Theory of the Ablative Richtmyer-Meshkov Instability

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Theory of the ablative Richtmyer-Meshkov instability is presented. It is shown that the main stabilizing mechanism of the ablation-front perturbations during the shock transit time is the dynamic overpressure that causes perturbation oscillations. The amplitude of the oscillation is proportional to $c_s/\sqrt{V_a V_{bl}}$ and its frequency is $\omega = k\sqrt{V_a V_{bl}}$, where k is the wave number, and c_s, V_a, and V_{bl} are sound speed, ablation, and blow-off plasma velocities, respectively. [S0031-9007(99)08686-X]

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In inertial confinement fusion (ICF) implosions, a laser irradiation induces a shock wave propagating through the target. During the shock transit time, the ablation front travels at a constant velocity, and any surface perturbations could grow due to the Richtmyer-Meshkov (RM)-like instability [1–4]. It is important to study such a growth because it determines the seed of the Rayleigh-Taylor (RT) instability that develops during the acceleration phase of implosion.

Similarity between the classical RM instability and the instability of the corrugated ablation front during the shock transit time is apparent. In the first case, a shock wave interacts with a distorted interface between two fluids [1]. As a result, the interface perturbation starts to grow because the transmitted shock creates a pressure excess behind the concave part of the shock ripple and a pressure deficiency behind the convex part. Such a pressure disturbance accelerates one fluid into another and leads to an interface instability. Theory of the classical RM instability [1] shows that the interface perturbations asymptotically grow linearly in time $\eta(kc_s t \gg 1) \sim \eta_0 k c_s t$, where η is the perturbation amplitude, k is the mode wave number, and c_s is the sound speed. In the case of corrugated ablation front, ablation pressure generates a rippled shock which also induces a pressure perturbation that could lead to an interface instability similar to the RM instability; we refer to such an instability as the "ablative RM instability." During the last two years, several researchers have made attempts to develop an analytic theory of the ablative Richtmyer-Meshkov instability. In Ref. [3] the perturbation evolution was derived by using the Chapman-Jouguet deflagration model. This model idealizes the region between the sonic point and the ablation front as a surface of discontinuity. However, as will be shown later, the thermal conduction inside such a region creates a restoring force that suppresses the perturbation growth. Because the deflagration model developed in [3] fails to capture the main stabilizing mechanism, such a model cannot be used to carry out the stability analysis of ablation fronts. In Ref. [4] saturation of the perturbation growth was found (in agreement with the results of Ref. [3]); at the ablation front, however, the authors used the so-called "Landau-Darrieus" boundary condition (prefront velocity normal to the ablation front remains unchanged) which is not applicable in the presence of the finite heat conduction [5] (see also the discussion later in the text). In this paper, we develop a sharp-boundary model to study the imposed mass-perturbation growth during the shock-transit time. The boundary conditions at the shock front are derived by using the Hugoniot relations. At the ablation front the result of the self-consistent analysis [5–7] is applied, and it is shown that the asymptotic behavior of the ablation-front perturbations is quite different from the earlier theoretical predictions [3,4]. In particular, the finite thermal conduction in the hot plasma corona produces a dynamic overpressure that causes perturbation oscillations (in agreement with the numerical results [2,4]) with the frequency $\omega = k\sqrt{V_a V_{bl}}$ and the amplitude $\eta_0 c_s/\sqrt{V_a V_{bl}}$, where V_a and V_{bl} are the ablation and the characteristic blow-off velocities, respectively. In addition, the mass ablation and vorticity convection damp the oscillation amplitude on a time scale $1/kV_a \gg 1/\omega$.

