Bose-Einstein Condensates in an Optical Lattice

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We calculate the quantum motion of a Bose-Einstein condensate in an optical lattice generated by a standing wave of laser light. We show how to boost a stationary condensate into motion or stop a moving condensate by manipulating the optical lattice and how to achieve Bloch oscillations of the condensate in an accelerating optical lattice. We show how atomic interactions affect these processes and discuss conditions for possible experimental realization. [S0031-9007(99)08636-6]

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Bose-Einstein condensates (BEC) [1–3] in dilute atomic gases provide a good opportunity for controlled study and manipulation of their dynamics, which has not been possible for He superfluids. Much work has already been done along this line of research, including the studies of nonlinear response to time-dependent modulations of the trap potential [4] and condensates in spatially periodic potentials [5,6].

In this Letter, we investigate the possibility of manipulating the condensate by a periodic potential, which may be created by a standing wave of laser light [7]. In particular, we show how to boost a stationary condensate to a finite velocity and study how a moving condensate may be stopped by a stationary potential. We also show how Bloch oscillations of the condensate arise in an accelerating potential. The motion of ultracold but non-BEC atoms in accelerating potentials have been studied extensively and can be understood in a model of noninteracting atoms [8–10]. Here we are interested in the effect of atomic interactions on the quantum transport of the condensate.

Modeling of the system.—Instead of using trapped gases, we study a model of a free condensate released from a trapping potential after the ground state BEC is achieved. The typical size of a BEC wave packet is of order 10 μm, which expands with a typical time scale of 10 ms. The wavelength of our standing wave will be much smaller than this size, and the proposed experimental processes are also of much shorter duration than the expansion time. It is then reasonable to model the condensate dynamics as a one-dimensional problem, where the system varies in the direction of the standing wave and is uniform in the perpendicular directions. Another case of interest is a BEC strongly confined in a long cigar shape [11,12]. The density profile in the transverse directions is held fixed by the trap while the motion along the longitudinal direction can be considered free. When the standing wave of the laser light is applied along the longitudinal direction, it is sufficient to consider only that direction, with the caution that the effective scattering length between the particles is renormalized by a factor of half due to the transverse confinement [13]. In both cases we can model the motion as a one-dimensional problem and take the initial state to be uniform before the standing wave is turned on.

Our study of the BEC dynamics will be based on the nonlinear Schrödinger equation. The equation has been successfully applied to the calculation of stable BEC states, the expansion of BEC, and collective excitations [14–20]. It can be derived from the mean-field theory, with the atom-atom interaction modeled by a repulsive δ-function potential, and should be very accurate for the dilute, near-zero temperature condensate [21,22]. Specifically, we consider the following 1D equation:

$$i\hbar\frac{\partial\phi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\phi}{\partial x^2} + V_0\cos(2k_Lx)\phi + \frac{4\pi\hbar^2 a}{m}|\phi|^2\phi\,, \tag{1}$$

where m is the atomic mass, k_L is the wave vector of the laser light, a is the s-wave scattering length between atoms, and V_0 is the magnitude of the potential which is proportional to the light intensity. The normalization of the wave function is such that $|\phi|^2$ represents the number of atoms per unit volume.

We rescale Eq. (1) by introducing dimensionless variables, $\tilde{x} = 2k_Lx$, $\tilde{t} = \frac{\hbar}{m}4k_L^2 t$, $\tilde{\phi} = \phi/\sqrt{n_0}$, $\tilde{V}_0 = \frac{m}{\hbar^2}\times(\frac{1}{4k_L^2})V_0$, and $C = \frac{\pi n_0 a}{k_L^2}$, where n_0 is the density of BEC. Then we obtain the dimensionless equation (replacing $\tilde{x}$ by x, etc.),

$$i\frac{\partial\phi}{\partial t} = -\frac{1}{2}\frac{\partial^2\phi}{\partial x^2} + V_0\cos(x)\phi + C|\phi|^2\phi\,. \tag{2}$$

We set $V_0 = 0.1$–0.4 and calculate the response of the solutions to the external potential for various values of C. As in Ref. [14], we use the Crank-Nicholson method [23,24] for the numerical solution of Eq. (2). This method preserves the unitarity of the time evolution and yields good convergence of the solutions for moderate values of

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the nonlinear coupling strength C ($C \leq 1$ mostly in this work).

