Counterterms and M-Theory Corrections to $D = 11$ Supergravity

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We construct a local on-shell invariant in $D = 11$ supergravity from the nonlocal four-point tree scattering amplitude. Its existence, together with earlier arguments, implies nonrenormalizability of the theory at lowest possible, two loop, level. This invariant, whose leading bosonic terms are exhibited, may also express the leading, "zero-slope," M-theory corrections to its $D = 11$ supergravity limit. [S0031-9007(99)08703-7]

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With the advent of M physics, of which it is the local limit, $D = 11$ supergravity [1], has regained a central role. This connection adds a further motivation to our quest for its explicit on-shell supersymmetric invariants: Not only would their existence describe specific candidate counterterms, completing a recent argument for the field theory's nonrenormalizability, but they would also exemplify concrete "zero-slope" corrections from the full M theory (whatever its ultimate form) similar to the corresponding string corrections to their limiting, $D = 10$, supergravities. That such invariants had not been given earlier is due to the absence of a systematic supersymmetric calculus, or even of a practical way to verify candidate terms. Indeed, it was only very recently [2,3] that the tensorial structure of the relevant invariant's four graviton sector was found, by explicit one-loop calculations. The present effort originated in trying to generalize techniques known from lower-dimensional models, such as the use of gravitational Bel-Robinson (BR) tensors as currents in constructing $D = 4$ supergravity invariants [4]; despite the strong degeneracy in the number of such currents at $D = 4$, their extensions will indeed play a key part in our $D = 11$ construction. A major step forward in this area was recently made [5] through beautiful use of the Yang-Mills supersymmetry/supergravity (YM SUSY/SUGRA) open/closed string correspondence, analytically extended to its maximal ($D = 10$) dimension. Although there is no underlying $D = 11$ YM SUSY model, we shall argue that the construction of [5] together with the invariant provided here, lend strong credence to a two-loop nonrenormalizability verdict for $D = 11$ supergravity.

Our construction is a physical one, with manifest supersymmetry: we calculate the tree-level four-point scattering amplitudes of the $D = 11$ theory. This procedure has several merits: First, there is no fermion-boson mixing in the tree diagrams; hence we are free just to calculate the bosonic contributions knowing that they are part of a guaranteed SUSY invariant, namely the total four-point S matrix. Second, because SUSY transformations are linear to leading order, there is no mixing with higher-point amplitudes. Third, we will see that one can uniformly extract the desired, local, invariant from the nonlocal S matrix without loss of SUSY. The basis for our computations is the full action of [1], expanded to the order required for obtaining the four-point scattering amplitudes among its two bosons, namely the graviton and the three-form potential $A_{\mu\nu\alpha}$ with field strength $F_{\mu\nu\alpha\beta} \equiv 4\partial_{[\mu}A_{\nu\alpha\beta]}$, invariant under the gauge transformations $\delta A_{\mu\nu\alpha} = \partial_{[\mu}\xi_{\nu\alpha]}$. From the bosonic truncation of this action (omitting obvious summation indices),

$$I_{11}^{B} = \int d^{11}x \left[-\frac{\sqrt{g}}{4\kappa^2} R(g) - \frac{\sqrt{g}}{48} F^2 + \frac{2\kappa}{144^2} \epsilon^{1\cdots 11} F_{1\ldots}F_{5\ldots}A_{\ldots 11} \right], \quad (1)$$

we extract the relevant vertices and propagators; note that κ^2 has dimension $[L]^9$ and that the (P,T) conserving cubic Chern-Simons (CS) term depends explicitly on κ but is (of course) gravity independent. The propagators come from the quadratic terms in $\kappa h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}$ and $A_{\mu\nu\alpha}$; they need no introduction. There are three cubic vertices, namely graviton, pure form and mixed-form graviton that we schematically represent as

$$V_3^g \sim (\partial h \partial h) h \equiv \kappa T_g^{\mu\nu} h_{\mu\nu}, \quad V_3^{gFF} \equiv \kappa T_F^{\mu\nu} h_{\mu\nu}, \qquad V_3^F \sim \kappa \epsilon AFF \equiv \kappa A_{\mu\nu\alpha} C_F^{\mu\nu\alpha}, \quad (2a)$$

$$T_g^{\mu\nu} \equiv G_{(2)}^{\mu\nu}, \quad T_F^{\mu\nu} \equiv F^{\mu}F^{\nu} - \frac{1}{8}\eta^{\mu\nu}F^2,$$

$$C_F^{\rho\sigma\tau} \equiv \frac{2}{(12)^4} \epsilon^{\rho\sigma\tau\mu_1\cdots\mu_8} F_{\mu_1\ldots}F_{\cdots\mu_8}. \quad (2b)$$