As mentioned in the introduction, the interface perturbations subject to the classical RM instability grow with a constant velocity ($\eta \sim t$). In the presence of ablation, there are several physical mechanisms that suppress such a growth [5–8]. To specify the stabilizing mechanisms, next we turn our attention to the process of ablation itself. During the ICF implosion, the laser energy is absorbed near the critical surface and transported by thermal conduction towards the cold target material increasing the target temperature and pressure. Then, the heated material (plasma) expands, creating a mass flow in the direction opposite to the direction of the heat wave. Because the ablative heat wave propagates at a speed much less than the sound speed $V_a \ll c_s$, the expansion region follows immediately after the heat front (target material ablates from the heat wave interface). The velocity of the expanding plasma is referred to as a "blow-off velocity." The ablative process in the vicinity of the heat front can be described by the diffusion equation $\rho c_p D_t T = -\nabla \mathbf{q} = \nabla(\kappa \nabla T)$, where $D_t = \partial_t + \mathbf{v}\nabla$, $\mathbf{q}$ is the heat flux, $\kappa \sim T^\nu$ is the nonlinear thermal conductivity, and c_p is specific heat at constant pressure (any pressure variation terms in the energy equation are neglected because they scale as the ablation Mach number $M_a = V_a/c_s \ll 1$). This equation shows that after the heat front passes through a region of the thickness $L = V_a \Delta t$ (V_a is the heat front velocity or ablation velocity), the enthalpy $h = c_p T$ of that region increases $m \Delta h \simeq (\kappa \partial_y T)_R \Delta t$, where the subscript R denotes the boundary through which the heat front enters into the region (right boundary), and $m = \rho L$ is the region mass. This enthalpy increase goes to the $p\, dV$ work of the expanding plasma (note that according to the law of thermodynamics, $\Delta e = \Delta h + P \Delta \rho / \rho^2$, it is energetically more efficient for plasma to expand towards the lower density corona than towards the higher density target material, hence such an expansion does not change the velocity of the heat front V_a). Assuming the constant pressure ($\rho T = \text{const}$), the enthalpy variation can be rewritten as $\rho \Delta h \simeq -c_p T \Delta \rho$; and using the mass-conservation equation $\Delta m = \Delta \rho L = -\rho V \Delta t$, the blow-off velocity becomes $V_{bl} \simeq (\kappa \partial_y T)_R / P$. Next, we consider propagation of the perturbed heat front through the same region of thickness L. As a result of perturbation growth, the peak of the heat front distortion protrudes into the hot plasma corona, and the front trough moves towards the cold material. This leads to a slight steepening of the temperature profile at the peak and flattening at the trough. The relative change in the temperature gradient along the heat front results in the following two effects. First, because the blow-off velocity is proportional to the heat flux, the region adjacent to the front peak expands faster then the region behind the front trough. The resulting dynamic overpressure creates a restoring force that stabilizes perturbations (observe that in the deflagration model developed in [3], the described effect of increasing blow-off velocity behind the perturbation peak is absent because such a model treats the ablation front as a Chapman-Jouguet point, i.e., the fluid right behind the ablation front moves with the local sound speed). The second effect is an increase in the ablation velocity in the region of higher temperature gradients. Indeed, the ablation velocity can be estimated from the diffusion equation. If we assume that the temperature distribution in the heat-front frame of reference is steady, $T = T(y + V_a t)$, the diffusion equation becomes $c_p \rho V_a T / L_T \approx \kappa_a T / L_T^2$, where L_T is the temperature gradient scale length and κ_a is the thermal conduction calculated at the density maximum. Then, the ablation velocity is $V_a \sim \kappa_a / (\rho L_T c_p)$. One can conclude that steepening of the temperature profile at the ablation front increases the ablation velocity. Thus, the perturbation peak ablates faster than the perturbation trough (perturbations change the prefront velocity, as opposite to the "Landau-Darrieus" boundary condition [4]). This effect leads to an additional stabilization.