Numerical results and theoretical analysis.—First, we consider how the current of a moving condensate degrades when a stationary periodic potential is turned on. Our initial wave function is taken to be $\phi = e^{ik_0 x}$ which has a current k_0. (Later, we will show how such a state may be prepared.) The potential is then turned on adiabatically to a strength V_0 in a time t_0 and stays constant afterwards. From the solution, we obtain the condensate current $j = (\frac{\hbar}{m})\,\mathrm{Im}(\phi^* \frac{d\phi}{dx})$ as shown in Fig. 1, where we have taken $k_0 = 1/4$, $V_0 = 0.05$, and $t_0 = 60$. The current decreases as the potential is turned on and settles down to new values depending on the strength of the atomic interaction C. For small C, the current decreases dramatically, but for $C \geq 1.0$, the current stays almost constant [also see Fig. 2(a)]. These results show that the ability for the condensate to maintain its current depends crucially on the strength of interaction between the atoms.

This strong dependence is explained in terms of the effective potential $V_0 \cos(x) + C|\phi|^2$ seen by each atom. We view our system as a noninteracting gas in the effective potential, with the condensate wave function corresponding to the Bloch state in the lowest energy band of the effective potential and with initial wave number k_0. The Bloch wave number is conserved because the potential is periodic and the state lies in the lowest band because k_0 lies in the first Brillouin zone and the potential is turned on adiabatically. The Bloch state has a periodic density profile so that the periodicity of the external potential is preserved in the effective potential. The effective potential is reduced by the repulsive self-interaction because the density of atoms tends to be larger in the potential wells and smaller in the barrier regions, so that the second term in the effective potential tends to even out the first term which represents the external potential.

An explicit analytic expression for the effective potential can be calculated using perturbation theory as $V_{\mathrm{eff}} \times \cos(x) + \mathrm{const}$, where $V_{\mathrm{eff}} = V_0/(1 + 4C)$. This result is valid as long as the condensate density is nearly uniform, i.e., when $V_{\mathrm{eff}} \ll 1$, which is realized for either a weak external potential or a strong atomic interaction. From this effective potential, we can also calculate the current perturbatively, with the result, $j = k_0 - 8k_0 V_{\mathrm{eff}}^2/(4k_0^2 - 1)^2$. We plot the current and the effective potential as functions of C in Fig. 2, where we see that the analytical results agree very well with the numerical data.

The above picture of noninteracting condensate in a reduced effective potential also gives an idea of the time scale for adiabaticity. The relevant energy gap is that between the lowest two bands at the same Bloch wave number, which is about $\Delta E = 1/4$ for $k_0 = 1/4$ in the limit of small V_{eff}. In order to avoid excitations across the gap, we choose our turn-on time of the potential to satisfy the condition $\Delta t_0 > \frac{2\pi}{\Delta E} = 8\pi$. Tiny oscillations of the current in Fig. 1 are due to residual nonadiabatic excitations, as is evident from the fact that their oscillation frequency coincides with ΔE. These oscillations become even smaller if we use a longer turn-on time.

Next, we show how a stationary condensate can be boosted to a finite velocity. We first turn on a stationary potential adiabatically to a strength of 0.1, then accelerate

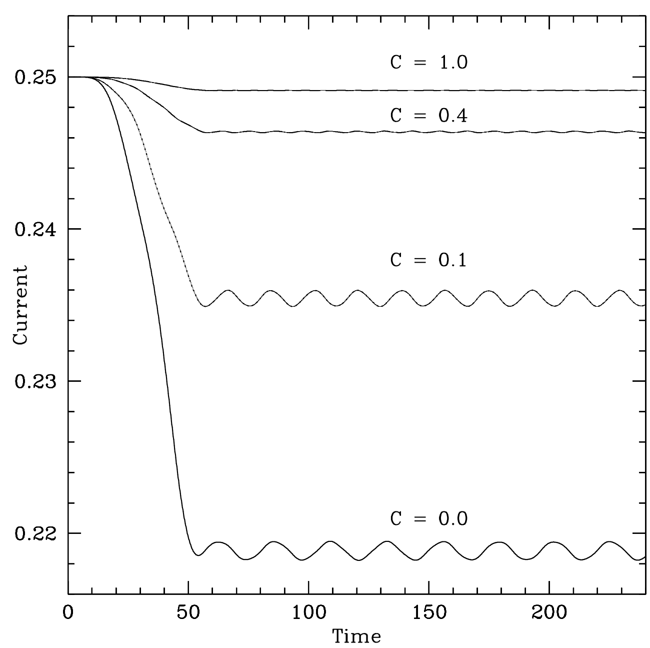

FIG. 1. Current as a function of time for the wave function with initial current $k_0 = 1/4$. $V_0 = 0.1$, and $t_0 = 60$. Results are shown for $C = 0.0$, 0.1, 0.4, and 1.0.

