End Point of the Hot Electroweak Phase Transition

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We study the hot electroweak phase transition by four-dimensional lattice simulations and give the phase diagram. A continuum extrapolation is done. We find that the phase transition is first order for Higgs-boson masses $m_H < 66.5 \pm 1.4$ GeV. Above this end point a rapid crossover occurs. Our result agrees with that of the dimensional reduction approach. It also indicates that the fermionic sector of the standard model (SM) may be included perturbatively. We obtain that the end point in the SM is 72.4 ± 1.7 GeV. Thus, the LEP Higgs-boson mass lower bound excludes any electroweak phase transition in the SM. [S0031-9007(98)08047-8]

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The observed baryon asymmetry is finally determined at the electroweak phase transition (EWPT) [1]. The understanding of this asymmetry needs a quantitative description of the phase transition. Unfortunately, the perturbative approach breaks down for the physically allowed Higgs-boson masses (e.g., $m_H > 70$ GeV) [2]. In order to understand this nonperturbative phenomenon a systematically controllable technique is used, namely, lattice Monte Carlo (MC) simulations. Since merely the bosonic sector is responsible for the bad perturbative features (due to infrared problems) the simulations are done without the inclusion of fermions. The first results dedicated to these questions were obtained on four-dimensional (4D) lattices [3]. Soon after, simulations of the reduced model in three dimensions were initiated as another approach [4]. This technique contains two steps. The first is a perturbative reduction of the original 4D model to a three-dimensional (3D) one by integrating out the heavy degrees of freedom. The second step is the nonperturbative analysis of the 3D model on the lattice, which is less CPU-time consuming than the MC simulation in the 4D model. The comparison of the results obtained by the two techniques is not only a useful cross-check on the perturbative reduction procedure for heavy bosonic modes, but also could give an indication that the fermions, which behave similarly to the heavy bosonic nodes, might be included perturbatively.

In the recent years exhaustive studies have been carried out both in the 4D [5] and in the 3D [6] sectors of the problem. These works determined several cosmologically important quantities such as the critical temperature (T_c), interface tension (σ), and latent heat ($\Delta\epsilon$).

Previous works show that the strength of the first order EWPT gets weaker as the mass of the Higgs-boson increases. Actually the line of the first order phase transitions separating the symmetric and Higgs phases on the m_H-T_c plane has an end point, $m_{H,c}$. There are several direct and indirect evidences for that. In four dimensions at $m_H \approx 80$ GeV the EWPT turned out to be extremely weak, even consistent with the no phase transition scenario on the 1.5σ level [7]. 3D results show that for $m_H > 95$ GeV no first order phase transition exists [8] and more specifically that the end point is $m_{H,c} \approx 67$ GeV [9,10]. In this Letter we present the analysis of the end point on 4D lattices. We study the thermodynamical limit of the first Lee-Yang zeros of the partition function [9,10]. In order to get rid of the finite lattice spacing effects a careful extrapolation to the continuum limit is performed. The end point value of the SU(2)-Higgs model is perturbatively transformed to the full standard model (SM).

We will study the 4D SU(2)-Higgs lattice model on asymmetric lattices, i.e., lattices with different spacings in temporal (a_t) and spatial (a_s) directions. Equal lattice spacings are used in the three spatial directions ($a_i = a_s,\ i = 1,2,3$) and another one in the temporal direction ($a_4 = a_t$). The asymmetry of the lattice spacings is given by the asymmetry factor $\xi = a_s/a_t$. The different lattice spacings can be ensured by different coupling strengths in the action for timelike and spacelike directions. The action reads

$$S[U,\varphi] = \beta_s \sum_{sp}\left(1 - \frac{1}{2}\,\mathrm{Tr}U_{pl}\right) + \beta_t \sum_{tp}\left(1 - \frac{1}{2}\,\mathrm{Tr}U_{pl}\right)$$
$$+ \sum_x \left\{ \frac{1}{2}\,\mathrm{Tr}(\varphi_x^+\varphi_x) + \lambda\left[\frac{1}{2}\,\mathrm{Tr}(\varphi_x^+\varphi_x) - 1\right]^2 - \kappa_s \sum_{\mu=1}^{3} \mathrm{Tr}(\varphi_{x+\hat{\mu}}^+ U_{x,\mu}\,\varphi_x) - \kappa_t \mathrm{Tr}(\varphi_{x+\hat{4}}^+ U_{x,4}\varphi_x)\right\}, \quad (1)$$

