Suppression of Transverse Bunch Instabilities by Asymmetries in the Chamber Geometry

A. Burov[1] and V. Danilov[2]
[1]*Fermi National Accelerator Laboratory, Batavia, Illinois 60510-221*
[2]*Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830-8218*
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The wake forces produced by a beam bunch can be reduced by making the vacuum chamber cross section axially asymmetric. Furthermore, the asymmetry results in a betatron tune shift for particles in the tail of the bunch. As a result, transverse instabilities of the bunch should be significantly suppressed for an asymmetric vacuum chamber. [S0031-9007(99)08709-8]

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An ultrarelativistic charged particle generates electromagnetic fields behind it in the vacuum chamber. The net effect of these fields on a following charge is determined by integrating the force over a structure period of the vacuum chamber L. The integrated transverse force $\mathbf{F}$ caused by a slight offset $\mathbf{r}_0$ of the leading particle from the axis of a round chamber is conventionally expressed in terms of the wake function [1]:

$$\int_L \mathbf{F}\, ds = -q^2 \mathbf{r}_0 W(z)\,, \qquad (1)$$

where q is the particle's charge and z is the distance between head and tail particles. The wake fields (1) set limits on the beam stability, and usually efforts are made to reduce them.

It can be shown using a particular example that the electrodynamic response to the beam offset (1) can be much smaller for an axially asymmetric chamber. Assume for this purpose that there is a small cylinder inside the chamber which lies closer to the beam than any other part of the chamber. In this case, the surface charges and currents responsible for the wake fields reside mainly on the surface of the cylinder. The value and the location of these screening charges are insensitive to the beam offsets because of the small size of this cylinder; that is, the wake force (1) is suppressed.

However, the influence of the asymmetry is not limited by this circumstance. Generally, the linear approximation for the wake force (1) contains an additional term, proportional to the tail offset $\mathbf{r}$. This term creates a betatron tune shift along the bunch, but it vanishes for the round chamber. Thus, all the particles in the bunch are in resonance with each other if the chamber is round. For axially asymmetric structures, however, the wake fields not only drive the oscillations of the tail particles but also detune them from the resonance with the driving force [2]. Similar electrodynamic properties of external rf fields in asymmetric structures were used in Ref. [3], where it was proposed to utilize simultaneous accelerating and focusing to provide the acceleration and Balakin-Novokhatsky-Smirnov damping [4] in linacs.

The importance of the betatron tune spread along a bunch in a storage ring was shown in Ref. [5]. It was demonstrated that this spread, introduced by means of an rf quadrupole, has a stabilizing role for the transverse bunch oscillations. It is natural to suppose that the tune spread produced by the wake fields is a stabilizing factor as well. If so, the detuning part of the wake may increase the thresholds of bunch transverse instabilities.

Driving and detuning wakes.—The transverse wake forces are regular functions of the transverse offsets of the leading and trailing particles, $\mathbf{r}_0$ and $\mathbf{r}$, and can be expanded in terms of these offsets [2]. Assuming for simplicity mirror symmetry for at least one transverse axis and neglecting the nonlinear terms, the forces can be presented as follows:

$$\begin{aligned} \textstyle\int_L F_x\, ds &= -q^2 x_0 W_x(z) + q^2 x D(z)\,, \\ \textstyle\int_L F_y\, ds &= -q^2 y_0 W_y(z) - q^2 y D(z)\,, \end{aligned} \qquad (2)$$

where insignificant constant terms are omitted. The first terms on the right hand sides describe the forces caused by the offsets of the leading particle; the functions $W(z)$ can be referred to as the driving wake functions. The second terms are responsible for the tune shifts of the tail particle; the function $D(z)$ can be called the detuning wake function. The detuning terms for the x and y axes are described in Eqs. (2) by the single function $D(z)$. This is a consequence of the Maxwell equations, which can be demonstrated by means of the Panofsky-Wenzel theorem [6]

$$\frac{\partial}{\partial z} \int_L \mathbf{F}_\perp\, ds = \nabla_\perp \int_L F_\parallel\, ds \qquad (3)$$

for the harmonic function $\int_L F_\parallel\, ds$. As follows from the form of Eqs. (2), there is no detuning for chambers invariant over a 90° rotation; $D(z) = 0$ in this case. To give examples, wake functions caused by the wall resistivity are presented below for three simplified cases, namely, for a round chamber, then, for an infinite plane, and, finally, for a small cylinder. The three cases are sketched in Figs. 1a, 1b, and 1c.

