Cosmic Microwave Background and Density Fluctuations from Strings plus Inflation

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In cosmological models where local cosmic strings are formed at the end of a period of inflation, the perturbations are seeded both by the defects and by the quantum fluctuations, with similar amplitudes. In a flat cosmology with zero cosmological constant and 5% baryonic component, strings plus inflation fit the observational data much better than each component individually. The large-angle cosmic microwave background spectrum is mildly tilted, for Harrison-Zeldovich inflationary fluctuations. It then rises to a thick Doppler bump, covering $\ell = 200-600$, modulated by soft secondary undulations. The standard cold dark matter antibiasing problem is cured. [S0031-9007(99)08611-1]

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The combination of recent data from cosmic microwave background (CMB) observations and large scale structure (LSS) surveys is proving a strong discriminant among cosmologies. The standard cold dark matter (sCDM) inflationary model [1], dominant for so long, is now disfavored as it predicts too high an amplitude for the CDM power spectrum at scales below about $30h^{-1}$ Mpc [2]. Cosmic strings and other topological defects [3], after resisting theoretical attack, have also failed the tests in the standard cosmology of $\Omega = 1$, $\Omega_\Lambda = 0$, $\Omega_b = 0.05$, and $H_0 = 50\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, as they cannot produce the required power on large scales [4–7]. Other cosmological parameters have been investigated in both inflationary [2] and defect [8] models, with the result that a cosmological constant improves matters in both scenarios. The recent type-1a supernova data [9] support this move away from an Einstein–de Sitter Universe.

However, the failings of the inflationary and defect sCDM models are to a certain extent complementary, and an obvious question is whether they can help each other to improve the fit to CMB and LSS data. Using our recent calculations for local cosmic strings [7] and the by now familiar inflationary calculations [10], we are able to demonstrate that the answer is yes. Even with Harrison-Zeldovich initial conditions and no inflation produced gravitational waves, the large-angle CMB spectrum is mildly tilted, as preferred by the Cosmic Background Explorer (COBE) data [11]. The CMB spectrum then rises into a thick Doppler bump, covering the region $\ell = 200-600$, modulated by soft secondary undulations. More importantly the standard CDM antibiasing problem is cured, giving place to a slightly biased scenario of galaxy formation.

It may seem baroque to invoke both inflation and strings to explain the cosmological perturbations. However, there is a very attractive inflationary model, namely, D-term inflation [12,13], which necessarily produces strings, and in which the perturbation amplitudes are of similar amplitude [14] (see also [15–17]). D-term inflation requires the existence of an extra gauged U(1) symmetry, which is broken at the end of inflation, thereby resulting in the formation of a network of cosmic strings. If the symmetry is not anomalous, the strings will be local; otherwise they are global [12]. The inflaton corresponds to a flat direction in the potential, where the energy density is set by the U(1) symmetry-breaking scale. Radiative corrections lift the flatness and force the fields eventually to the U(1)-breaking supersymmetric minimum. A big attraction of the model is its naturalness in the technical sense: the flat direction is present as a result of symmetry, and the model avoids having to fine-tune any coupling constants.

The strings appear as a result of the breaking of the U(1) symmetry, although the details of the process are far from certain. It may be that the evolution of the homogeneous inflaton causes the U(1)-symmetric point in the potential to become unstable [18], or there may also be a nonthermal phase transition after inflation ends, during a "preheating" phase [19]. However, subject to the condition that the fields making the strings are uncorrelated at large distances, the subsequent evolution of the string network is thought to be independent of the formation process [3].

The ensuing structure formation scenario is highly exotic and worth studying just by itself. Regarded in the abstract, structure formation may be due to two types of mechanism: active and passive perturbations. Passive fluctuations are due to an apparently acausal imprint in the initial conditions of the standard cosmic ingredients, which are then left to evolve by themselves. Active perturbations are due to an extra cosmic component, which evolves causally (and often nonlinearly), and drives perturbations in the standard cosmic ingredients at all times. Inflationary fluctuations are passive. Defects are the quintessential active fluctuation.