where $U_{x,\mu}$ denotes the SU(2) gauge link variable, U_{sp} and U_{tp} the path-ordered product of the four $U_{x,\mu}$ around a space-space or space-time plaquette, respectively; φ_x stands for the Higgs field. It is useful to introduce the hopping parameter $\kappa^2 = \kappa_s \kappa_t$ and $\beta^2 = \beta_s \beta_t$. The anisotropies $\gamma_\beta^2 = \beta_t/\beta_s$ and $\gamma_\kappa^2 = \kappa_t/\kappa_s$ are functions of the asymmetry ξ. These functions have been determined perturbatively [11] and nonperturbatively [12] demanding the restoration of the rotational symmetry in different channels. In this paper we use the asymmetry parameter $\xi = 4.052$, which gives $\gamma_\kappa = 4$ and $\gamma_\beta = 3.919$. The reason for choosing $\xi > 1$ is that while a_t fixes the scale of the temperature, a_s determines the number of lattice points for a given (necessarily large in our case) physical volume. The number of lattice points is constrained by computer resources. Thus, simulation on asymmetric lattices is of principal importance for the present investigation. Details of the simulation techniques can be found in [5].

We have performed our simulations on finer and finer lattices, moving along the lines of constant physics (LCP). In our case there are three bare parameters (κ, β, λ). The bare parameters are chosen in a way that the zero temperature renormalized gauge coupling g_R is held constant and the mass ratio for the Higgs- and W-bosons $R_{\mathrm{HW}} = m_H/m_W$ corresponds to the Higgs mass at the end point of first order phase transitions: $R_{\mathrm{HW},c}$. These two conditions determine a LCP as a one-dimensional subspace in the original space of bare parameters. The position on the LCP gives the lattice spacing a. As the lattice spacing decreases, $R_{\mathrm{HW},c} \rightarrow R_{\mathrm{HW,cont}}$. A schematic illustration is shown in Fig. 1. The LCP (solid line) defined by the end point represents the above idea. The short dashed lines give the projections to the λ-κ plane. The increasing numbers on the LCP show the temporal extensions of the lattice, thus corresponding to smaller and smaller lattice spacings. The dotted lines represent phase transition points of theories with fixed renormalized g^2 and L_t but different R_{HW} values. Along the dotted lines one can observe first order phase transitions up to the LCP defined by the end point condition. Note, however, that this end point LCP is not the same as the LCP defined by the constant $R_{\mathrm{HW}} = R_{\mathrm{HW,cont}}$ value (long dashed line). They merge for decreasing lattice spacings, but at larger a the difference is the result of the "poor realization" of Wilson's renormalization group transformations with only three terms and parameters in the action. It is worth mentioning that the SU(2)-Higgs model is trivial for small gauge couplings, therefore, the $a \rightarrow 0$ limit cannot be performed. Even the points on the end point LCP do not define continuum theories. The second order phase transitions on it merely reflect a finite temperature phenomenon; the corresponding zero temperature SU(2)-Higgs theory is still trivial.

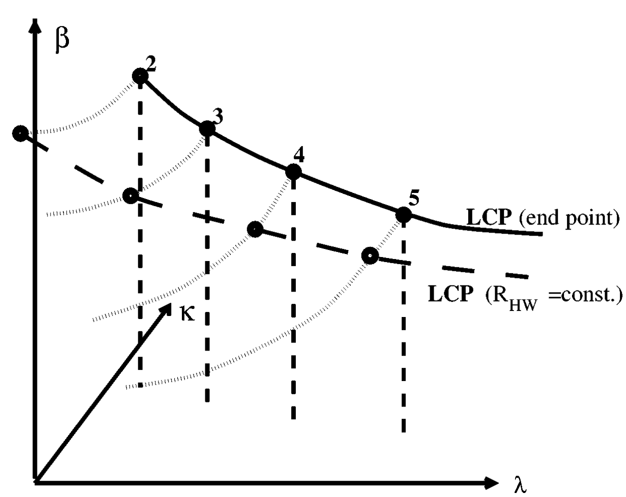

FIG. 1. Schematic view of the phase diagram. The solid line represents the LCP defined by the end point condition. The numbers on the line correspond to the temporal extension for which the end point is realized (the dashed lines show their projection to the κ-λ plane). The dotted lines running into these points correspond to first order phase transitions for $g_R^2 =$ const but different $R_{\mathrm{HW}'\mathrm{s}}$. A LCP defined by a constant R_{HW} value is shown by the long dashed line.