The form's current C_F and stress tensor T_F are both manifestly gauge invariant. In our computation, two legs of the three-graviton vertex are always on linearized Einstein shell; we have exploited this fact in writing it in the simplified form (2), the subscript on the Einstein tensor denoting its quadratic part in h. [Essentially, the on-shell legs are the ones in $T_g^{\mu\nu}$, the off-shell one multiplies it.] To achieve coordinate invariance to correct, quadratic, order one must also include the four-point contact vertices

$$V_4^g \sim \kappa^2(\partial h \partial h)hh, \qquad V_4^{gF} = \kappa^2 \frac{\delta I^F}{\delta g_{\alpha\beta}\delta g_{\mu\nu}} h_{\alpha\beta}h_{\mu\nu} \tag{3}$$

when calculating the amplitudes; these are the remedies for the unavoidable coordinate variance of the gravitational stress tensor $T_g^{\mu\nu}$ and the fact that $T_F^{\mu\nu} h_{\mu\nu}$ is only first order coordinate-invariant. The gravitational vertices are not given explicitly, as they are both horrible and well known [6,7]. We reiterate that gravitinos are decoupled at tree level; while four-point amplitudes involving them would mix with bosonic ones under supersymmetry transformations, this would merely provide a (useful) check on our arithmetic.

We start with the four-graviton amplitude, obtained by contracting two V_3^g vertices in all three channels [labeled by the Mandelstam variables (s,t,u)] through an intermediate graviton propagator (that provides a single denominator), adding the contact V_4^g and then setting the external graviton polarization tensors on free Einstein shell. The resulting amplitude $M_4^g(h)$ will be a nonlocal (precisely thanks to the local V_4^g contribution) quartic in the Weyl tensor. (We do not differentiate in notation between Weyl and Riemann here and also express amplitudes in covariant terms for simplicity, even though they are valid only to lowest relevant order in the linearized curvatures.) Explicit calculation is, of course, required to discover the exact R^4 combinations involved and things are much more complicated in higher dimensions than in $D = 4$, where there are exactly two possible local quartics in the Weyl tensor—for example, the squares of Euler ($E_4 \equiv R^*R^*$) and Pontryagin ($P_4 \equiv R^*R$) densities ($^\star R \equiv 1/2\epsilon R$). The special property of the Einstein action (that also ensures its supersymmetrizability) is that this amplitude must be maximally helicity conserving (treating all particle as incoming), thereby fixing its local part to be [8] $(E_4 - P_4)(E_4 + P_4)$. This invariant is also, owing to identities peculiar to $D = 4$, expressible [4] as the square of the (unique in $D = 4$) BR tensor $B_{\mu\nu\alpha\beta} = (RR + R^*R^*)_{\mu\nu\alpha\beta}$. But $D = 4$ is a highly degenerate case in both respects: generically, there are seven independent quartic monomials [9] in the Weyl tensor for $D \geq 8$ and an intrinsically three-parameter family of BR tensors; as might be expected, there is no longer any simple equivalence between $(\mathrm{BR})^2$ forms and helicity (though it might be fruitful to explore its extensions to generic D). Still, these descriptions are robust: for example, one hint for the gravitational amplitude is provided by its diagrammatic origin in terms of $T_{\mu\nu}^g$ because there is a (highly gauge-dependent) identity of the schematic form $B_{\mu\nu\alpha\beta} \sim \partial_{\alpha\beta}^2 T_{\mu\nu}^g$. Within our space limitations, we cannot exhibit the actual calculation here; fortunately, this amplitude has already been given (for arbitrary D) in the pure gravity context [6]. It can be shown, using the basis of [9], to be of the form

$$\begin{aligned} M_4^g &= \kappa^2(4stu)^{-1} t_8^{\mu_1\cdots\mu_8} t_8^{\nu_1\cdots\nu_8} \\ &\quad \times R_{\mu_1\mu_2\nu_1\nu_2} R_{\mu_3\mu_4\nu_3\nu_4} R_{\mu_5\mu_6\nu_5\nu_6} R_{\mu_7\mu_8\nu_7\nu_8} \\ &\equiv (stu)^{-1} L_4^g \,, \end{aligned} \tag{4}$$