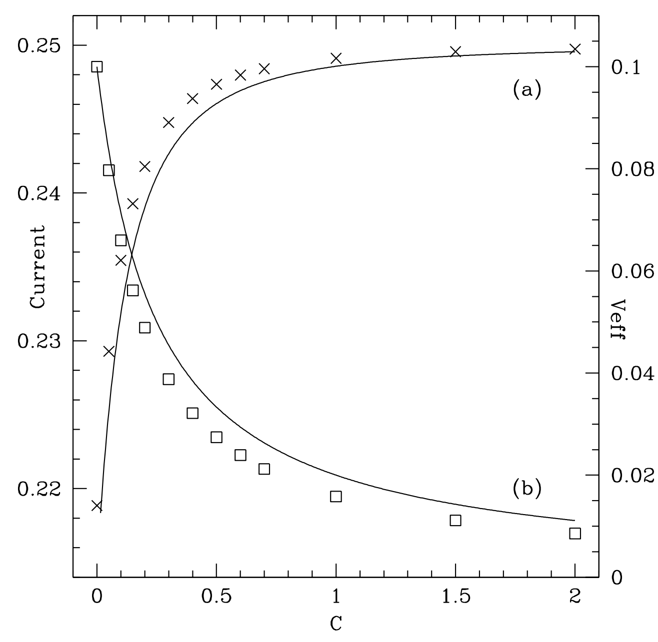

FIG. 2. Average current (a) and the strength of the effective potential V_{eff} (b) as a function of C after the turn-on of the potential for the runs in Fig. 1. Open squares and crosses are numerical results. Solid lines are analytic results.

the potential to a final velocity of v_f. The induced current (the average velocity) of the condensate is shown in Fig. 3 for $v_f = 0.2$ and for various values of atomic interaction C. For $C = 0$, the condensate follows the motion of the potential, acquiring the same velocity as the potential. For nonzero C, the current is lower, implying a leakage of the atoms through the potential. For larger C, very little current is dragged by the potential. These results can again be simply understood in terms of the effective potential reduced by the self-interaction. The insensitivity of the motion of the condensate with strong atomic interaction to an external potential reminds us of the property of a superfluid.

A real three-dimensional BEC has an inhomogeneous density distribution. Our results should be valid when the spatial variation of the density over a lattice constant is small. When one tries to drag the BEC by a periodic potential, it may happen that only the low density region is affected while the high density portion of the BEC remains undragged due to the self-interaction of the atoms.

Finally, we show in Fig. 4 Bloch oscillations of the condensate when the potential is accelerated at a constant rate a. The average slope of the current is given by the acceleration, meaning that the condensate follows the potential on average. The oscillatory modulations can be understood by the following arguments. In the comoving frame, the potential is stationary but the atoms feel an inertial force, which makes the Bloch wave number drifting at a rate of $-a$. If the lowest band of the effective potential has the dispersion $\epsilon(k)$, then the velocity in the comoving frame is given by $\epsilon'(-at)$. Because of the periodicity of the Bloch band, this velocity has a zero mean and an oscillation period of $1/a$, which agree with the results in the figure.

The size and shape of the modulations (excluding the fast oscillations to be discussed below) in Fig. 4 can also be explained in terms of Bloch oscillations in the effective potential [9]. For $C = 0$, the potential of strength $V_0 = 0.4$ is known to produce a narrow band with a cosine energy dispersion, which explains the small and sinusoidal modulations in that case. For large C, the effective potential is weak, and the energy dispersion is parabolic (as $k^2/2$ in the free case) except near the Brillouin zone edge $k = \pm\frac{1}{2}$, where it becomes flat due to Bragg reflection. The acceleration of the condensate in the comoving frame is given by $-a\epsilon''(-at)$, which nearly cancels the acceleration of the potential everywhere, except when k is near the zone edge. The velocity of the condensate should then follow a stair case curve, with the steps coinciding with the occurrence of Bragg reflections. The fast oscillations for the cases of $C = 0.3$ and 0.5 in Fig. 4 are due to Landau-Zener tunneling [25] through the gap between the lowest two bands of the effective potential. The critical acceleration for the tunneling is $\pi V_{\mathrm{eff}}^2/2$ [8], which is smaller for larger values of C and becomes comparable to the acceleration used in the calculation for the above two cases. A detailed study of Zener tunneling of a BEC will be reported in the future.