A conventional approach to these problems includes Fourier transformation over the longitudinal coordinate and treatment of the wall resistivity as a perturbation. The fields caused by the resistivity are harmonic functions in the transverse plane xy; the tangential electric field

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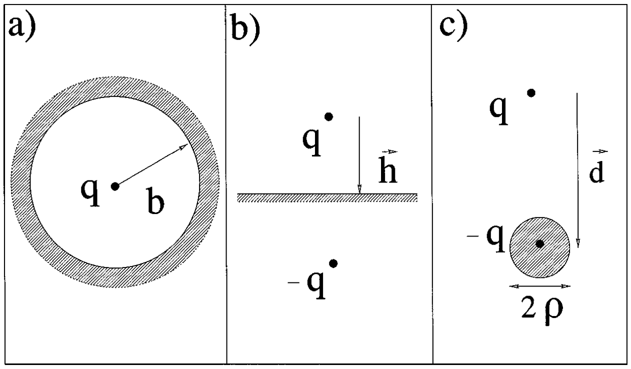

FIG. 1. Three types of beam surroundings.

satisfies the Leontovich boundary condition at the metal surface [7]:

$$\mathbf{E}_t = \zeta \mathbf{H}_t \times \mathbf{n}, \tag{4}$$

where $\mathbf{H}_t$ is the magnetic field calculated for perfect conductivity, and $\mathbf{n}$ is a unit normal vector pointed inside the wall. The so-called surface impedance

$$\zeta = \sqrt{\frac{\omega}{8\pi\sigma}}(1 - i) \tag{5}$$

is determined by the finite conductivity σ at the given field frequency ω; this factor is assumed to be small, $|\zeta| \ll 1$.

For the first case of the round vacuum chamber the wake functions can be found in Ref. [1]:

$$W_x(z) = W_y(z) = -\frac{2}{\pi b^3}\sqrt{\frac{c}{\sigma z}}L, \quad D(z) = 0, \tag{6}$$

where c is a velocity of light and b is the vacuum chamber radius.

For the case of the resistive plane, the boundary condition (4) can be presented in the following form:

$$E_\parallel = 2\zeta E_n', \tag{7}$$

where $E_\parallel$ is the longitudinal component and E_n' is the normal component of the image charge electric field. Taking into account that the field E_n' satisfies the Laplace equation, it can be concluded that Eq. (7) describes the resistive wall field $E_\parallel$ not only on the metal surface but everywhere in the volume. After the longitudinal field is found, the transverse wake forces are calculated from the Panofsky-Wenzel theorem (3), which gives

$$\mathbf{F}_\perp(z) = \frac{2q^2}{\pi}\sqrt{\frac{c}{\sigma z}}\nabla\frac{\mathbf{n}(\mathbf{r} - \mathbf{r}_0')}{(\mathbf{r} - \mathbf{r}_2')^2} \tag{8}$$

with $\mathbf{r}_0' = (x_0, -y_0)$ standing for the image charge position (the y axis here is assumed to be normal to the plane).

To find the wakes (2), the work corresponding to the force (8) has to be expanded over the leading and trailing particle offsets:

$$W_x(z) = W_y(z) = D(z) = -\frac{L}{2\pi h^3}\sqrt{\frac{c}{\sigma z}}, \tag{9}$$

where $h = |\mathbf{h}|$ is the distance from the beam to the plane. This geometry demonstrates the possibility for the detuning wake to be equal to the driving wake.

