Renormalized SO(5) Symmetry in Ladders with Next-Nearest-Neighbor Hopping

E. Arrigoni and W. Hanke
Institut für Theoretische Physik, Universität Würzburg, D-97074 Würzburg, Germany
(Received 19 December 1997)

We study the occurrence of SO(5) symmetry in the low-energy sector of two-chain Hubbard-like systems by analyzing the flow of the running couplings (g-ology) under renormalization group in the weak-interaction limit. It is shown that SO(5) is asymptotically restored for low energies for rather general parameters of the bare Hamiltonian. This holds also with the inclusion of a next-nearest-neighbor hopping which explicitly breaks particle-hole symmetry if one accounts for a different single-particle weight for the quasiparticles of the two bands of the system. The physical significance of this renormalization SO(5) symmetry is discussed. [S0031-9007(99)08601-9]

PACS numbers: 71.10.–w, 11.10.Hi, 11.30.Ly

Recently, it was shown that both the two-dimensional (2D) Hubbard and the t-J model enjoy an approximate SO(5) symmetry [1,2], unifying antiferromagnetism (AF) with d-wave superconductivity (dSC) [3]. This symmetry principle gives a definite microscopic description of the AF $\rightarrow$ dSC transition as the chemical potential is varied. From the SO(5) multiplet structure verified by exact cluster diagonalization, one can see how the SO(5) superspin vector is rotated from the AF to dSC direction and can show that, at the critical chemical potential, the energy barrier ΔE between AF and dSC states is an order of magnitude smaller ($\sim J/10$) than the exchange coupling J, i.e., the natural parameter in the model. This finding is clearly of importance: While it is well established that both t-J and Hubbard models reproduce very successfully the "high"- and "medium"-energy physics of order U and $J \sim t^2/U$ of the cuprates, the low-energy content of order of the superconducting gap has so far eluded theoretical investigations. The variance ΔE of the multiplet splitting is a well-defined measure of "how good" the SO(5) symmetry is realized in the bare model considered for a given system size. It then seems natural to ask whether this deviation from exact SO(5) symmetry remains small or even vanishes if one goes to the infinite-volume limit and to lower energies, i.e., under renormalization-group (RG) flow.

In this Letter, we demonstrate that the low-energy regime of rather general Hubbard-type models, including finite-ranged interactions (provided they are weak) between one and two dimensions, i.e., ladders [4], is indeed dominated by the scaling towards an SO(5)-invariant model. This is a remarkable result, since it is the first model which is non-SO(5) invariant at the bare starting (microscopic) level, where the existence of SO(5) symmetry is proven for low energies. We first show that this holds for the *particle hole* (ph) symmetric case with nearest-neighbor hopping only [5,6]. In addition, we consider the effect of a next-nearest-neighbor (intrachain) hopping t_2 which produces an explicit breaking of the ph symmetry. We demonstrate that, also in this case, the system becomes SO(5) symmetric for low energies (at least up to order t_2^2) provided one considers a *renormalized* SO(5) transformation [7] which takes into account a different renormalization of the single-particle weight of the bonding and antibonding bands. This result is of importance because of two points: (i) It sheds light on the effect of the longer-range hoppings in general on the fate of SO(5) symmetry. This issue is also under intensive discussion in the case of 2D systems [2,8,9]: t_2 is known to strongly affect AF correlations and the Fermi-surface topology in the cuprates. (ii) In addition, the renormalized SO(5) representation, introduced here for the first time, is likely to be realized in a significantly larger class of physical systems allowing, for example, for asymmetries in the AF and dSC phases, such as different transition temperatures.

Specifically, we consider two coupled chains in the band representation with total "low-energy" action $S = S_0 + S_I$, where the noninteracting part S_0 can be written at a given point τ in the RG flow as

$$S_0 = \sum_{\mathbf{k},\sigma} C^\dagger_{\mathbf{k}\sigma} Z^{-1}_{k_\perp \tau}[i\omega - \nu V_{k_\perp \tau} k_\parallel] C_{\mathbf{k}\sigma} \,. \tag{1}$$