A scenario combining active and passive perturbations would bypass most of the current wisdom on what to expect in either scenario. It is believed that the presence or absence of secondary Doppler peaks in the CMB power spectrum tests the very fundamental nature of inflation, whatever its guise [20]. In the mixed scenarios we shall

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consider whether inflationary scenarios could produce spectra with any degree of secondary oscillation softening.

We shall now recap some of the main features of the D-term inflation model in which the strings plus inflation scenario find an attractive expression. The simplest model [13] has three chiral fields, S, Φ_+, and Φ_-, which have charges 0, 1, and -1 under an extra $\mathrm{U}(1)_X$ symmetry. If one imposes an R symmetry, the only possible superpotential is $W = \lambda S \Phi_+ \Phi_-$, and one assumes a Kähler potential with minimal form $K = |S|^2 + |\Phi_+|^2 + |\Phi_-|^2$. The scalar potential for the bosonic components s, ϕ_+, and ϕ_- is then

$$V = \lambda^2 |s|^2 (|\phi_+|^2 + |\phi_-|^2) + \lambda^2 |\phi_+ \phi_-|^2 + \frac{1}{2} g^2 (|\phi_+|^2 - |\phi_-|^2 + \xi)^2, \qquad (1)$$

while the kinetic terms have the normal canonical form. The vacuum $s = \phi_+ = 0$, $|\phi_-| = \sqrt{\xi}$ is supersymmetric but breaks the U(1) symmetry. The field s is massless at tree level, while the fields $\phi_\pm$ have masses $m_\pm^2 = \lambda^2 |s|^2 \pm g^2 \xi$. Thus for $|s| > g\sqrt{\xi}/\lambda$, and $\phi_\pm = 0$, the potential is flat in the s direction and has a positive curvature in the $\phi_\pm$ directions. As a consequence of the broken supersymmetry, there are radiative corrections to the potential along this flat direction from $\phi_\pm$ and their fermionic partners, resulting in an effective potential [13],

$$V_{\mathrm{eff}} = \frac{1}{2} g^2 \xi^2 \left(1 + \frac{g^2}{16\pi^2} \ln \frac{\lambda^2 |s|^2}{\Lambda^2} \right), \qquad (2)$$

where Λ is the renormalization scale.

It is also possible that the U(1) symmetry is anomalous. However, the gauged U(1) symmetry is already effectively broken, leaving behind a global U(1) symmetry, which is broken when the charged fields gain expectation values. This results in the formation of global strings [12], to which our calculations do not strictly apply.

The two major pieces of data to which we wish to compare the theory are the mean square temperature fluctuation in the multipole ℓ, or $\ell(\ell + 1)C_\ell/2\pi$, and the power spectrum of matter density fluctuations $P(k) = \langle |\delta(k)| \rangle^2$. In order to compute CMB and LSS power spectra we note that fluctuations due to cosmic strings are imprinted long after their formation. The string network is produced at the end of inflation, and it is conceivable that the inflationary fluctuations and the initial configuration of strings are correlated. However strings are highly incoherent [21], meaning that all string modes become decorrelated with themselves in time. Incoherence is due to the nonlinear interactions present in the string evolutions, which lead to any Fourier mode being driven by all others. Hence the string network which produces the CMB and LSS fluctuations is surely uncorrelated with the string network produced at the end of inflation and therefore with the inflationary fluctuations.

The evolution of radiation, neutrinos, CDM, and baryons is linear for both string driven and inflationary perturbations. The fact that these two types of fluctuations are uncorrelated means that we can simply add the power spectra in CMB and LSS produced by each component separately.