The final example treats the case of the beam passing along a small resistive cylinder, Fig. 1c; the detuning wake is shown to dominate here. Taking into account that the image charge is located at the position $\mathbf{r}' = \mathbf{r}\rho^2/r^2$ and assuming the cylinder radius ρ to be much smaller than the distance between the beam and the cylinder, $\rho \ll r_0 = |\mathbf{d}| = d$, the longitudinal electric field is

$$E_s = -\frac{2\zeta q}{\rho}\left(1 + \frac{\rho^2 \mathbf{r}\mathbf{r}_0}{r^2 r_0^2}\right) + C\ln(r/\rho), \tag{10}$$

which includes an arbitrary constant C. The small dipole term in the brackets reflects a weak dependence of the fields on the source position $\mathbf{r}_0$. To find the constant C, an additional boundary condition is needed. It can be assumed that this system is bounded by a conducting cylinder with the radius $R \gg r_0$. Then the constant C is found by equating the expression for the monopole part of E_s to zero at this remote surface, giving

$$C = \frac{2\zeta q}{\rho\ln(R/\rho)}.$$

Using (3) one can obtain the integrated transverse force and finally the wakes:

$$D(z) = -\frac{L}{\pi d^2 \rho \ln(R/\rho)}\sqrt{\frac{c}{\sigma z}} \tag{11}$$

$$W_x(z) = W_y(z) = -\frac{L\rho}{\pi d^4}\sqrt{\frac{c}{\sigma z}}. \tag{12}$$

Introducing the detuning factors $\kappa_x = D(z)/W_x(z)$, $\kappa_y = D(z)/W_y(z)$, the results for the various geometries are expressed as

$$\kappa_x = \begin{cases} 0, & \text{axial symmetry,} \\ 1, & \text{plane wall } \mathbf{n}_x \perp \mathbf{h}, \\ -1, & \text{plane wall } \mathbf{n}_x \parallel \mathbf{h}, \\ d^2/[\rho^2\ln(R/\rho)], & \text{small cylinder } \mathbf{n}_x \perp \mathbf{d}, \\ -d^2/[\rho^2\ln(R/\rho)], & \text{small cylinder } \mathbf{n}_x \parallel \mathbf{d}. \end{cases} \tag{13}$$

Here $\mathbf{n}_x$ is the unit vector in the x direction, and the vectors $\mathbf{h}, \mathbf{d}$ are defined in Fig. 1.

The driving wake function $W(z)$ for the small cylinder (12) is a factor $\propto \rho/d \ll 1$ smaller than the wake functions of the round chamber (6) or parallel plates (9) with the same aperture. This result demonstrates how the transverse instability can be suppressed by the decrease of the driving wake function. The detuning wakes work in the same direction; they damp the instability even more.

Finally, note that the plane wall result $\kappa_x = \pm 1$ is valid not only for the resistive wall wake. It applies as well to the wake generated by a longitudinal variation of the chamber cross section, when the cross section is a significantly elongated figure such as a rectangle or ellipse.

Coherent stabilization by the detuning wake.—The detuning wake modulates the betatron frequencies along the bunch. Such a modulation introduced by means of an rf quadrupole was studied in Ref. [5]. It was shown there that the transverse instabilities can be strongly damped in this case because the particles are kept out of resonance with each other. Following Ref. [5], the numerical results for the influence of the detuning wake on the transverse mode coupling instability are presented below.

Assuming the bunch to consist of particles with the same synchrotron amplitude a and a homogeneous distribution over the synchrotron phase (the so-called *air-bag model* [1]), the transverse equation of motion is written

$$\frac{d^2x(\phi)}{dt^2} + \omega_b^2 x(\phi) = F_x(\phi),$$

$$F_x(\phi) = -\frac{Nq^2}{2\pi\gamma mL}\int_{-|\phi|}^{|\phi|}[W(z)x(\phi') - D(z)x(\phi)]\,d\phi',$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \omega_s\frac{\partial}{\partial\phi}, \qquad z = a\cos\phi - a\cos\phi'. \tag{14}$$