The technical implementation of the above LCP idea has been done as follows. By fixing $\beta = 8.0$ in the simulations, we have observed that g_R is essentially constant within our errors. For the small differences in g_R we have performed perturbative corrections. We have carried out $T \neq 0$ simulations on $L_t = 2, 3, 4, 5$ lattices (for the finite temperature case one uses $L_t \ll L_x, L_y, L_z$) and tuned κ to the transition point. This condition fixes the lattice spacings: $a_t = a_s/\xi = 1/(T_c L_t)$ in terms of the transition temperature T_c in physical units. The third parameter λ, finally specifying the physical Higgs mass in lattice units, has been chosen in a way that the transition corresponds to the end point of the first order phase transition subspace.

In this paper $V = L_t L_s^3$ type 4D lattices are used. For each L_t we had eight different lattices, each of them had approximately twice as large a lattice volume as the previous one. The smallest lattice was $V = 2 \times 5^3$ and the largest one was $V = 5 \times 50^3$. We collected quite a large statistics and the Ferrenberg-Swendsen reweighting [13] was used to obtain information in the vicinity of a simulation point.

The determination of the end point of the finite temperature EWPT is done by the use of the Lee-Yang zeros of the partition function Z [14]. Near the first order phase transition point the partition function reads

$$Z = Z_s + Z_b \propto \exp(-Vf_s) + \exp(-Vf_b), \quad (2)$$

where the indices $s(b)$ refer to the symmetric (Higgs) phase and f stands for the free-energy densities. Near the phase transition point we also have

$$f_b = f_s + \alpha(\kappa - \kappa_c), \quad (3)$$

since the free-energy density is continuous. One gets that

$$Z = 2\exp[-V(f_s + f_b)/2]\cosh[-V\alpha(\kappa - \kappa_c)], \quad (4)$$

which shows that for complex κ $\mathcal{Z}$ vanishes at

$$\mathrm{Im}(\kappa) = \pi(n - 1/2)/V\alpha \qquad (5)$$

for integer n. In case a first order phase transition is present, these Lee-Yang zeros move to the real axis as the volume goes to infinity. In case a phase transition is absent, the Lee-Yang zeros stay away from the real κ axis. Thus, the way the Lee-Yang zeros move in this limit is a good indicator for the presence or absence of a first order phase transition [14]. Denoting κ_0 as the lowest zero of $\mathcal{Z}$, i.e., the position of the zero closest to the real axis, one expects in the vicinity of the end point the scaling law $\mathrm{Im}(\kappa_0) = c_1(L_t, \lambda)V^{\nu} + c_2(L_t, \lambda)$. In order to pin down the end point we are looking for a λ value for which c_2 vanishes. In practice we analytically continue $\mathcal{Z}$ to complex values of κ by reweighting the available data. Also small changes in λ have been done by reweighting. As an example, the dependence of c_2 on λ for $L_t = 3$ is shown in Fig. 2. To determine the critical value of λ, i.e., the largest value, where $c_2 = 0$, we have performed fits linear in λ to the non-negative c_2 values.

Having determined the end point $\lambda_{\mathrm{crit}}(L_t)$ for each L_t we calculate the $T = 0$ quantities (R_{HW}, g_R^2) on $V = (32L_t)(8L_t)(6L_t)^2$ lattices, where $32L_t$ belongs to the temporal extension and extrapolates to the continuum limit. All the $T = 0$ simulations were performed at $\lambda = 0.000\,178$ and an extrapolation to the $\lambda_{\mathrm{crit}}(L_t)$ has been made. The parameters and results of the simulations are collected in Table I, while Table II shows the R_{HW} values extrapolated to the $\lambda_{\mathrm{crit}}(L_t)$. Having established the correspondence between $\lambda_{\mathrm{crit}}(L_t)$ and R_{HW}, the L_t dependence of the critical R_{HW} is easily obtained. Figure 3 shows the dependence of the end point R_{HW} values on $1/L_t^2$. For our bosonic theory a linear extrapolation in $1/L_t^2$ yields the continuum limit value of the end point R_{HW}. We obtain 66.5 ± 1.4 GeV, which is our final result.