The spectrum of the perturbations from D-term inflation is calculable [14], and can be expressed in terms of N, the number of e foldings between the horizon exit of cosmological scales today and the end of inflation, which occurs at $|\eta| = 1$. One finds

$$\frac{\ell(\ell + 1)C_\ell^{\mathrm{I}}}{2\pi T_{\mathrm{CMB}}^2} \simeq \frac{1}{4} |\delta_H(k)|^2 \simeq \frac{(2N + 1)}{75} \left(\frac{\xi^2}{M^4} \right), \qquad (3)$$

where $T_{\mathrm{CMB}} = 2.728$ K is the temperature of the microwave background, and $\delta_H(k)$ is the matter perturbation amplitude at the horizon crossing. The corrections to this formula, which is zeroth order in slow roll parameters, are not more than a few percent. The inflationary fluctuations in this model are almost scale invariant (Harrison-Zeldovich) and have a negligible tensor component [13].

The string contribution is uncorrelated with the inflationary one and is proportional to $(G\mu)^2$, where μ is the string mass per unit length, given by $\mu = 2\pi\xi$. We can write it as

$$\frac{\ell(\ell + 1)C_\ell^{\mathrm{S}}}{2\pi T_{\mathrm{CMB}}^2} = \frac{\mathcal{A}^{\mathrm{S}}(\ell)}{16} \left(\frac{\xi^2}{M^4} \right), \qquad (4)$$

where the function $\mathcal{A}^{\mathrm{S}}(\ell)$ gives the amplitude of the fractional temperature fluctuations in units of $(G\mu)^2$. Currently, there is no accurate analytic way of calculating $\mathcal{A}^{\mathrm{S}}(\ell)$, and we must resort to numerical simulations [8]. Our simulations use large string networks to compute the unequal time correlators of the energy momentum tensor [4], which are used as sources in a code which solves the general relativistic perturbation equations in the tight-coupling approximation. There are two main approximations in our approach: first, we neglect Hubble damping on the string network, which means that we overestimate the string density, and we cannot model the radiation-matter transition, during which the string density decreases by about 25%. Second, the tight coupling approximation is accurate only to about 10% in the region of the Doppler peaks in the CMB spectrum. However, these approximations are justified by the gains in computational efficiency, and by the fact that even with Hubble damping, string network simulations are subject to systematic errors of uncertain size due to lattice cutoff dependencies [22]. Furthermore, our work exploits the general scaling property of a string network to give us an excellent dynamic range, and we believe it represents an improvement over previous work [6], which reports $\mathcal{A}^{\mathrm{S}}(\ell) \simeq 60$ on large angular scales, with little dependence on ℓ. Our simulations give $\mathcal{A}^{\mathrm{S}}(\ell) \simeq 120$, with a fairly strong tilt. The source of the difference is not altogether clear: as mentioned above, our code neglects the energy losses of the strings through Hubble damping. The simulations of Allen *et al.* do include Hubble damping, which would tend to reduce the string density and hence the normalization. However, they have a problem of a lack of dynamic range, and therefore may be missing some power from strings at early times, and therefore higher ℓ.

Jeannerot [14] took the Allen-Shellard normalization and $N \simeq 60$, and found that the proportion of strings to inflation is roughly 3:1. With our normalization, the approximate ratio is 4:1. In any case this ratio is far from a robust prediction in strings plus inflation models, as it depends on the number of e foldings, and the string normalization, both of which are uncertain. We will therefore leave it as a free parameter. For definiteness we shall parametrize the contribution due to strings and inflation by the strings to inflation ratio R_{SI}, defined as the ratio in C_ℓ at $\ell = 5$; that is $R_{\mathrm{SI}} = C_5^{\mathrm{S}}/C_5^{\mathrm{I}}$.

In Figs. 1 and 2 we present power spectra in CMB and CDM produced by a sCDM scenario, by cosmic strings, and by strings plus inflation. We have assumed the traditional choice of parameters, setting the Hubble parameter $H_0 = 50\ \mathrm{km\,sec^{-1}\,Mpc^{-1}}$, the baryon fraction to $\Omega_b = 0.05$, and assumed a flat geometry, no cosmological constant, three massless neutrinos, standard recombination, and cold dark matter. The inflationary perturbations have a Harrison-Zeldovich or scale invariant spectrum, and the amount of gravitational radiation (tensor modes) produced during inflation is assumed to be negligible. The cosmic strings are assumed to attain scaling by losing their energy into gravitational radiation or some other noninteracting radiation fluid. Other assumptions for the equation of state of the decay products may be made [7]: however, a relativistic equation of state produces the worst bias problem at large scales and thus represents the "worst case" string scenario.