up to a possible contribution from the quartic Euler density E_8, which is a total divergence to this order (if present, it would only contribute at R^5 level). The result (4) is also the familiar superstring zero-slope limit correction to $D = 10$ supergravity, where the $t_8^{\mu_1\cdots\mu_8}$ symbol originates from the $D = 8$ transverse subspace [10]. [Indeed, the "true" origin of the ten dimensional analog of (4) was actually traced back to $D = 11$ in the one-loop computation of [2,3].] Note that the local part, L_4^g, is simply extracted through multiplication of M_4^g by stu, which in no way alters SUSY invariance, because all parts of M_4 behave the same way.

In many respects, the form (4) for the four-graviton contribution is a perfectly physical one. However, in terms of the rest of the invariant to be obtained below, one would like a natural formulation with currents that encompass both gravity and matter in a unified way as in fact occurs in, e.g., $N = 2$, $D = 4$ supergravity [11]. This might also lead to some understanding of other SUSY multiplets. Using the quartic basis expansion, one may rewrite L_4^g in various ways involving conserved BR currents and a closed four-form $P_{\alpha\beta\mu\nu} = 1/4 R^{ab}_{[\mu\nu} R_{\alpha\beta]ab}$, for example

$$\begin{aligned} L_4^g = 48\kappa^2 \Big[& 2B_{\mu\nu\alpha\beta}B^{\mu\alpha\nu\beta} - B_{\mu\nu\alpha\beta}B^{\mu\nu\alpha\beta} \\ & + P_{\mu\nu\alpha\beta}P^{\mu\nu\alpha\beta} + 6B^\rho_{\mu\rho\alpha}B^{\mu\sigma\alpha}_\sigma \\ & - \frac{15}{49}(B^{\mu\nu}_{\mu\nu})^2 \Big], \end{aligned} \tag{5a}$$

$$\begin{aligned} B_{\mu\nu\alpha\beta} \equiv\; & R_{(\underline{\mu}\rho\alpha\sigma}R^{\rho\sigma}_{\underline{\nu})\beta} - \frac{1}{2} g_{\mu\nu}R_{\alpha\rho\sigma\tau}R_\beta^{\rho\sigma\tau} \\ & - \frac{1}{2} g_{\alpha\beta}R_{\mu\rho\sigma\tau}R_\nu^{\rho\sigma\tau} \\ & + \frac{1}{8} g_{\mu\nu}g_{\alpha\beta}R_{\lambda\rho\sigma\tau}R^{\lambda\rho\sigma\tau}, \end{aligned} \tag{5b}$$

where () means symmetrization with weight one of the underlined indices. At $D = 4$, $P_{\mu\nu\alpha\beta}$ obviously reduces to $\epsilon_{\mu\nu\alpha\beta}P_4$, and L_4^g can easily be shown to have the correct B^2 form, as must be the case from brute force dimensional reduction arguments.

Let us now turn to the pure form amplitude, whose operative currents are the Chern-Simons $C^F_{\mu\nu\alpha}$ and the stress tensor $T^F_{\mu\nu}$, mediated, respectively, by the A and graviton propagators; each contribution is separately invariant. By dimensions, the building block will be $\kappa\partial F\partial F$; getting hints from $D = 4$, however, would require using (unwieldy) $N = 8$ models. Instead, we computed the two relevant, $C_F C_F$ and $T_F T_F$, diagrams directly, resulting in the four-point amplitude $M_4^F = (stu)^{-1} L_4^F = (stu)^{-1}\kappa^2(\partial F)^4$, again with an overall (stu) factor. (An independent calculation of M^F has just been reported in [12]; we have not compared details.) An economical way to organize L_4^F is in terms of matter BR tensors and corresponding C^F extensions, prototypes being the "double gradients" of $T^F_{\mu\nu}$ and of C^F,

$$\begin{aligned} B^F_{\mu\nu\alpha\beta} &= \partial_\alpha F_\mu \partial_\beta F_\nu + \partial_\beta F_\mu \partial_\alpha F_\nu \\ &\quad - \frac{1}{4}\,\eta_{\mu\nu}\partial_\alpha F \partial_\beta F\,, \\ \partial^\mu B^F_{\mu\nu\alpha\beta} &= 0\,, \end{aligned} \tag{6a}$$

