 + i\lambda\frac{u'}{u}\right)\psi_1 = 0, \quad (7)$$

where the prime stands for the derivative in x. Then, the eigenvalue problem (6) can be solved exactly in the three different regions, i.e., for $x > L/2$,

$$\psi(x,0) = \frac{C_1}{u_0}\begin{pmatrix} u_0 \\ (\lambda - i\kappa)e^{-i\phi} \end{pmatrix} e^{-\kappa x}, \quad (8)$$

where $\kappa^2 = u_0^2 - \lambda^2 > 0$; for $|x| < L/2$,

$$\psi(x,0) = \frac{C_2}{u_1}\begin{pmatrix} u_1 \\ \lambda - \eta \end{pmatrix} e^{i\eta x} + \frac{C_3}{u_1}\begin{pmatrix} u_1 \\ \lambda + \eta \end{pmatrix} e^{-i\eta x}, \quad (9)$$

where η is defined as a solution of the algebraic equation, $\eta^2 = \alpha\eta + (\lambda^2 - \alpha\lambda - |u_1|^2)$; and for $x < -L/2$,

$$\psi(x,0) = \frac{C_4}{u_0}\begin{pmatrix} u_0 \\ (\lambda + i\kappa)e^{i\phi} \end{pmatrix} e^{\kappa x}. \quad (10)$$

Matching the solutions (8) to (10) at $x = \pm L/2$, we obtain an eigenvalue equation for λ. We present it here in a simplified form, taking formally $\alpha = 0$,

$$(\xi - \nu)[\nu \sin(\xi + \phi) + \sqrt{S_0^2 - \nu^2} \cos(\xi + \phi)] + 2S_0 S_1 \sin\xi = (\xi + \nu)[\nu \sin(\xi - \phi) - \sqrt{S_0^2 - \nu^2} \cos(\xi - \phi)], \quad (11)$$

where $S_0 = u_0 L$, $S_1 = u_1 L$, $\nu = \lambda L$, and $\xi = \sqrt{\nu^2 - S_1^2}$. Real solutions ν_n of Eq. (11) satisfying the condition $S_1 < \nu < S_0$ correspond to dark solitons with $\lambda_n = \nu_n / L$. Equation (11) *includes all known results* on the generation of dark solitons. When $u_1 = \phi = 0$, this problem predicts only pairs of dark solitons [11,12]. In general, Eq. (11) describes generation of both even and odd numbers of dark solitons, depending on the values of the parameters.

In most experiments, including the case of magnetic solitons [6], the input dark pulse is generated by switching off the cw signal almost completely for the time interval $T = L/v_g$. In such a case $u_1 \approx 0$, $S_1 \approx 0$, and S_0 is the total dimensionless area of the input dark pulse. Then, the eigenvalue equation, even for $\alpha \neq 0$, simplifies to

$$\nu \sin(\nu - \phi) = \sqrt{S_0^2 - \nu^2} \cos(\nu - \phi), \quad (12)$$

and its solutions can be investigated analytically. We present the number of real solutions of Eq. (12) on the parameter plane (ϕ, S_0). When $\phi = 0$, Eq. (12) has only pairs of real solutions, so that for $\pi N < S_0 < \pi(N + 1)$, where N is an integer, $2(N + 1)$ dark solitons are generated [11]. In the case $\phi = \pi/2$, when the total phase jump $2\phi = \pi$, the solutions are similar to those discussed earlier for the initial condition $u(x, 0) = u_0 \tanh(ax)$, i.e., for $\pi N/2 < S_0 < \pi(N + 1)/2$ there exist exactly $(1 + 2N)$ dark solitons, one fundamental soliton with zero intensity at the center and N symmetric pairs [2]. The number of dark solitons in the case of arbitrary ϕ $(0 < \phi < \pi)$ is shown in Fig. 1 on the plane (ϕ, S_0). The dotted lines, $\phi = \pi M \pm S_0$ $(M = 0, \pm 1, \ldots)$, separate the domains with different numbers of dark solitons. The main *qualitatively new result* of our analysis is that any nonzero value of the phase shift $\phi \neq 0$ can lead to the consecutive generation of both odd and even number of dark solitons, when the area S_0 of the input dark pulse grows. Thus, the cases $\phi = 0$ and $\phi = \pi/2$ are degenerate.