To perform a quantitative stability analysis of the ablation front, one has to solve the system of conservation equations inside the following four regions: (1) $y < y_s$, undriven target; (2) $y_s < y < y_a$, material compressed by the shock; (3a) $y_a < y < y_a + L_a$, ablation region; and (3) $y > y_a + L_a$, blow-off plasma. Then, asymptotically matching the solution at the boundaries of each region, the temporal evolution of the shock and ablation front perturbations can be derived. For modes with $kL_a > 1$ the problem is too complicated to be analytically solved; however, in the regime when the density-gradient stabilization is not important, $kL_a \ll 1$, the ablation region (but not the entire conduction zone [3]) can be approximated as a surface of discontinuity. Because of the sharp interfaces at the shock and ablation fronts, such a model is commonly referred to as a "sharp-boundary model" (SBM) [7,8]. The SBM is solved in the standard fashion. First, all perturbed quantities are decomposed in the Fourier space $Q_1 = \tilde{Q}(y,t) e^{ikx}$. Then, in the frame of reference moving with the compressed-region velocity, the linearized conservation equations are combined into a single partial differential equation for the pressure perturbation $\tilde{p}$ [1,3,9], $\partial_t^2 \tilde{p} - c_s^2 \partial_y^2 \tilde{p} + k^2 c_s^2 \tilde{p} = 0$, where c_s is the sound speed of the compressed material. The boundary conditions are derived by integrating the conservation equations across the shock front and the ablation region and taking the limit of $L_a \to 0$. Integration across the shock front gives the standard Hugoniot jump conditions for the oblique shocks [9]. At the ablation front, it is straightforward to integrate the mass- and momentum-conservation equations. However, the jump condition derived from the energy equation (jump in the transverse velocity) contains an additional unknown: the perturbed heat flux in the blow-off region. This problem was addressed in Ref. [8] where the SBM to study the RT instability was developed. it was shown that approximating the ablation front by an isotherm, first, it is possible to define the perturbed heat flux in the blow-off region, and, second, the result of the SBM (with the appropriately chosen value of the blow-off velocity) reproduces the result of the self-consistent stability analysis of accelerated ablation fronts [5–7]. In the present model we use the same approximation of the isothermal ablation front, and derive the same jump condition for the transverse velocity [8]: $\tilde{v}_3(y_a) - \tilde{v}_2(y_a) = (V_{bl} - V_a) k \eta_a$, where η_a is the ablation front perturbation, and the subscript 3(2) denotes the region number. An increase in transverse velocity, as mentioned earlier, is due to the finite thermal conductivity and the consequence of the fact that heat flux increases at the perturbation peak leading to an additional plasma expansion and an increase in the blow-off velocity.

Next, solving the partial differential equation for the pressure perturbation and applying the appropriate boundary conditions, we derive the ablation-front evolution. The details of such a derivation will be published elsewhere; here we report the final result. The asymptotic behavior ($kc_s t \gg 1$) of the ablation-front perturbations takes the following form:

$$\frac{\eta_a}{\eta_0} \simeq \eta_v(t) + \{\Sigma_2 \sin \omega t + [\Sigma_1 - \eta_v(0)] \cos \omega t\} e^{-2kV_a t}, \tag{1}$$

where

$$\eta_v(t) = \frac{2c_s}{V_{bl}} e^{kV_a t} \int_\infty^{kV_a t} e^{-\eta} \Omega(\eta)\, d\eta\,,$$

$\omega = k\sqrt{V_a V_{bl}}$, $\Omega(y) = i(\nabla \times \mathbf{v})_z/(kc_s)$ is the normalized vorticity created by the rippled shock,

$$\Omega(y) \simeq \frac{(M_s^2 - 1)\Sigma_0 \cosh\theta_s}{2M_s^2(\gamma + 1)\sinh^2\theta_s} J_1(\hat{y})\,,$$

and

$$\Sigma_0 = \frac{16(M_s^2 - 1)}{6\gamma M_s^2 - \gamma + 3}\,,$$

$$\Sigma_1 = 1 - 2\Sigma_0 \frac{M_s^2(5\gamma - 1) + 2(\gamma + 3)}{M_s^2(17\gamma - 7) + 2(\gamma + 9)}\,,$$

$$\Sigma_2 = \frac{kc_s}{3\omega}(1 + \Sigma_0 - \Sigma_1)\,,$$

$$\theta_s = \tanh^{-1}\sqrt{\frac{2 + (\gamma - 1)M_s^2}{2\gamma M_s^2 - (\gamma - 1)}}\,.$$