$$\begin{aligned} C^F_{\rho\sigma\tau;\alpha\beta} &= \frac{1}{(24)^2}\,\epsilon_{\rho\sigma\tau\mu_1\cdots\mu_8}\partial_\alpha F^{\mu_1\cdots\mu_4}\partial_\beta F^{\mu_5\cdots\mu_8}, \\ \partial^\rho C^F_{\rho\sigma\tau;\alpha\beta} &= 0\,. \end{aligned} \tag{6b}$$

From (6) we can construct L_4^F as

$$\begin{aligned} L_4^F = &\frac{\kappa^2}{36}\,B^F_{\mu\nu\alpha\beta}B^F_{\mu_1\nu_1\alpha_1\beta_1}G^{\mu\mu_1;\nu_1\nu}K^{\alpha\alpha_1;\beta_1\beta} \\ &- \frac{\kappa^2}{12}\,C^F_{\mu\nu\rho;\alpha\beta}C^{F\mu\nu\rho}_{\alpha_1\beta_1}K^{\alpha\alpha_1;\beta_1\beta}. \end{aligned} \tag{7}$$

The matrix $G^{\mu\nu;\alpha\beta} \equiv \eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\nu\alpha}\eta^{\mu\beta} - 2/(D-2)\eta^{\mu\nu}\eta^{\alpha\beta}$ is the usual numerator of the graviton propagator on conserved sources. The origin of $K^{\mu\nu;\alpha\beta} \equiv \eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\nu\alpha}\eta^{\mu\beta} - \eta^{\mu\nu}\eta^{\alpha\beta}$ can be traced back to "spreading" the stu derivatives: for example, in the s channel we can write $tu = -1/2K^{\mu\nu;\alpha\beta}p^1_\mu p^2_\nu p^3_\alpha p^4_\beta$; the analogous identities for the other channels can be obtained by crossing. [It is convenient to define $s \equiv (p_1p_2), t \equiv (p_1p_3), u \equiv (p_1p_4)$, with $p_1 + p_2 = p_3 + p_4$. Note also the absence of (G,K) factors from (5), they are already incorporated into the BR's.] It is these identities that enabled us to write M_4's universally as $(stu)^{-1}L_4$'s: Originally the M_4 have a single denominator (from the intermediate specific exchange, s, t, or u channel); we uniformize them all to $(stu)^{-1}$ through multiplication of, say, s^{-1} by $(tu)^{-1}(tu)$. The extra derivatives thereby distributed in the numerators have the further virtue of turning all polarization tensors into curvatures and derivatives of forms, as we have indicated. It is worth noting that the matter $(\mathrm{BR})^2$ form (7) is in fact valid for any matter-matter four-point amplitude mediated by a graviton through minimal coupling, simply because of the $h_{\mu\nu}T^{\mu\nu}_{\mathrm{matt}}$ vertex and the $\mathrm{BR}_{\mathrm{matt}} \sim \partial^2 T_{\mathrm{matt}}$ relation. In particular, one can easily give natural extensions of the bosonic results both for the pure fermionic four-point function, since it too has an associated BR tensor $\sim\partial^2_{\alpha\beta}T^\psi_{\mu\nu}$ and for mixed Fermi-boson contributions. For example, the former resembles (7), with a $B^\psi B^\psi$ part as well as a $C^\psi C^\psi$ part from the nonminimal $\bar\psi\Gamma\psi F$ coupling in I_{11}. Indeed "current-current" terms are generically present for any amplitude generated by any gauge-field-current coupling, as evidenced by these ubiquitous CC contributions.