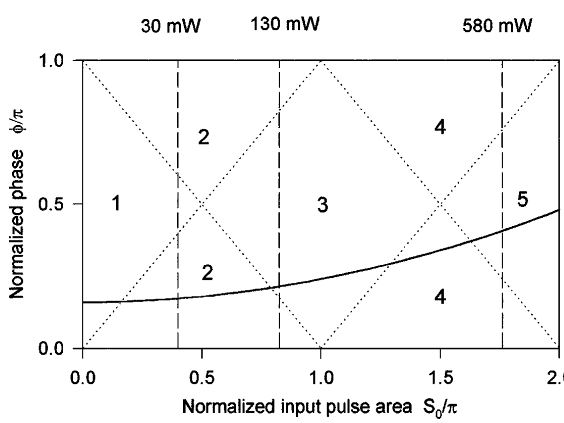

FIG. 1. Number (shown in the domains marked by dotted lines) of dark solitons generated by the initial pulse $u_0(x)$. The solid line is the phase shift (3) calculated for the experimental data from Ref. [6]. The vertical dashed lines give the values of the input pulse area S_0 corresponding to the input powers used in the experiment.

Now, to see how many solitons are generated when the input power varies, we define the phase shift by Eq. (3) and show it as a solid line in Fig. 1. As follows from Fig. 1, if the spatial phase shift is described by Eq. (3), the variation of the amplitude A of the cw background can change the number of solitons from 1 to 5.

To provide a clearer comparison of our theory with the experimental results on the generation of dark solitons [6], we assume a linear relation $|A_m|^2 = BP_{\text{in}}$, where P_{in} is the experimental input power and $|A_m|$ is the amplitude of the excited magnetostatic wave normalized to the saturation magnetization [4]. We choose the coefficient $B = 8.5 \times 10^{-3}\ W^{-1}$ to obtain a typical value of the amplitude $|A| = 7 \times 10^{-2}$ (see, e.g., Ref. [13]) for the maximum input power $P_{\text{in}} = 580$ mW used in the experiment [6]. The dashed vertical lines in Fig. 1 show the values of the input pulse area S_0 corresponding to the experimental input powers. The positions of the crossing points of the dashed lines with the solid curve $\phi = f(S_0)$ indicate the number of dark solitons that are expected to be generated from the input pulse at a given power.

Eigenvalues λ_n and corresponding amplitudes κ_n of the generated dark solitons have been calculated for the parameters from [6], and the amplitudes are presented in Fig. 2. As follows from Fig. 2, at $P_{\text{in}} = 30$ mW we expect to obtain an asymmetric pair of dark solitons, whereas at $P_{\text{in}} = 130$ mW a similar pair with larger amplitudes and a very small third soliton should emerge. At the largest power $P_{\text{in}} = 580$ mW [6], our theory predicts a dark soliton with *almost zero intensity at*

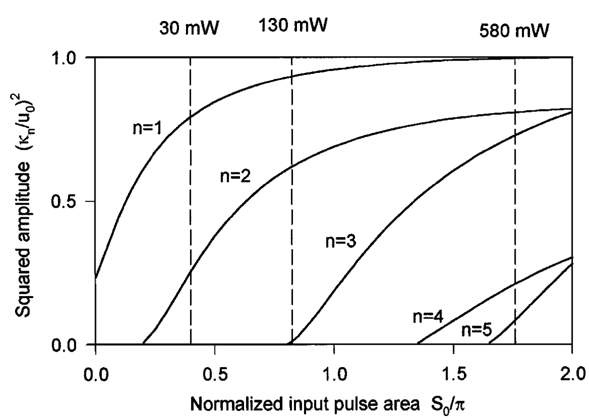

FIG. 2. Normalized amplitudes κ_n of the generated dark solitons as calculated from Eq. (11) for the parameters of the experiment [6].