Here ϕ is the synchrotron phase, ω_b and ω_s are, respectively, the betatron and the synchrotron frequencies, and N is the number of particles in the bunch. An expansion of the deviation $x(\phi)$ over the synchrotron harmonics

$$x(\phi) = e^{-i\omega_b t}\sum_{n=-\infty}^{+\infty} x_n e^{-i\alpha\omega_s t + in\phi}, \tag{15}$$

reduces Eq. (14) to a set of algebraic equations for the eigenvector components x_n and the eigenvalues α:

$$x_n(\alpha - n) = K\sum_{m=-\infty}^{+\infty} x_m K_{nm}, \qquad K = \frac{Nq^2}{2\pi^2\gamma m\omega_b\omega_s L},$$

$$K_{nm} = \int_0^{\pi}\cos(n\phi)\,d\phi\int_0^{\phi}W(z)\cos(m\phi')\,d\phi' - \int_0^{\pi}\cos[(n-m)\phi]\,d\phi\int_0^{\phi}D(z)\,d\phi', \tag{16}$$

where the influence of the coherent interaction is taken to be small in comparison with the transverse focusing, $\alpha\omega_s \ll \omega_b$. To resolve such equations, the sum has to be truncated to a finite number of the modes. In the numerical calculations, five modes were taken initially; then, the results were compared with 9- and 15-mode truncations. All the resistive wall wake functions have the following form:

$$W(z) = -Q/\sqrt{z}, \qquad D(z) = -\kappa Q/\sqrt{z},$$

where Q is the geometry factor. The examples for the detuning factor κ are given by Eq. (13).

Figure 2 presents plots for dimensionless eigenvalues α as functions of the dimensionless intensity parameter

$$\mathcal{I} = KQ\sqrt{a} \tag{17}$$

at various detuning factors κ. The dependence of the mode behavior on this factor is seen to be significant.

The mode coupling instability threshold is least for the symmetric case, $\kappa = 0$. At $\kappa = 1$, coupling and decoupling thresholds merge (degenerate case) and the beam is stable for any current. This result is valid for any mode truncation, so it appears to be an exact property. A small coupling-decoupling instability area appears again at higher κ.

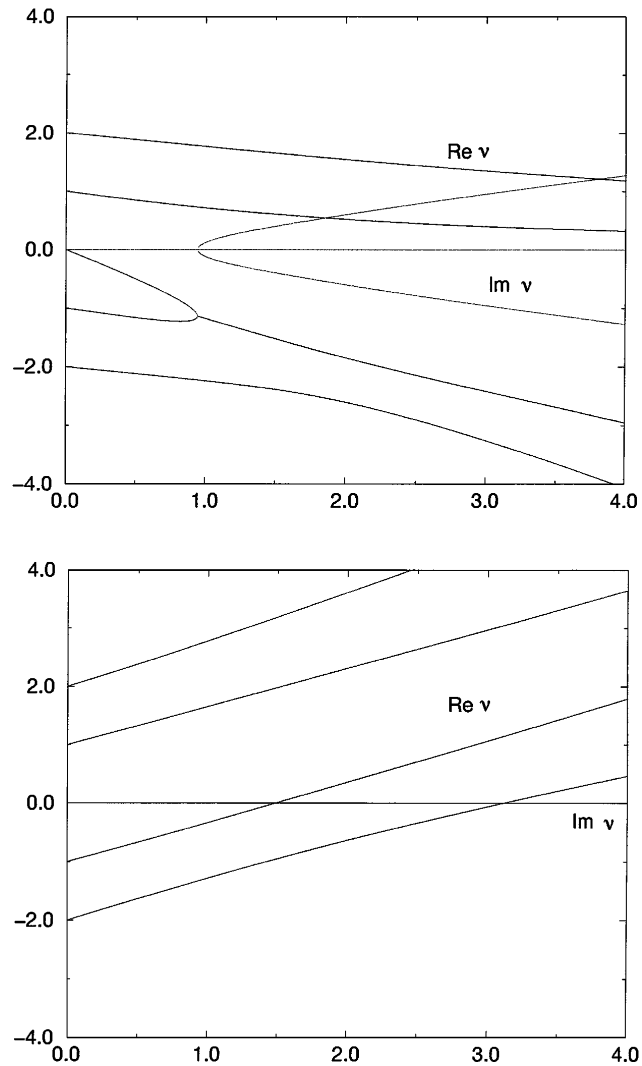

FIG. 2. Eigenvalues α versus the intensity parameter $\mathcal{I}$ for various detuning factors $\kappa = 0$ (top) and $\kappa = 1$ (bottom).