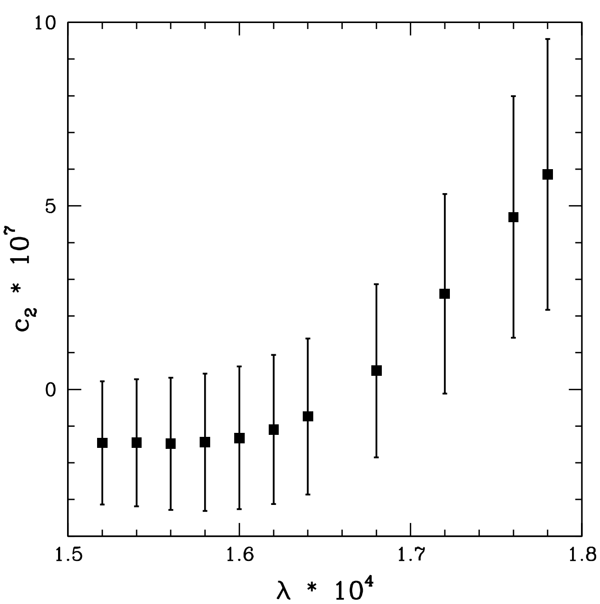

FIG. 2. Dependence of c_2 on λ for $L_t = 3$.

TABLE I. Summary of simulation parameters and results on R_{HW} and g_R^2 at $T = 0$.

L_t	λ_{sim}	κ_{sim}	R_{HW}	g_R^2
2	0.000 178	0.107 733	0.934(10)	0.569(4)
3	0.000 178	0.106 988	0.913(12)	0.575(3)
4	0.000 178	0.106 620	0.905(8)	0.585(5)
5	0.000 178	0.106 497 4	0.867(36)	0.566(30)

Comparing previous 4D and 3D results [10,15–17] it was believed that there is a discrepancy between the end point obtained in 3D [9,10] and the higher value indicated by the $L_t = 2$ 4D simulations [7,18]. Indeed increasing L_t results in a decrease of the end point mass value (cf. Fig. 3). However, our continuum result presented in this Letter completely agrees with that of the 3D analysis of [10]. Since the error bars on the end point determinations are on the few percent level, the uncertainty of the dimensional reduction procedure is also in this range. This indicates, although does not prove, that the analogous perturbative inclusion of the fermionic sector results also in few percent error on the end point M_H.

Based on our published data [5,12] and the results of this paper we are now able to draw the precise phase diagram of the SU(2)-Higgs model in the (T_c/m_H-R_{HW}) plane. This is shown in Fig. 4. The continuous line—representing the phase boundary—is a quadratic fit to the data points. The T_c/m_H extrapolation to the continuum limit is done as described in [5], for $m_H \approx 35$ GeV, cf. Fig. 10 of [5] as an example. (Note that T_c/m_H increases in the continuum extrapolation; however, it decreases for small L_t's in the asymmetric lattice case.)

Finally, we determine what is the end point value in the full SM. Our nonperturbative analysis shows that the perturbative integration of the heavy modes is correct within our error bars. Therefore we use perturbation theory [19] to transform the SU(2)-Higgs model end point value to the full SM. We obtain 72.4 ± 1.7 GeV, where the error includes the measured error of $R_{\mathrm{HW,cont}}$, g_R^2 and the estimated uncertainty [15] due to the different definitions of the gauge couplings between this paper and [19]. Although it is a matter of principle to use the same definition of g_R^2 both in the lattice simulation and in the perturbative calculation, however, a conservative error estimate [15] shows that the resulting error is small. The

TABLE II. Critical λ corresponding to the end point of phase transition as a function of L_t and the corresponding value of R_{HW}.