We now highlight the main features in the resulting CMB and LSS power spectra. The CMB power spectrum shape in these models is highly exotic. The inflationary contribution is close to being Harrison-Zeldovich. Hence it produces a flat small ℓ CMB spectrum. The admixture of strings, however, imparts a tilt. Depending on R_{SI} one may tune the CMB plateau tilt between 1 and about 1.4, without invoking primordial tilt and inflation produced gravity waves. This may be seen as a positive feature, as a flat spectrum is not favored by the COBE data [11].

The proverbial inflationary Doppler peaks are transfigured in these scenarios into a thick Doppler bump, covering the region $\ell = 200-600$. The height of the peak is similar for sCDM and strings, with standard cosmological parameters. Fitting two of the Saskatoon points therefore requires fiddling with cosmological parameters. The Doppler bump is modulated by small undulations, which cannot truly be called secondary peaks. By tuning R_{SI} one may achieve any degree of secondary oscillation softening. This provides a major loophole in the argument linking inflation with secondary oscillations in the CMB power spectrum. If these oscillations were not observed, inflation could still survive, in the form of the models discussed in this Letter.

The CMB fluctuations in these models combine a Gaussian component, produced by inflationary fluctuations, and a non-Gaussian component, due to strings. The superposition of Gaussian and non-Gaussian maps often leads to rather subtle non-Gaussian structures, visually indistinguishable from Gaussian maps [23]. Sophisticated statistics would certainly be required to recognize the non-Gaussian signal in these theories [23,24].

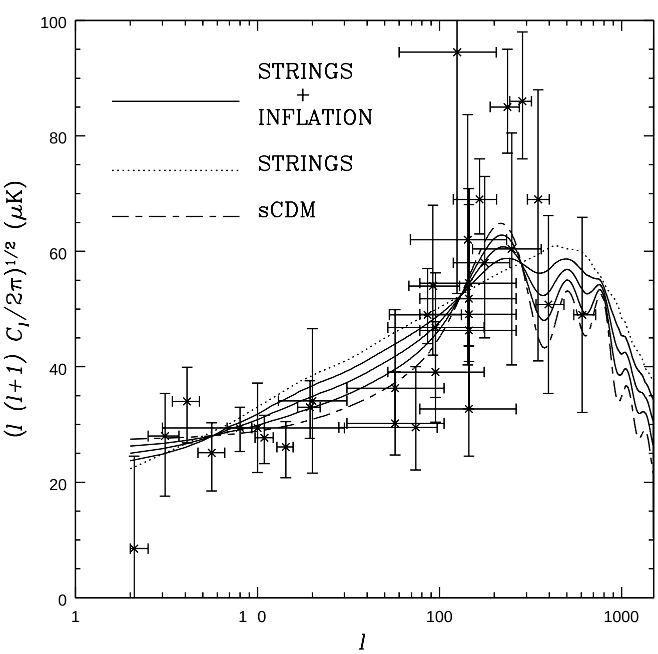

FIG. 1. The CMB power spectra predicted by cosmic strings, sCDM, and by inflation and strings with $R_{\mathrm{SI}} = 0.25$, 0.5, and 0.75. The large-angle spectrum is always slightly tilted. The Doppler peak becomes a thick Doppler bump at $\ell = 200-600$, modulated by mild undulations.