Here, $C_{\mathbf{k}\sigma}$ $(C^\dagger_{\mathbf{k}\sigma})$ are Grassmann variables associated with the destruction (creation) of a fermion, and σ is the spin. $\mathbf{k} \equiv \{i\omega, k_\perp, k_\parallel, \nu\}$ is a shorthand notation for the Matsubara frequency $i\omega$, and the momentum perpendicular $[k_\perp = (0, \pi)]$ and parallel $(k_\parallel)$ to the chain direction. The latter momentum is measured relative to the Fermi point $\nu k_{F,k_\perp}$ associated with the "band" $k_\perp$ with Fermi velocity $V_{k_\perp \tau}$, and $\nu = \pm 1$ refers to right- and left-moving fermions, respectively. This action is restricted to modes with $|k_\parallel| < \Lambda$ with $\Lambda = \Lambda_0 e^{-\tau}$. This weak-coupling RG method has previously been applied to obtain the phase diagram of the two-chain Hubbard model [4]. In order to study the occurrence of SO(5) symmetry in Hubbard-like models, it is convenient to rewrite the interaction part S_I of the action in terms of SO(5)-invariant and SO(5)-breaking terms [10]. Defining the SO(5) spinor as in Ref. [11],

$$\Psi_{\mathbf{k}} \equiv \{C_{\mathbf{k}\uparrow}, C_{\mathbf{k}\downarrow}, -\cos k_\perp C^\dagger_{\bar{\mathbf{k}}\uparrow}, -\cos k_\perp C^\dagger_{\bar{\mathbf{k}}\downarrow}\}^t, \tag{2}$$

where $\overline{\mathbf{k}}$ stands for $\{-i\omega, -k_\perp + \pi, -k_\parallel, \nu\}$ and the interacting action S_I can be shown to be expressable in terms of the 4×4 charge-rotation Dirac matrix Γ^{15} [11]:

$$S_I = \frac{1}{2\beta N} \sum_{\mathbf{k}_1,\ldots,\mathbf{k}_4}{}' g_0(\cdots)\Psi^\dagger_{\mathbf{k}_1}[1 + a(\cdots)\Gamma^{15}]\Psi_{\mathbf{k}_2} \times \Psi^\dagger_{\mathbf{k}_3}[1 + b(\cdots)\Gamma^{15}]\Psi_{\mathbf{k}_4} + (k_1, k_3) \leftrightarrow (k_2, k_4)\,. \tag{3}$$

Here, $(\cdots)$ represents the sets of variables on which the couplings g_0, a, and b depend. As usual, each coupling can be considered as dependent only on the Fermi momenta closest to where the corresponding process takes place, i.e., the $(\cdots)$ are labeled by $(\nu_1, k_{\perp 1}; \nu_2, k_{\perp 2} \,|\, \nu_3, k_{\perp 3}; \nu_4, k_{\perp 4})$. Moreover, $\sum'$ denotes a sum with conservation of frequency and lattice momentum.

The SO(5) symmetric part $S_I^{(0)}$ of S_I is given by Eq. (3) with $a(\cdots) = b(\cdots) = 0$ [12,13]. For a general SO(5)-invariant action $S_I^{(0)}$, it can be shown that one can restrict oneself to the seven independent couplings $g_0^{(4)}$, $g_0^{(4;0-\pi)}$, $g_0^{(2)}$, $g_0^{(2;0-\pi)}$, $g_0^{(1)}$, $g_0^{(1t,0-\pi)}$, and $g_0^{(1,0-\pi)}$, defined in analogy with the g-ology formalism [14–16]. The SO(5)-breaking term $S_I^{(1)}$ of the interacting action with ph symmetry can be demonstrated to be the term proportional to $g_0(\cdots)a(\cdots)b(\cdots)$ in (3), and thus we define the corresponding SO(5)-breaking couplings as $g_1(\cdots) \equiv g_0(\cdots)a(\cdots)b(\cdots)$. In this case, we can restrict ourselves to only five independent couplings (it can be shown that the others are redundant), namely, $g_1^{(4)}$, $g_1^{(2)}$, $g_1^{(1)}$, $g_1^{(1t;0-\pi)}$, and $g_1^{(1;0-\pi)}$ [14]. At half-filling and with $t_2 = 0$, the Hamiltonian is ph symmetric, since the velocities of the two bands $V_{k_\perp=0}$ and $V_{k_\perp=\pi}$ are equal. Since the ph-breaking terms in (3) are proportional to Γ_{15}, one can set in the ph-symmetric case $a(\cdots) = -b(\cdots)$, and consider the RG flow of the couplings $g_0^{()}$ and $g_1^{()}$ only.