Experimental realization and future directions.—In typical experiments to date, the relevant parameters are given by $n_0 = 10^{20}\ \mathrm{m}^{-3}$, $a = 5.4$ nm, and $k_L = 2\pi/\lambda = 8.06 \times 10^6\ \mathrm{m}^{-1}$ for Rb [26] and $n_0 = 3 \times 10^{21}\ \mathrm{m}^{-3}$, $a = 2.65$ nm, and $k_L = 1.07 \times 10^7\ \mathrm{m}^{-1}$ for

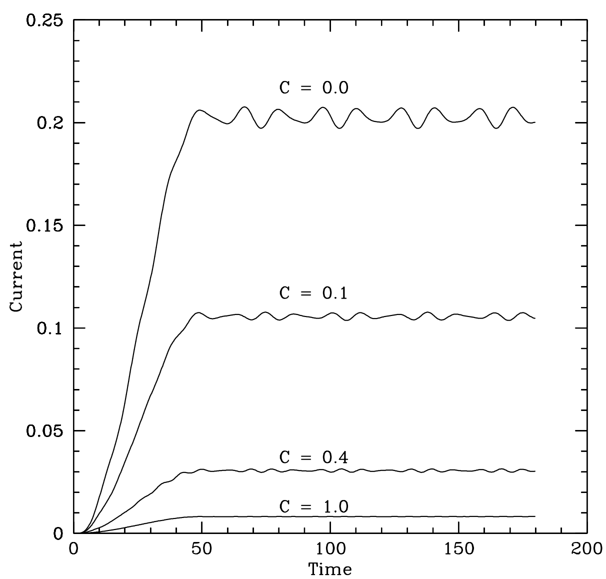

FIG. 3. Current as a function of time for the wave function with zero initial current. $V_0 = 0.1$ and the acceleration occurs between $t = 0$ and $t = 50$. Results are shown for $C = 0.0$, 0.1, 0.4, and 1.0.

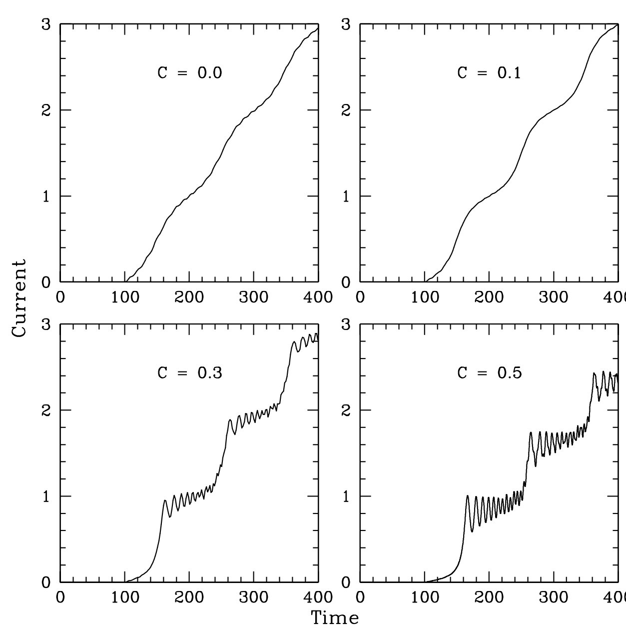

FIG. 4. Current as a function of time for the wave function with zero initial current. Parameters are $V_0 = 0.4$ and $t_0 = 70$. The acceleration is $a = 0.01$ for $t \geq 100$. Results are shown for $C = 0.0$, 0.1, 0.3, and 0.5.

Na [12]. The strength of atom-atom interaction is given by $C = \frac{\pi n_0 a}{k_L^2} = 2.6 \times 10^{-2}$ for Rb and $C = 2.2 \times 10^{-1}$ for Na. Larger values of C may be achieved by a higher density of the condensate, a higher a, and a smaller k_L. All three parameters can be changed independently. A higher density may be achieved by a factor of 5 enhancement without rendering the condensate's lifetime too short for the processes discussed here. a can be tuned as well by a Feshbach resonance [12,27,28]. A smaller k_L may be achieved by adjusting the relative angle between the two beams of interfering light without changing the laser frequency. Therefore, the phenomena discussed in this Letter should be observable within the current experimental capability.