L_t	λ_{crit}	κ_{crit}	$R_{\mathrm{HW},c}$
2	0.000 177 3(14)	0.107 729 2(2)	0.932(11)
3	0.000 166 4(27)	0.106 958 1(2)	0.884(12)
4	0.000 159 0(44)	0.106 631 6(3)	0.841(26)
5	0.000 166 4(20)	0.106 494 8(6)	0.833(36)

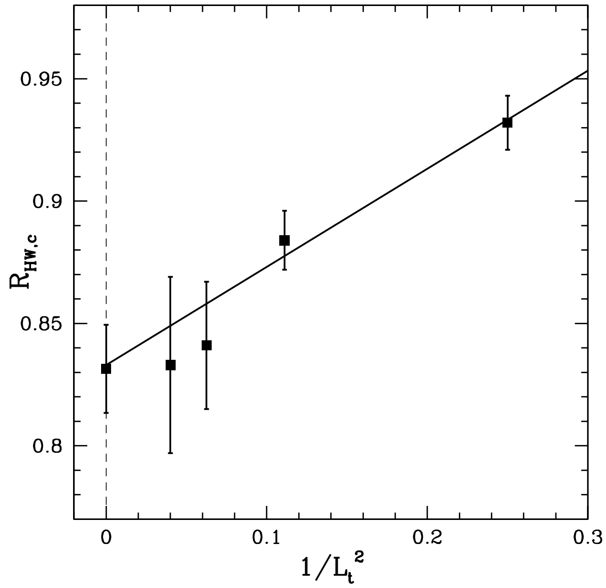

FIG. 3. Dependence of $R_{\mathrm{HW},c}$, i.e., R_{HW} corresponding to the end point of first order phase transitions on $1/L_t^2$ and extrapolation to the infinite volume limit.

dominant error comes from the uncertainty on the position of the end point.

In conclusion, we have determined the end point of the hot EWPT with the technique of Lee-Yang zeros from simulations in 4D SU(2)-Higgs model. The phase diagram has been also presented. The phase transition is first order for Higgs masses less than 66.5 ± 1.4 GeV, while for larger Higgs masses only a rapid crossover is expected. One of the most important results of the present Letter is that integrating out the heavy modes perturbatively is precise as shown by a comparison to our nonperturbative results. Thus the above 66.5 ± 1.4 GeV value can be perturbatively transformed to the full SM. We obtain 72.4 ± 1.7 GeV for the end point Higgs mass. As pointed out above the perturbative inclusion of the fermionic sector of the SM is expected to be correct to a few percent.

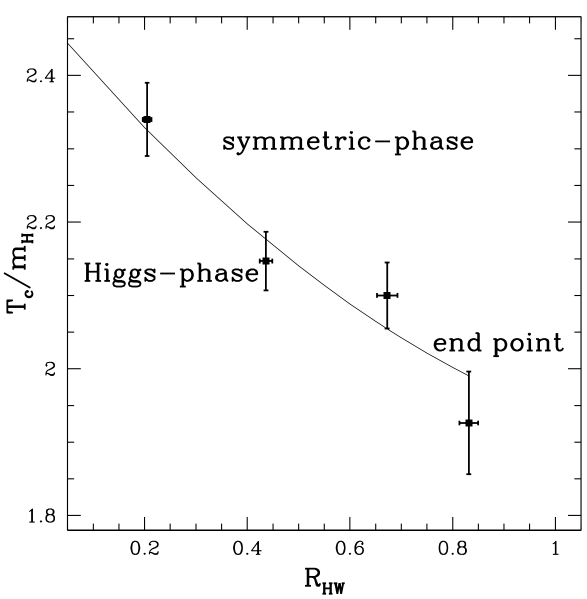

FIG. 4. Phase diagram of the SU(2)-Higgs model in the T_c/m_H-R_{HW} plane. The continuous line—representing the phase boundary—is a quadratic fit to the data points.

The present experimental lower limit of the SM Higgs-boson mass is 89.8 GeV [20]. Taking into account all errors our end point value excludes the possibility of any EWPT in the SM. This also means that the SM baryogenesis in the early Universe is ruled out.

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