Here M_s is shock Mach number, γ is the ratio of specific heats, $J_1(x)$ is the Bessel function, and $\hat{y} = y/\sinh\theta_s$. Equation (1) shows that the ablation substantially modifies perturbation behavior: linear in time asymptotic growth ($\eta \sim t$ for $V_a = 0$) in the presence of ablation turns into surface oscillations. Such oscillations are caused by the finite thermal conductivity which increases fluid velocity behind the perturbation peak (see discussion earlier in the text); a higher fluid velocity leads to a dynamic overpressure ΔP_d and a restoring force ($F_r \sim -\partial_y \Delta P_d \sim -k\rho_2 V_a \Delta\tilde{v} \sim -k^2\rho_2 V_a V_{bl}\eta_a$) that stabilizes perturbations. The perturbation amplitude in this case obeys a simple differential equation $\rho_2 d_t^2 \eta_a = F_r = -k^2\rho_2 V_a V_{bl}\eta_a$ or

$$d_t^2\eta_a + k^2 V_a V_{bl}\eta_a = 0 \qquad (2)$$

that describes oscillations with the frequency $\omega = k\sqrt{V_a V_{bl}}$. The estimate of the oscillation frequency can be also obtained by using the result of the self-consistent theory of the ablative RT instability [5–7]. For the ablation fronts with large Froude numbers [$\mathrm{Fr} = V_a^2/(gL_a)$], the perturbation growth rate is $\gamma \simeq \sqrt{kg - k^2 V_a V_{bl}} - 2kV_a$. Taking the limit of $g \to 0$ ($\mathrm{Fr} \to \infty$) in the last expression gives the oscillation frequency $\omega = i\gamma = k\sqrt{V_a V_{bl}}$, in agreement with Eq. (1). Next, we turn our attention to the term η_v [see Eq. (1)] which is due to the vorticity convection from the rippled shock towards the ablation front (in the ablation front frame of reference, the compressed material moves with the ablation velocity). The vorticity convection from the shock produces a destabilizing effect because it brings additional perturbations (in particular, the perturbations of the velocity gradients) towards the ablation region. Velocity gradients lead to a dynamic pressure gradient $\partial_y P_d \simeq \rho V(\partial_y \tilde{v})_s$ that generates a destabilizing force. Such a force modifies Eq. (2) to

$$d_t^2\eta_a + \omega^2\eta_a = V_a(\partial_y\tilde{v})_s\,. \qquad (3)$$

In addition to the dynamic overpressure stabilization, the difference in the ablation velocity at the perturbation peak and trough (see discussion earlier in the text) and also the vorticity convection away from the ablation front damp the oscillation amplitude. These effects, similar to the case of the ablative RT instability, introduce a damping term in the wave equation [5–8],

$$d_t^2\eta_a + 4kV_a d_t\eta_a + \omega^2\eta_a = V_a(\partial_y\tilde{v})_s\,.$$

As a result, the oscillation amplitude decays exponentially in time [factor $e^{-2kV_a t}$ in Eq. (1)], and the late-time perturbation evolution is determined by a balance of the dynamic overpressure and the vorticity forces. The last oscillates in time with a decaying amplitude $\sim\sin(kV_a t)/\sqrt{kV_a t}$. The fact that the planar shock is stable and the shock ripple and the perturbations inside the compressed region oscillate in time is well known [1,3,4,9]. The stabilizing mechanism is due to the created by the shock lateral flow that increases pressure behind the concave part of the shock front and decreases it behind the convex part. As pressure increases (decreases), the shock speeds up (slows down), reducing the front distortion. The frequency of the shock oscillations is proportional to kc_s, and the oscillation amplitude decays in time as $1/\sqrt{kc_s t}$. Note that the decaying rate is determined by the symmetry of the sound waves transporting the pressure disturbances. In the planar foil, the cylindrical pressure waves attenuate as $1/\sqrt{r}$ ($r = c_s t$ is the radius of the wave front), thus the overall pressure perturbation behind the shock front decays as $1/\sqrt{c_s t}$. Because of the shock front oscillations, the perturbations inside the compressed region oscillate in space $\tilde{Q} \sim \sin[kc_s(y/U_s)]/\sqrt{ky}$, where U_s is the shock speed; hence at the ablation front $y = V_a t$ hydrodynamic quantities evolve according to $\tilde{Q} \sim \sin(kV_a t)/\sqrt{kV_a t}$. Applying the last formula to the velocity gradient $\partial_y\tilde{v}$, we recover the asymptotic limit of η_v.