Apart from a detailed study of the Landau-Zener tunneling mentioned above, future theoretical investigations are needed to explore other possibilities of the condensate motion such as Wannier-Stark ladders and quantum chaos, which have been observed on cold but non-BEC atoms. Because of the nonlinearity due to atomic interactions, spontaneous breaking of translational symmetry of the condensate can occur under certain conditions as is seen in a primitive study of ours. Further improvement of the theoretical framework is also needed to include the effects of thermal and quantum fluctuations.

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Note added.—During the revision of this manuscript, we received a preprint (now published) from Anderson and Kasevich, reporting the observation of Bloch oscillations and Zener tunneling of a BEC in a stationary optical lattice under gravity [29]. In the free-falling frame of reference, this experimental system is equivalent to the one described here and offers a potentially good testing ground for our theoretical predictions.

[1] M. H. Anderson *et al.*, Science **269**, 198 (1995).
[2] C. C. Bradley *et al.*, Phys. Rev. Lett. **75**, 1687 (1995).
[3] K. B. Davis *et al.*, Phys. Rev. Lett. **75**, 3969 (1995).
[4] M. Brewczyk, K. Rzążewski, and C. W. Clark, Phys. Rev. A **57**, 488 (1998).
[5] K. Berg-Sørensen and K. Mølmer, Phys. Rev. A **58**, 1480 (1998); K.-P. Marzlin and Weiping Zhang, cond-mat/9810085; M. J. Steel and Weiping Zhang, cond-mat/9810284.
[6] D. Jaksch *et al.*, Phys. Rev. Lett. **81**, 3108 (1998).
[7] M. Raizen, C. Salomon, and Q. Niu, Phys. Today **50**, No. 7, 30 (1997).
[8] Q. Niu, X.-G. Zhao, G. A. Georgakis, and M. G. Raizen, Phys. Rev. Lett. **76**, 4504 (1996).
[9] M. Ben-Dahan *et al.*, Phys. Rev. Lett. **76**, 4508 (1996).
[10] S. R. Wilkinson, C. F. Bharucha, K. W. Madison, Qian Niu, and M. G. Raizen, Phys. Rev. Lett. **76**, 4512 (1996).
[11] M.-O. Mewes *et al.*, Phys. Rev. Lett. **77**, 416 (1996).
[12] S. Inouye, M. R. Andrews, J. Stenger, H.-J. Miesner, D. M. Stamper-Kurn, and W. Ketterle (to be published).
[13] V. M. Pérez-García, H. Michinel, and H. Herrero, Phys. Rev. A **57**, 5 (1998).
[14] P. A. Ruprecht, M. J. Holland, K. Burnett, and M. Edwards, Phys. Rev. A **51**, 4704 (1995).
[15] A. Smerzi and S. Fantoni, Phys. Rev. Lett. **78**, 3589 (1997).
[16] M. Edwards and K. Burnett, Phys. Rev. A **51**, 1382 (1995).
[17] M. Edwards *et al.*, Phys. Rev. A **53**, R1950 (1996).
[18] M. Edwards *et al.*, Phys. Rev. Lett. **77**, 1671 (1996).
[19] S. Stringari, Phys. Rev. Lett. **77**, 2360 (1996).
[20] K. G. Singh and D. S. Rokhsar, Phys. Rev. Lett. **77**, 1667 (1996).
[21] V. L. Ginzburg and L. P. Pitaevskii, Zh. Eksp. Teor. Fiz. **34**, 1240 (1958).
[22] E. P. Gross, J. Math. Phys. (N.Y.) **4**, 195 (1963).
[23] W. H. Press *et al.*, *Numerical Recipes: The Art of Scientific Computing* (Cambridge University Press, Cambridge, England, 1992), 2nd ed.
[24] A. R. Mitchell and D. F. Griffiths, *The Finite Difference Method in Partial Differential Equations* (Wiley, Chichester, 1980).
[25] C. Zener, Proc. R. Soc. London A **137**, 696 (1932).
[26] E. A. Burt *et al.*, Phys. Rev. Lett. **79**, 337 (1997).
[27] J. M. Vogels *et al.*, Phys. Rev. A **56**, R1067 (1997).
[28] Ph. Courteille, R. S. Freeland, D. J. Heinzen, F. A. van Abeelen, and B. J. Verhaar (to be published).
[29] B. P. Anderson and M. A. Kasevich, Science **282**, 1686 (1998).