The remaining amplitudes are the form "bremsstrahlung" M^{FFFg} and the graviton-form scattering M_4^{Fg}. The M^{FFFg} amplitude represents radiation of a graviton from the CS term, i.e., contraction of the CS and $T^F_{\mu\nu}h^{\mu\nu}$ vertices by an intermediate A line, yielding

$$\begin{aligned} M_4^{FFFg} &= (stu)^{-1}L_4^{FFFg}, \\ L_4^{FFFg} &= -\frac{\kappa^2}{3}\,C^F_{\mu\nu\rho;\alpha\beta}C^{RF\mu\nu\rho}_{\alpha_1\beta_1}K^{\alpha\alpha_1;\beta_1\beta}, \end{aligned} \tag{8a}$$

$$C^{RF}_{\mu\nu\rho;\alpha\beta} \equiv 4\partial_\lambda(R^{\sigma[\lambda}_{(\alpha\beta)}F^{\mu\nu\rho]}_\sigma) - \frac{2}{3}\,R^{\sigma\lambda}_{(\alpha\beta)}\partial_\lambda F^{\mu\nu\rho}_\sigma. \tag{8b}$$

The off-diagonal current C^{RF} has antecedents in $N = 2$ $D = 4$ theory [11]; it is unique only up to terms vanishing on contraction with C^F. While its (8b) form is compact, there are more promising variants, with better conservation and trace properties. The $M^{Fg}, \sim \kappa^2R^2(\partial F)^2$, has three distinct diagrams: mixed T^FT^g mediated by the graviton; gravitational Compton amplitudes $\sim(hh)T_FT_F$ with a virtual A line, and finally the four-point contact vertex $FFhh$. The resulting M_4^{Fg} is again proportional to $(stu)^{-1}$,

$$\begin{aligned} M_4^{Fg} &= (stu)^{-1}L_4^{Fg}, \\ L_4^{Fg} &= \frac{\kappa^2}{3}\,(\tfrac{1}{4}B^g_{\mu\nu\alpha\beta}B^F_{\mu_1\nu_1,\alpha_1\beta_1}G^{\mu\mu_1;\nu\nu_1} \\ &\quad - C^{RF}_{\mu\nu\rho;\alpha\beta}C^{RF\mu\nu\rho}_{\alpha_1\beta_1})K^{\alpha\alpha_1;\beta_1\beta}, \end{aligned} \tag{9}$$

up to subleading terms involving traces. The complete bosonic invariant, $L_4 \equiv L_4^F + L_4^g + L_4^{Fg} + L_4^{FFFg}$, is not necessarily in its most unified form, but it suggests some intriguing possibilities, especially in the matter sector. For example, it is worth noting that the "C" currents can be unified into a unique current, which is the sum of the two, and their contributions to the invariant are simply its appropriate square. The corresponding attempt for the BR sector, unfortunately, does not quite work, at least with our choice of currents. We hope to return to this point elsewhere; instead we discuss some important consequences of the very existence of this invariant, where elegance of its presentation is irrelevant.