In order to apply Eq. (1) to the flat foils commonly used in ICF experiments, one needs to estimate the value of blow-off velocity V_{bl}. In general, the velocity of ablated plasma is not uniform, and it increases in the direction towards the plasma corona. However, as shown in [6–8], the appropriate value of the blow-off velocity to be substituted into the SBM is $V_{bl} = V_a/[\mu(\nu)(kL_0)^{1/\nu}]$, where ν is the power index for the thermal conduction, L_0 is the characteristic thickness of ablation front, $\mu = (2/\nu)^{1/\nu}/\Gamma(1 + 1/\nu) + 0.12/\nu^2$, and $\Gamma(x)$ is the Gamma function. The parameters ν and L_0 can be determined by using the fitting procedure described in Ref. [10]. For plastic (CH) targets directly driven by a flattop laser pulse with an intensity of 50–200 TW/cm^2, such a procedure gives $L_0 \simeq 0.1\ \mu$m and $\nu \simeq 1$, thus the oscillation period is $T_{\mathrm{CH}} = 2\pi/\omega \simeq 3/[V_a(\mu\mathrm{m/ns})\sqrt{k(\mu\mathrm{m}^{-1})}]$ ns.

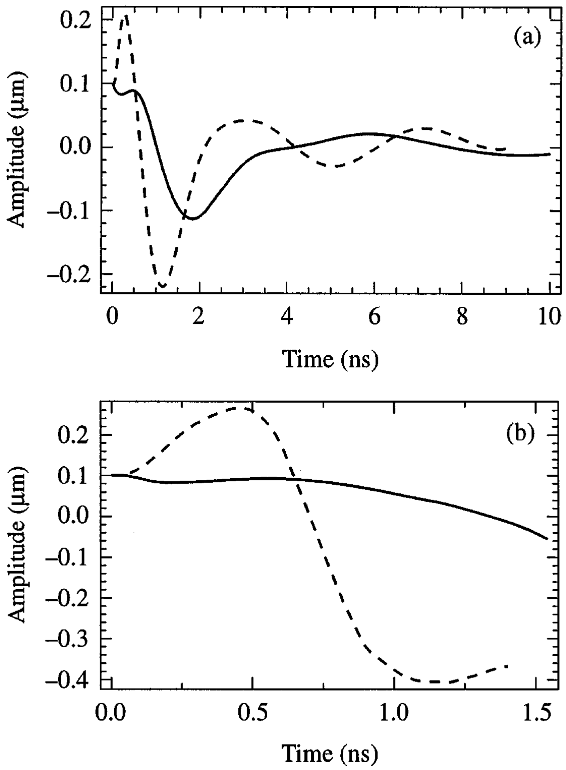

FIG. 1. Time evolution of the ablation-front perturbatin of CH (solid line) and solid DT (dashed line) foils calculated using the result of the SBM (a) and 2D ORCHID simulations (b).

Cryogenic (DT) targets have much smaller density-gradient scale length $L_0 \simeq 0.01\ \mu\text{m}$, $\nu \simeq 2$, and $T_{\text{DT}} \simeq 2/[V_a(\mu\text{m/ns})k^{3/4}(\mu\text{m}^{-1})]$ ns. Figure 1(a) shows the ablation-front evolution (as predicted by the SBM) of the solid DT and CH foils driven by a square pulse with an intensity of $100\ \text{TW/cm}^2$. The initial amplitude of perturbation is $0.1\ \mu$m, and its wavelength is 20 μm. Observe the change in the oscillation frequency after the vorticity force balances the dynamic overpressure force ($t_{\text{CH}} > 3.5$ ns, $t_{\text{DT}} > 2.5$ ns). For comparison, the results of the 2D ORCHID [11] simulations are plotted in Fig. 1(b). Although the oscillation frequency is well reproduced by the model, the predicted amplitude is lower then the one observed in the simulation. This could be related to the fact that the size of the conduction zone changes in time at the beginning of implosion, and the corrections due to a time variation in the ablation and the blow-off velocities become important. This problem will be addressed in future work.

In summary, the analytic theory of the ablative Richtmyer-Meshkov instability was developed. It was shown that the main stabilizing mechanism of the ablation-front perturbations is the dynamic overpressure created by the finite thermal conduction.

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