To begin with, we have evaluated the RG equations for the $g_0^{()}$ and $g_1^{()}$ couplings [16] at one loop by using the standard g-ology procedure (cf. Refs. [4,15]), including the interband umklapp processes. As already shown for the two-chain case [4], the system always flows to strong coupling, i.e., the g's diverge at a value of $\tau = \tau_c \propto 1/g(\tau = 0)$, $g(\tau)$ being the scale of the interaction [proportional to the maximum of all $g_i^{()}(\tau)$]. This signals an instability towards some gapped state. Nevertheless, the striking new result is that, even in non-SO(5)-invariant system, such as, e.g., the Hubbard model, the SO(5)-invariant couplings $g_0^{()}$ dominate with respect to the symmetry-breaking couplings $g_1^{()}$, when approaching τ_c [17]. This can be seen from the ratio of the maxima of these two types of couplings, $g_1^{(\max)}(\tau)/g_0^{(\max)}(\tau)$, going to zero, as shown in the inset of Fig. 1. Here, $g_i^{(\max)}(\tau)$ is defined as the largest absolute value, and thus the scale of the couplings of a given type i ($i = 0, 1, ph$) at a given τ. This result implies that *the low-energy modes of the system can be described by an effective* SO(5)-*symmetric action, at least for sufficiently small* $g(\tau = 0)$ [5,17]. In fact, we have verified that this occurs for very general values of the Hamiltonian, including longer-ranged interactions.

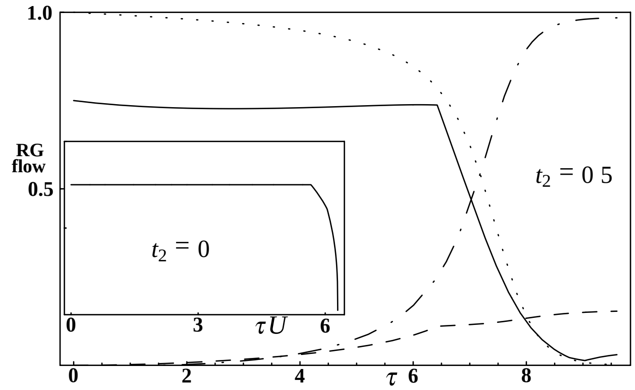

FIG. 1. RG flow of the SO(5)-breaking terms $3g_1^{(\max)}/g_0^{(\max)}$ (solid line), $3g_{ph}^{(\max)}/g_0^{(\max)}$ (dashed line), $\Delta V_\tau/\Delta V_{\tau=0}$ (dotted line), and ΔZ_τ^{-1} (dash-dotted line), as a function of $\tau = -\log(\Lambda/\Lambda_0)$ for $U = 1$, $t = t_\perp = 1$, $t_2 = 0.5$, and half-filling. Here, $\Delta V_\tau = V_{0\tau} - V_{\pi\tau}$, and $\Delta Z_\tau^{-1} \equiv [(Z_{0\tau}^{-1}/Z_{\pi\tau}^{-1})^2 - 1]/(V_{0\tau=0}/V_{\pi\tau=0} - 1)$. The inset shows $3g_1^{(\max)}/g_0^{(\max)}$ vs τU for $t_2 = 0$.

A next-nearest-neighbor hopping t_2 breaks ph symmetry explicitly and requires the introduction of a ph-breaking interaction $S_I^{(ph)}$. Here, $a(\cdots) \neq -b(\cdots)$ in Eq. (3), and thus we need extra couplings $g_{ph}(\cdots)$, which we have defined as $g_{ph}(\cdots) = g_0(\cdots)[a(\cdots) + b(\cdots)]/2$. In this case, one can show that the couplings can be restricted to $g_{ph}^{(4)}$, $g_{ph}^{(2)}$, and $g_{ph}^{(1)}$ [14,16]. The initial ($\tau = 0$) source of ph-symmetry breaking for $t_2 \neq 0$ stems from the noninteracting part of the action S_0, due to the difference of the Fermi velocities ΔV_0 of the two bands. In the following, we will show that SO(5) symmetry is restored [at least up to $\mathcal{O}(t_2)^2$], at low energies, even in the presence of this ph [and thus SO(5)]-breaking term.