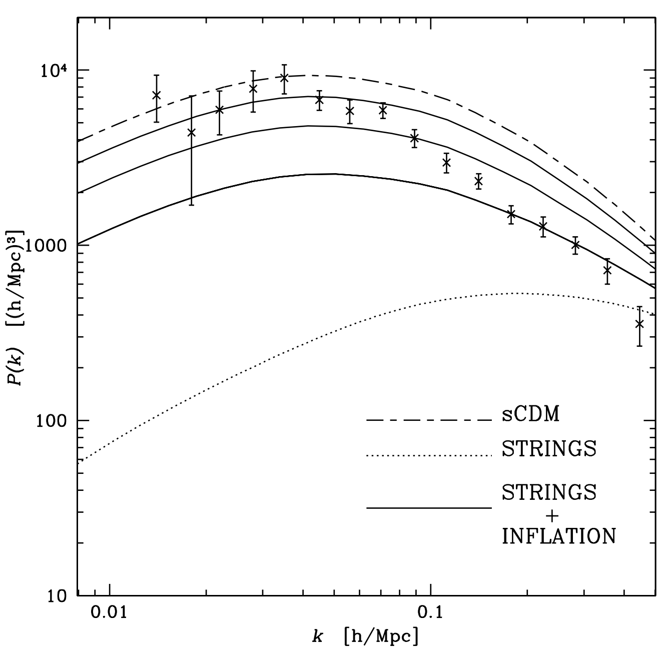

FIG. 2. The power spectra in CDM fluctuations predicted by cosmic strings, sCDM, and by inflation and strings with $R_{\mathrm{SI}} = 0.25$, 0.5, and 0.75.

In these scenarios the LSS of the Universe is almost all produced by inflationary fluctuations. However COBE scale CMB anisotropies are due to both strings and inflation. Therefore COBE normalized CDM fluctuations are reduced by a factor $(1 + R_{\mathrm{SI}})$ in strings plus inflation scenarios. This is equivalent to multiplying the sCDM bias by $\sqrt{1 + R_{\mathrm{SI}}}$ on all scales, except possibly the smallest scales, where the string contribution may be non-negligible. Given the fact that sCDM scenarios produce too much structure on small scales (too many clusters) this is a desirable feature.

We find that the bias required to fit the power spectrum of Peacock and Dodds [25] at the $8h^{-1}$ Mpc scale is $b_8 = \sigma_8^{\mathrm{PD}}/\sigma_8 = 0.7$, 0.8, and 1.0, for $R_{\mathrm{SI}} = 0.25$, 0.5, and 0.75, respectively. In $100h^{-1}$ Mpc spheres one requires the bias $b_{100} = \sigma_{100}^{\mathrm{PD}}/\sigma_{100} = 1.0$, 1.2, and 1.6 to match the observations. None of these values is uncomfortable. If most of the objects used to estimate $P(k)$ evolved from high peaks of the density field, one should have biasing, not antibiasing, and the bias should also be of the order of 1. Strings plus inflation with $R_{\mathrm{SI}} > 0.75$ complies with these requirements, whereas each separate component does not. However, with the simplest choice of cosmological parameters, the bias must be scale dependent in these models.

The model naturally inherits most of the sCDM good features. For instance, it fits the Lyman break galaxy clustering found in [26] and the constraints inferred from damped Lyman-α systems [27].

If we are prepared to combine inflation with more unusual string scenarios, in which, say, the strings lose energy in a direct channel into CDM [7], then the major novelty is that strings will be responsible for the LSS on scales below about $30h^{-1}$ Mpc. We will discuss elsewhere how this may have interesting implications for the time evolution of the CDM power spectrum [26]. Active models drive fluctuations at all times, and therefore produce a time dependence in $P(k)$ different from passive models. In such models there would also be intrinsic non-Gaussianity at the scale of clusters, with interesting connections with the work of [28].

In summary, mixing cosmic string and sCDM spectra smooths the hard edges of either separate component, leaving a much better fit to LSS and CMB power spectra (see [29] for another happy marriage). It is perhaps rather ironic that inflation and strings, often presented in opposition to one another, should find such a fruitful union. We do not wish to claim that the outcome is perfect: the shape of the power spectrum still goes wrong in the standard cosmology. The purpose of this work is to start the investigation of a new set of cosmological models, those which combine inflation and defects. As the data improve we will be able to constrain them more heavily, particularly as the first peak in the CMB power spectrum begins to be traced. Meanwhile, we are left with an intriguing hybrid model.

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