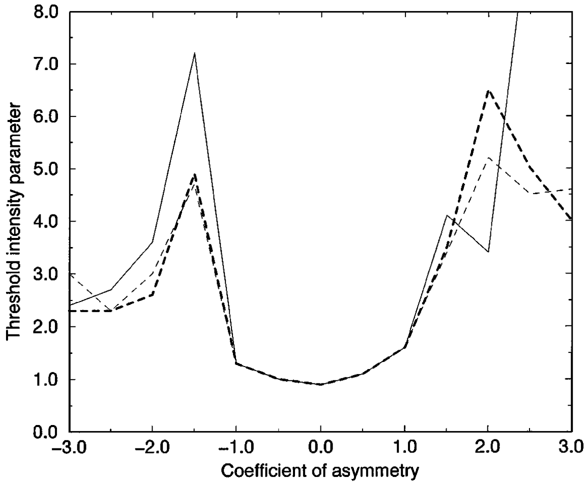

FIG. 3. Intensity threshold of the transverse mode coupling instability I versus the detuning factor κ for the 5-mode (solid line), 9-mode (thick dashed line), and 15-mode (thin dashed line) truncations.

Figure 3 shows the threshold behavior versus coefficient of asymmetry κ for the 5, 9, and 15 modes calculation. The instability threshold has its minimum for the symmetric chamber, $\kappa = 0$. Then it increases with the absolute value of the detuning factor and has two asymmetrical maxima at $\kappa \approx -1.5$ and $\kappa \approx 2$.

The results shown in this figure should be interpreted carefully, taking into account that an asymmetry not only introduces the detuning wake but also changes the driving wake. For instance, the thresholds for the resistive wall, examples (a) and (b) with $h = b$ (Fig. 1), differ approximately by a factor of $4 \times 1.5 = 6$, where the factor 4 is related to the driving wake damping and the factor 1.5 is the benefit due to the detuning for $\kappa = -1$, according to the Fig. 3.

Conclusions.—Only one kind of wake function, called here the driving wake function, has been conventionally taken into account for the beam stability analysis. It has been shown that this conventional approach can lead to significant underestimation of the beam stability thresholds.

It has been demonstrated here that the strength of the detuning wake function depends on the geometry of the chamber cross section. For any kind of wake, the ratio of the detuning function to the driving function is zero for round cross sections. This ratio is ± 1 when the cross section is an elongated rectangle or ellipse whose height may vary with the longitudinal coordinate. For the resistive wall wake, a geometry was found in which the detuning wake function is much higher than the driving wake. It is probable that cross sections with such a property exist for the wakes driven by geometry variations as well.

It can be concluded that for asymmetric vacuum chamber elements, which are usual in practice, the detuning wake function must be taken into account; conventional codes like MAFIA need to be improved accordingly.

For all of the examples here, an asymmetry-driven increase of the detuning wake combines with a decrease of the conventional wake; both of these factors favor beam stability. These properties of asymmetric cross sections look promising for the design of future accelerators.

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[1] A. Chao, *Physics of Collective Beam Instabilities in High Energy Accelerators* (John Wiley Sons, Inc., New York, 1993).
[2] S. Heifets, A. Wagner, and B. Zotter, "Generalized Impedances and Wakes in Asymmetric Structures," SLAC Report No. SLAC/AP110, 1998.
[3] W. Schnell, "Microwave Quadrupoles for Linear Colliders," CLIC Note 34, CERN Report No. CERN-LEP-RF/87-24, 1987.
[4] V. Balakin, A. Novokhatsky, and V. Smirnov, in *Proceedings of the 12th International Conference on High Energy Accelerators* (Fermilab, Batavia, 1983), p. 119.
[5] V. Danilov, "On Possibility to Increase the TMCI Threshold by RF Quadrupole," Fermilab Report No. TM-2033, 1997; Phys. Rev. ST Accel. Beams **1**, 041301 (1998).
[6] W. K. H. Panofsky and W. A. Wenzel, Rev. Sci. Instrum. **27**, 967 (1956).
[7] L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media* (Pergamon, Oxford, New York, 1984).