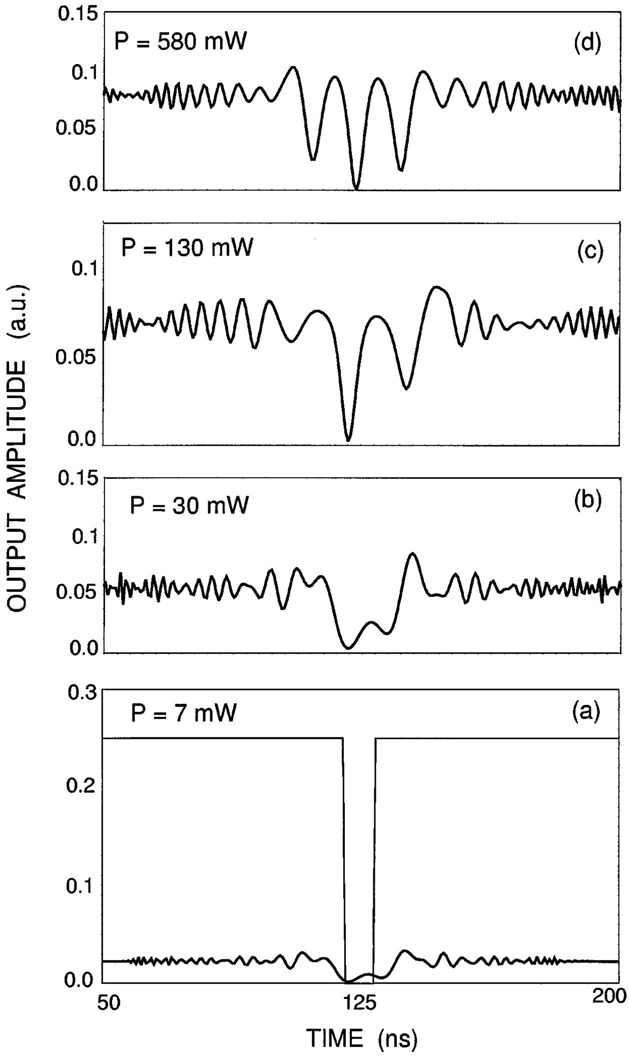

FIG. 3. Results of numerical simulations of dark solitons generated in a dissipative medium due to the nonlinearity-induced phase shift. The four cases correspond to the input powers from Ref. [6], as marked in the plots. The input boxlike pulse is shown in (a).

its center and two pairs of slightly asymmetric grey solitons. These results are in a good qualitative agreement with the experimental output envelopes presented in Fig. 1(a) of Ref. [6] for the corresponding values of the input power. To provide further confirmation of this correspondence, in Fig. 3 we present the results of our numerical simulations carried out for the dissipative NLS equation with the parameters taken directly from the experiment. A remarkable similarity between our Fig. 3 and Fig. 1(a) of Ref. [6] can be noticed, especially for the cases (a) to (c), thus providing a strong justification of our concept of the accumulated nonlinear phase.

In conclusion, we have shown that a localized nonlinear wave excited by a source at a fixed location acquires a nonlinearity-induced spatial phase shift during the process of its generation in a dispersive nonlinear medium. Such a phase shift can be large for nonlinear waves in solids (e.g., for spin waves), leading to the generation of different numbers of dark solitons by a pulse of the same shape and duration at different input powers. The results allow us to explain the specific features of the magnetic dark soliton generation reported earlier. They can be useful for interpreting experiments on the generation of solitons in other solid-state systems.

The authors are indebted to C. Patton, B. Kalinikos, and M. Peyrard for useful discussions. This work has been supported by the AvH Stiftung, by the Deutsche Forschungsgemeinschaft through SFB 185 "*Nichtlineare Dynamik,*" by the International Science Foundation (SPU-062034), by the National Science Foundation (DMR-9701640), by the Oakland University Foundation, and by the Australian Academy of Science.

[1] A. Hasegawa and Y. Kodama, *Solitons in Optical Communications* (Oxford University Press, Oxford, 1995).
[2] Yu. S. Kivshar and B. Luther-Davies, Phys. Rep. **298**, 81 (1998).
[3] See, e.g., B. A. Kalinikos, N. G. Kovshikov, and A. N. Slavin, JETP Lett. **38**, 413 (1983); M. A. Tsankov, M. Chen, and C. E. Patton, J. Appl. Phys. **76**, 4274 (1994); J. M. Nash, C. E. Patton, and P. Kabos, Phys. Rev. B **51**, 15 079 (1995); A. N. Slavin, H. Benner, K. J. Foos, and T. Lesperance, J. Phys. (Paris) **7**, C1-459 (1997).
[4] For an overview, see A. N. Slavin, B. A. Kalinikos, and N. G. Kovshikov, in *Nonlinear Phenomena and Chaos in Magnetic Materials,* edited by P. E. Wigen (World-Scientific, Singapore, 1994), pp. 209–248.
[5] M. Planat, in *Continuum Models and Discrete Systems,* edited by G. A. Maugin (Longman Scientific & Technical, Essex, 1991), Vol. 2, pp. 339–349.
[6] M. Chen, M. A. Tsankov, J. M. Nash, and C. E. Patton, Phys. Rev. Lett. **70**, 1707 (1993).
[7] A. K. Zvezdin and A. F. Popkov, Zh. Eksp. Teor. Fiz. **84**, 606 (1983) [Sov. Phys. JETP **57**, 350 (1983)].
[8] A. D. Boardman, S. A. Nikitov, K. Xie, and H. Mehta, J. Magn. Magn. Mater. **145**, 358 (1995).
[9] D. J. Kaup, J. Math. Phys. **25**, 277 (1984); D. J. Kaup and P. J. Hansen, Physica (Amsterdam) **18D**, 77 (1986).
[10] Yu. S. Kivshar *et al.,* Phys. Rev. B **43**, 1098 (1991).
[11] V. E. Zakharov and A. B. Shabat, Zh. Eksp. Teor. Fiz. **64**, 1627 (1973) [Sov. Phys. JETP **37**, 823 (1973)].
[12] S. A. Gredeskul and Yu. S. Kivshar, Phys. Rev. Lett. **62**, 977 (1989).
[13] A. N. Slavin and G. M. Dudko, J. Magn. Magn. Mater. **86**, 115 (1990).