Consider first the issue of renormalizability of $D = 11$ supergravity. As we mentioned at the start, the work of [5] formally regarded as an analytic continuation to $D = 11$, states that the coefficient of a two-loop candidate counterterm is nonzero. Our result exhibits this invariant explicitly; taken together, they provide a compelling basis for the theory's nonrenormalizability. In this connection a brief review of the divergence problem may be useful. For clarity, we choose to work in the framework of dimensional regularization, in which only logarithmic divergences appear and, consequently, the local counterterm must have dimension zero (including dimensions of the coupling constants in the loop expansion). Now a generic gravitational loop expansion proceeds in powers of κ^2 (we will separately discuss the effect of the additional appearance of κ in the CS vertex). At one loop, one would have $\triangle I_1 \sim \kappa^0 \int dx^{11} \triangle L_1$; but there is no candidate $\triangle L_1$ of dimension 11, since odd dimension cannot be achieved by a purely gravitational $\triangle L_1$, except at best through a "gravitational" $\sim\epsilon\Gamma RRRR$ or "form-gravitational" $\sim\epsilon ARRRR$ CS term [13], which would violate parity: Thus, if present, they would represent an anomaly, and so be finite anyway. [In this connection we also note that the presence of a Levi-Civita symbol ϵ usually does not invalidate the use of dimensional regularization (or reduction) schemes to the order we need. In any case, our conclusions would also apply, in a more complicated way, in other regularization schemes that preserve SUSY.] The two-loop term would be $\triangle L_2 \sim \kappa^2 \int d^{11}x \triangle L_2$, so that $\triangle L_2 \sim [L]^{-20}$ which can be achieved (to lowest order in external lines) by $\triangle L_2 \sim \partial^{12} R^4$, where ∂^{12} means twelve explicit derivatives spread among the four curvatures. There are no relevant two-point $\sim\partial^{16}R^2$ or three-point $\sim\partial^{14}R^3$ terms because the R^2 can be field-redefined away into the Einstein action in its leading part (to h^2 order, E_4 is a total divergence in any dimension) while R^3 cannot appear by SUSY. This latter fact was first demonstrated in $D = 4$ but must therefore also apply in higher D simply by the brute force dimensional reduction argument. So the terms we need are, for their four-graviton part, L_4^g of (5) with twelve explicit derivatives. The companions of L_4^g in L_4^{tot} will simply appear with the same number of derivatives. It is easy to see that the additional ∂^{12} can be inserted without spoiling SUSY; indeed they appear as naturally as did multiplication by stu in localizing the M_4 to L_4: for example, ∂^{12} might become, in momentum space language, $(s^6 + t^6 + u^6)$ or $(stu)^2$. This establishes the structure of the four-point local counterterm candidate. As we mentioned, its coefficient (more precisely that of R^4) is known and nonvanishing at $D = 11$ when calculated in the analytic continuation framework of [5], which is certainly correct through $D = 10$. Consider lastly possible invariants involving odd powers of κ arising from the CS vertex. One might suppose that there is a class of one-loop diagrams, consisting of a polygon (triangle or higher) with form/graviton segments and appropriate emerging external bosons at its vertices, that could also have local divergences. The simplest example would be a form triangle with three external F-lines $\sim\kappa^3 \int d^{11}x\, \partial^9 \epsilon AFF$. This odd number of derivatives cannot be achieved and still yield a local scalar. This argument also excludes the one-loop polygon's gravitational or form extensions such as F^2R, FR^2, or even F^3R at this κ^3 level. One final comment: nonrenormalizability had always been a reasonable guess as the fate of $D = 11$ supergravity, given that it does not share the $N = 4$ YM SUSY theory's conformal invariance, because of the dimensional coupling constant κ. The opposite guess, however, that some special (M-theory related?) property of this "maximally maximal" model might keep it finite (at least to some higher order) could also have been reasonably entertained *a priori*, so this was an issue worth settling.

Perhaps more relevant to the future than the field theory's ultraviolet behavior is the light that can be shed on "nearby" properties of M theory, whatever its ultimate form. Given that $D = 11$ supergravity is its local limit, one would expect that there are local, "zero-slope" corrections that resemble the corrections that $D = 10$ string theories make to their limiting $D = 10$, supergravities. Among other things, various brane effects might become apparent in this way. Our local invariant (quite apart from the ∂^n factors inserted for counterterm purposes) is then the simplest such possible correction. As we saw, it shares with $D = 10$ zero-slope limits the same $t_8 t_8 R^4$ pure graviton term, but now acquires various additional form-dependent and spinorial contributions as well. A detailed version of our calculations will be published elsewhere.

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[1] E. Cremmer, B. Julia, and J. Scherk, Phys. Lett. B **79**, 409 (1978).
[2] M.B. Green, M. Gutperle, and P. Vanhove, Phys. Lett. B **409**, 177 (1997); M.B. Green, Nucl. Phys. (Proc. Suppl.) **68**, 242 (1998).
[3] J.G. Russo and A.A. Tseytlin, Nucl. Phys. **B508**, 245 (1997).
[4] S. Deser, J.H. Kay, and K.S. Stelle, Phys. Rev. Lett. **38**, 527 (1977).
[5] Z. Bern, L. Dixon, D.C. Dunbar, M. Perelstein, and J.S. Rozowsky, Nucl. Phys. **B530**, 401 (1998).
[6] S. Sannan, Phys. Rev. D **34**, 1749 (1986).
[7] A.E.M. van de Ven, Nucl. Phys. B **378**, 309 (1992).
[8] S.M. Christensen, S. Deser, M.J. Duff, and M.T. Grisaru, Phys. Lett. B **84**, 411 (1979).
[9] S.A. Fulling, R.C. King, B.G. Wybourne, and C.J. Cummins, *Classical Quantum Gravity* **9**, 1151 (1992).
[10] J.H. Schwarz, Phys. Rep. **89**, 223 (1982).
[11] S. Deser and J.H. Kay, Phys. Lett. B **76**, 400 (1978).
[12] J. Plefka, M. Serone, and A. Waldron, hep-th/9809070.
[13] M.J. Duff and D. Toms, in *Unification of the Fundamental Particle Interactions II* (Plenum Press, New York, 1982).