Comment on "$\Delta I = 4$ Bifurcation in Ground Bands of Even-Even Nuclei and the Interacting Boson Model"

Toki and Wu (TW) [1] analyzed the ground-band energies in a number of deformed nuclei and claimed to find a staggering pattern, which they interpreted in the interacting boson model (IBM) by allowing a much larger deformation of the boson systems than hitherto assumed. Both the experimental results and their theoretical interpretation come as a surprise and deserve a closer examination.

To measure the purported staggering effects in normally deformed nuclei requires measurements of extended sequences of γ-ray transition energies to a precision and accuracy much better than $\sim$0.1 keV. This is a delicate experimental problem that depends on factors such as the linearity of the electronics employed, the energy dispersion of the analog-to-digital conversion, and being certain that the γ-ray peaks observed are free from contamination. If the original workers were not concerned about such effects at the required level, one could be misled by uncritically accepting compiled data.

We have examined the experimental evidence for staggering in the nuclei mentioned by TW. We find that the experimental situation is at best intriguing but by no means compelling. In virtually all cases the error bars on the staggering parameter are larger than the apparent staggering effect, and the data can be equally well described with no staggering. In the few cases where the staggering parameter deviates significantly from zero, there is no $\Delta I = 4$ bifurcation pattern and alternative explanations should be sought.

IBM Hamiltonians leading to very large deformations have not been used in the literature previously. TW focused only on the ground band and did not consider the β and γ bands, which set the energy scale and hence are essential for a consistent description of spectra. We address this issue using the standard quadrupole Hamiltonian and the intrinsic state for the N boson system,

$$H = -\kappa Q \cdot Q, \qquad Q = [s^\dagger \tilde{d} + d^\dagger \tilde{s}]^{(2)} + \chi[d^\dagger \tilde{d}]^{(2)},$$
$$|\phi\rangle = (N!\,(1 + \beta^2)^N)^{-1/2}[s^\dagger + \beta d_0^\dagger]^N|0\rangle. \quad (1)$$

The solution for the IBM deformation parameter β follows from $\beta^2 - \overline{\chi}\beta - 1 = 0$, where $\overline{\chi} = -\sqrt{2/7}\,\chi$ [2] (see Fig. 1a). Analytic expressions for the energy and $E2$ transitions involving the ground, β, and γ bands have been derived using angular momentum projected mean field theory, which leads to a $1/N$ expansion [3]. To leading order in N, the relevant energies are given by

$$E_2 = \kappa(1 + \beta^2)^2/2\beta^2, \qquad E_\beta = 2N\kappa(1 + \beta^2),$$
$$E_\gamma = 2N\kappa[\beta(\beta + \overline{\chi}) - (1 - \overline{\chi}\beta)^2]/(1 + \beta^2). \quad (2)$$

Similar expressions are available for the $E2$ transitions, of which we quote here $R = NB(E2; 2_\gamma \to 0_g)/B(E2; 2g \to 0_g)$, obtained using the Q operator in Eq. (1),

$$R = 2(1 - \overline{\chi}\beta)^2/[\beta^2(1 + \beta^2)]. \quad (3)$$

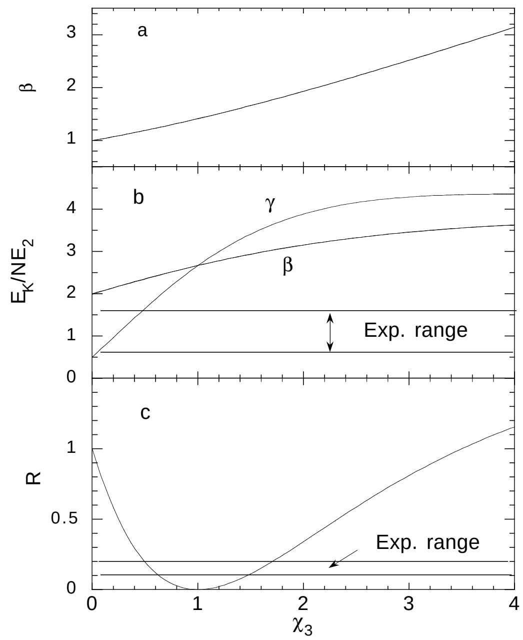

FIG. 1. (a) Deformation β as a function of $\chi_3 = \chi/(-\sqrt{7}/2)$ normalized to the SU(3) value; (b) energy ratios for the β and γ bands; (c) $B(E2)$ ratio R for the γ band.

In Fig. 1, we show how the above quantities change with χ (and hence deformation). Note that the ratios plotted are independent of N and the interaction strength κ. Experimental energy ratios in the rare earths and actinides are indicated in Fig. 1b. The deformation used by TW ($\beta = 2.6$) corresponds to $\chi_3 = 3.2$, that is 3.2 times the SU(3) value. It is clear from Fig. 1b that such large deformations are not compatible with the experimental systematics. In fact, for a consistent description of data, one has to include a d-boson term in (1) (besides using $\chi_3 < 1$), which reduces β further [4]. Finally, for large deformations, the $B(E2)$ ratio in Fig. 1c is an order of magnitude larger than the experimental values in the actinides. We conclude that the staggering phenomena proposed by TW do not have a sound experimental or theoretical basis.

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[1] H. Toki and L.A. Wu, Phys. Rev. Lett. **79**, 2006 (1997).
[2] J.N. Ginocchio and M.W. Kirson, Nucl. Phys. **A350**, 31 (1980).
[3] S. Kuyucak and I. Morrison, Ann. Phys. (N.Y.) **181**, 79 (1988); **195**, 126 (1989).
[4] S. Kuyucak and S.C. Li, Phys. Lett. B **354**, 189 (1995).