These results are obtained on the basis of two complementary RG calculations. Calculation (i) considers the RG flow of the self-energy parameters $V_{k_\perp\tau}$ and $Z_{k_\perp\tau}^{-1}$ at two loops, and of the coupling parameters $g_i^{()}(\tau)$ at one loop, taking the τ dependence of all of the parameters at each RG step fully into account. This first calculation (i), although not rigorously controlled (see below), is motivated by the fact that we are interested in studying the RG flow of the self-energy, which is the leading symmetry-breaking term when t_2 is included [16]. In a second calculation (ii), we will show how our main results about the renormalized $\widetilde{\mathrm{SO}}(5)$ symmetry obtained within this first procedure can also be achieved in an alternative, more controlled way, where we consider only the renormalization of the $g_i^{()}$. Nevertheless, the first calculation (i) is instructive, in order to provide a physical interpretation for the single-particle renormalization factors Z as discussed in the conclusions. Indeed, in procedure (i) the Z factors, and thus the renormalized SO(5) transformation, derive naturally from the RG flow, while in (ii) they are introduced right at the outset.

In calculation (i), the relevant part of the renormalized action has the form of Eq. (1) with τ-dependent Fermi velocities and single-particle weights.

The flow of these parameters is shown in Fig. 1. As the bare ($\tau = 0$) Hamiltonian, we take the half-filled Hubbard ladder with isotropic intrachain and interchain hoppings $t = t_\perp = 1$, next-nearest-neighbor hopping $t_2 = 0.5$ (corresponding to $\Delta V_{\tau=0} \approx 1.9$), and $U = 1$. Actually, we have verified that the results we discuss below are rather general and hold also in the presence of anisotropy $t_\perp \neq t$ and nearest-neighbor interactions ($\gtrsim -U$). The $t_2 = 0$ case [6], discussed above, is plotted in the inset for comparison.

For the $t_2 \neq 0$ case, ΔV_τ (Fig. 1, dotted line) flows to zero, but ΔZ_τ^{-1} (Fig. 1, dash-dotted line, initially zero) scales to unity. Therefore, the initial asymmetry between the bands due to the different Fermi velocities is transferred into a difference in the single-particle weights Z, such that, for large τ, $Z_{0\tau}^{-1}/Z_{\pi\tau}^{-1} \to \sqrt{V_{0\tau=0}/V_{\pi\tau=0}}$. In order to restore the coefficient of the $i\omega$ term in Eq. (1) to unity, the standard procedure [15] is to reabsorb this renormalization into the definition of new Grassmann variables $\tilde{C}_{\mathbf{k}\sigma}$ and to set $\sqrt{Z_{k_\perp\tau}^{-1}}\, C_{\mathbf{k}\sigma} = \tilde{C}_{\mathbf{k}\sigma}$. This standard procedure is dictated by the requirement to identify the *canonical* Fermi operators with correct anticommutation relations, as will be discussed later. In this way, the noninteracting part of the (renormalized) action will again be symmetric under the exchange of the two bands (and thus SO(5) symmetric in the new fields). This transformation, however, also affects the interaction part, and one should consistently redefine the renormalized SO(5) spinor in Eq. (2) to $\tilde{\Psi}_{\mathbf{k}}$, whereby the $C_{\mathbf{k}\sigma}$ are again replaced with the $\tilde{C}_{\mathbf{k}\sigma}$. The couplings defined in this way are of course different from the original ones and we will distinguish them with a tilde, i.e., $g_i^{()} \to \tilde{g}_i^{()}$. The remarkable result is that *the transformation which makes the noninteracting part of the action* SO(5) *symmetric also restores* SO(5) *in the interacting part.* This is demonstrated in Fig. 2a, which plots the ratio of the $\tilde{g}_i^{(\max)}$ as a function of the flow parameter τ. We note that the non-SO(5) couplings $\tilde{g}_1^{(\max)}$ and $\tilde{g}_{ph}^{(\max)}$ all flow to zero (relative to the $\tilde{g}_0^{(\max)}$). Thus, at large τ, SO(5) symmetry is restored for low energies in the "$\sim$" basis. However, at the energy scale where ΔV_τ starts to decrease and ΔZ_τ^{-1} starts to become finite ($\tau \sim 7$ in Fig. 1), the renormalized couplings can be shown to become large and the weak-coupling expansion is no longer controlled, as anticipated.

To support this physically appealing yet uncontrolled calculation, we verify, in terms of a *controlled* RG calculation [17] (i.e., at one loop), that $\widetilde{\mathrm{SO}}(5)$ symmetry is indeed recovered at least up to $\mathcal{O}(t_2^2)$. This alternative derivation clarifies why the single-particle weights

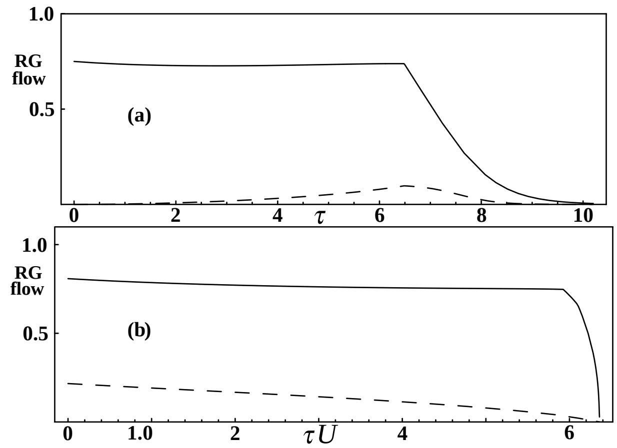

FIG. 2. $\widetilde{\mathrm{SO}}(5)$-breaking couplings $3\tilde{g}_1^{(\max)}/\tilde{g}_0^{(\max)}$ (solid line) and $3\tilde{g}_{ph}^{(\max)}/\tilde{g}_0^{(\max)}$ (dashed line) as a function of τ. (a) shows the results of the RG procedure (i) and (b) shows the results of (ii) as defined in the text. The parameters are the same as in Fig. 1.

renormalize proportionally to $(\sqrt{V_{\tau=0}})^{-1}$, as obtained asymptotically in the two-loop calculation. In this RG procedure (ii), we start from the action $S_0 + S_I$ and carry out the transformation $\tilde{T}$ on the Grassmann variables right *at the outset,* where $\tilde{T}$ is defined as $\tilde{T} C_{i\omega,k_\|,k_\perp} = 1/\sqrt{u_{k_\perp}}\, \tilde{C}_{i\omega',k_\|',k_\perp'}$, with $i\omega' = i\omega/u_{k_\perp}$ and $k_\|' = k_\| u_{k_\perp}$, and $u_{k_\perp} = \sqrt{V_{k_\perp}}$ [18]. Such a transformation, which is always possible with Grassmann variables, is motivated by our first calculation (i). By changing the sum over $i\omega$ and $k_\|$ into a sum over $i\omega'$ and $k_\|'$ separately for each band, the noninteracting part of the action again recovers its explicit SO(5) symmetry. Furthermore, by defining a new SO(5) spinor $\tilde{\Psi}_{\mathbf{k}}$ in terms of the $\tilde{C}$, we obtain new couplings $\tilde{g}_0^{()}$, $\tilde{g}_1^{()}$, and $\tilde{g}_{ph}^{()}$, as in step (i). In Fig. 2b, we show the corresponding RG flow of the ratios of the $\tilde{g}_i^{(\max)}$. With increasing RG parameter τ the ph-symmetry breaking term $\tilde{g}_{ph}^{(\max)}/\tilde{g}_0^{(\max)}$ vanishes (full line), while the $\widetilde{\mathrm{SO}}(5)$-breaking term $\tilde{g}_1^{(\max)}/\tilde{g}_0^{(\max)}$ goes to a finite but rather small value [19] (Fig. 2, dashed line). The $\widetilde{\mathrm{SO}}(5)$ symmetry thus is recovered up to a very high degree of precision for low energies [5]. In contrast with the results of procedure (i), the result of (ii) is *controlled* for small $g(\tau = 0)$ [17,20]. In this way, we have shown *in a controlled way that* $\widetilde{\mathrm{SO}}(5)$ is restored for low energies *at least* up to order t_2^2 for small $g(\tau = 0)$. Our two-loop calculation (i) further suggests that even this $\widetilde{\mathrm{SO}}(5)$-breaking term of order t_2^2 might be removed by the self-energy renormalization.

The renormalized $\widetilde{\mathrm{SO}}(5)$ symmetry introduced here, and the related renormalization of the single-particle weights $Z_{k_\perp\tau}$, can be understood in terms of a simplified scheme, which renormalizes the Hamiltonian, by restricting the Hilbert space to a subspace with energy ω smaller than a certain cutoff $\omega_0 \propto \Lambda_0 \exp(-\tau)$. In the restricted subspace, the total integrated spectral density (which we identify with $Z_{k_\perp \tau}$) will be less than 1. Since the spectral sum rule identifies $Z_{k_\perp \tau}$ with the anticommutator of the Fermi operators $C_{\mathbf{k}}$, the canonical Fermi operators with anticommutator equal to 1 in this subspace are the transformed field operators $\widetilde{C}_{\mathbf{k}}$ introduced above [21].

In conclusion, we have shown that the effective low-energy action (or Hamiltonian) of a ladder with weak interaction is asymptotically SO(5) symmetric [5]. With the inclusion of a next-nearest-neighbor hopping t_2 the action is invariant under a generalized $\widetilde{\mathrm{SO}}(5)$ transformation [7], which performs a "stretched" SO(5) rotation of the order parameters. Physically, this $\widetilde{\mathrm{SO}}(5)$ symmetry may be present in the low-energy sector of a *larger* and more *generic* class of physical systems than the ordinary SO(5). Moreover, since this stretched rotation does not conserve the norm of the superspin (order-parameter) vector [3], a renormalized $\widetilde{\mathrm{SO}}(5)$ theory can possibly admit asymmetries between the antiferromagnetic and superconducting phases, such as, for example, the difference in T_c's [3,8].

We are grateful to S. C. Zhang, B. Brendel, and M. G. Zacher for many useful discussions. E. A. was supported by the EC-TMR program ERBFMBICT950048 and W. H. by FORSUPRA and BMBF (05 605 WWA 6).

[1] S. Meixner, W. Hanke, E. Demler, and S.-C. Zhang, Phys. Rev. Lett. **79**, 4902 (1997).

[2] R. Eder, W. Hanke, and S. C. Zhang, Phys. Rev. B **57**, 13 781 (1998).

[3] S.-C. Zhang, Science **275**, 1089 (1997).

[4] M. Fabrizio, Phys. Rev. B **48**, 15 838 (1993); L. Balents and M. P. A. Fisher, Phys. Rev. B **53**, 12 133 (1996).

[5] The results of our RG calculations for $t_2 = 0$ imply that the low-energy asymptotic behavior of any correlation function $\mathcal{F}[C_{i\omega,k_\parallel,k_\perp}]$ (functional of the fermion fields) is identical with its SO(5)-transformed one, $\mathcal{R}\mathcal{F}[C_{i\omega,k_\parallel,k_\perp}]$, up to relative corrections vanishing for small $g(\tau = 0)$. For example, the ratio of the spin and the charge gap $\Delta_s/\Delta_c \to 1$ for $g(\tau = 0) \to 0$. In the *ph*-broken case, this holds for correlation functions transformed under $\widetilde{\mathcal{R}}$ [7] up to corrections of order t_2^2.

[6] For the *ph*-symmetric case, see also H.-H. Lin, L. Balents, and M. P. A. Fisher, Phys. Rev. B **58**, 1794 (1998).

[7] Throughout this paper we denote by $\widetilde{\mathrm{SO}}(5)$ the renormalized SO(5) symmetry, whenever the corresponding transformation $\widetilde{\mathcal{R}}$ is expressed in terms of the original fermion fields C. The transformation $\widetilde{\mathcal{R}}$ is obtained by first performing the $\widetilde{T}$ transformation (described in the text), then the SO(5) transformation on the new fields [5], and then transforming back to the original fermion fields, i.e., $\widetilde{\mathcal{R}} = \widetilde{T}^{-1}\mathcal{R}\widetilde{T}$. To illustrate what invariance under $\widetilde{\mathcal{R}}$ means, let's take the noninteracting case. The Green's function of the band $k_\perp = 0$ is $G(i\omega, k_\parallel, k_\perp = 0) = 1/(i\omega - V_0 k_\parallel)$. We choose a particular SO(5) transformation for $\mathcal{R}$, namely, the exchange of the two bands. We thus have $\widetilde{\mathcal{R}}G(i\omega, k_\parallel, 0) = \widetilde{T}^{-1}\mathcal{R}\, V_0^{-1/2} G(i\omega/\sqrt{V_0}, k_\parallel\sqrt{V_0}, 0) = \widetilde{T}^{-1} V_0^{-1/2} \times G(i\omega/\sqrt{V_0}, k_\parallel\sqrt{V_0}, k_\perp = \pi) = (V_0/V_\pi)^{-1/2} \times G(i\omega/\sqrt{V_0/V_\pi}, k_\parallel\sqrt{V_0/V_\pi}, \pi) = 1/(i\omega - V_0 k_\parallel)$. Our RG calculation additionally shows that the invariance under $\widetilde{\mathcal{R}}$ also holds (asymptotically) for the interacting case, which is a nontrivial result.

[8] C. L. Henley, Phys. Rev. Lett. **80**, 3590 (1998).

[9] S.-C. Zhang, cond-mat/9709289.

[10] Compare also D. G. Shelton and D. Sénéchal, Phys. Rev. B **58**, 6818 (1998).

[11] We use the notation of S. Rabello, H. Kohno, E. Demler, and S.-C. Zhang, Phys. Rev. Lett. **80**, 3586 (1998).

[12] One can show that it is not necessary to introduce tensor interactions for the SO(5)-invariant action (cf. [11]), as they are already included in the scalar form (3) due to the Fierz identity (see also Ref. [13]).

[13] D. Scalapino, S.-C. Zhang, and W. Hanke, Phys. Rev. B **58**, 443 (1998).

[14] The couplings depend on $(\nu_1, k_{\perp_1}; \nu_2, k_{\perp_2} \mid \nu_3, k_{\perp_3}; \nu_4, k_{\perp_4})$ and are defined in analogy with the g-ology of Ref. [15], as $g_i^{(4)} \equiv g_i(+,0;+,0 \mid +,0;+,0)$, $g_i^{(4;0-\pi)} \equiv g_i(+,0;+,\pi \mid +,\pi;+,0)$, $g_i^{(2)} \equiv g_i(+,0;+,0 \mid -,0;-,0)$, $g_i^{(2;0-\pi)} \equiv g_i(+,0;+,\pi \mid -,0;-,\pi)$ for the forward, and $g_i^{(1)} \equiv g_i(+,0;-,0 \mid -,0;+,0)$, $g_i^{(1t;0-\pi)} \equiv g_i(+,0;-,\pi \mid -,0;+,\pi)$, and $g_i^{(1;0-\pi)} \equiv g_i(+,0;-,\pi \mid -,\pi;+,0)$, where $i = 0$, 1, or *ph* (see also text and [16]).

[15] J. Sòlyom, Adv. Phys. **28**, 201 (1979).

[16] The full set of RG equations can be found in the following web site: http://theorie.physik.uni-wuerzburg.de/~arrigoni/ladder/rgequations.html.

[17] In the one-loop calculation, the g_0 couplings start to dominate with respect to the SO(5) symmetry-breaking couplings g_1 at, say, $\tau = \tau^*$ with $\tau^* \gg 1$. For small enough $g(\tau = 0)$, τ^* can be chosen sufficiently far from $\tau_c \propto g(\tau = 0)^{-1}$, such that one is still in the perturbative regime $g(\tau^*) \ll 1$. See, also E. Arrigoni, Phys. Status Solidi B **195**, 425 (1996). This one-loop calculation is thus sufficient to show that the ratio $g_1^{(\max)}(\tau)/g_0^{(\max)}(\tau)$ vanishes for $g(\tau = 0) \to 0$ for any $\tau > \tau^*$.

[18] In order not to confuse the reader, we have chosen for this new transformation the same symbol "$\sim$" as for the one performed in the procedure (i), since they have the same physical meaning. The two transformations differ for large τ by (a) an overall constant and (b) by a scaling of the frequencies and of the momenta. Notice, however, that, due to their frequency and momentum independence, the $\tilde{g}_i^{()}$ couplings defined in (ii) are equivalent to the $\tilde{g}_i^{()}$ defined in (i) apart from a common factor.

[19] We have verified that, as a function of t_2 and for different U, $\tilde{g}_1^{(\max)}/\tilde{g}_0^{(\max)} \to \approx 0.1 t_2^2$.

[20] The only remaining explicitly non-SO(5)-invariant term in the renormalized action is due to the different momentum and frequency conservation for the $k_\perp = 0$ and $k_\perp = \pi$ bands arising from the $\widetilde{T}$ transformation. However, one can show that this term also breaks the SO(5) symmetry by an amount of order t_2^2 at most.

[21] In the case of the ladders, where $Z_{k_\perp \tau}$ vanishes at τ_c due to a spin gap, this argument can be applied up to, say, $\tau = \tau^*$ for which $Z_{k_\perp \tau}$ is still appreciable and the SO(5) symmetry is still achieved (see [17]).