Stimulated Emission and Amplification in Josephson Junction Arrays

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We report measurements of mm-wavelength radiation from two-dimensional arrays of underdamped Josephson junctions. All of our samples emit coherently in a novel synchronized state, which is triggered by a resonance in the array structure. Measurements of the detected power as a function of the number N of active junctions show a threshold, suggesting population inversion. Above threshold, the power scales with N^2 up to an array size bigger than the free-space radiation wavelength. The highest measured conversion efficiency from dc to ac power is about 17%. Our data are consistent with stimulated emission causing coherence and cannot be explained by existing classical coupling mechanisms.
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Josephson junctions are natural voltage-to-frequency transducers. When a dc voltage V_{dc} is applied across a junction, the supercurrent oscillates at a frequency [1],

$$\nu = 2eV_{\mathrm{dc}}/h\,, \tag{1}$$

where e is the electron charge and h is the Planck constant. In 1970, Tilley [2] predicted that interconnected Josephson junctions could self-synchronize with a common radiation field and emit coherently. The underlying physical mechanism suggested for synchronization was a quantum one, completely analogous to the quantum description of a collection of superradiant atoms in a resonant cavity. One prediction of this paper was that the output power would scale as the square of the number of active junctions. In general, such a N^2 dependence is a signature of coherence for a system of N oscillators. The connection between a single Josephson junction and the quantum radiating two-level atom was further investigated by Rogovin and Scully in 1976 [3]. They found theoretically that spontaneous emission as well as stimulated emission and absorption should occur in Josephson junctions [3].

One-dimensional and two-dimensional arrays have been shown to emit coherently (but classically) in previous papers [4,5]. A necessary requirement for the operation of these devices was that they had to be small compared to the radiation wavelength. In this case, synchronization was understood by classically treating the interaction between the junctions and the electromagnetic field. In fact, the coupling was classically described (via Kirchhoff's laws) through the ac currents flowing in the external load [4,6], as distinguished from a quantum stimulated emission mechanism involving Cooper pairs and photons.

In this Letter we present measurements of 150 GHz radiation emitted by two-dimensional Josephson junction arrays. Our arrays are very different from typical arrays which are coherent through classical coupling. First, our devices are made of underdamped junctions (no shunt resistors are present and the dissipation in the arrays is orders of magnitude smaller than in the shunted arrays studied in Refs. [4,5]). Moreover, each junction is coupled to a high-Q cell resonance, while previous arrays are low-Q oscillators. Our data provide the first strong evidence that an array of many Josephson junctions, which are macroscopic and nonidentical objects, can radiate in a quantum coherent state in the same manner as atoms in a laser.

In our samples, the array size is comparable to or, in some devices, larger than the free-space radiation wavelength $\lambda \simeq 2$ mm [7]. Our emission frequency corresponds to a high-Q resonance in the structure formed by the array and the ground plane. The resonance causes sharp, self-induced steps in the current-voltage characteristic of the array. We measure the power coupled to a detector as a function of the number of array junctions biased on the resonant step. No detectable power is coupled into the detector if the number of active junctions is below a threshold N_T. (A threshold is found in lasers and occurs when gain exceeds losses, indicating population inversion.)

Above N_T, the detected power scales with the square of the number of active junctions, up to dc-to-ac power conversion efficiency of about 17%. By contrast, arrays which synchronize through classical coupling mechanisms have maximum ac-to-dc conversion efficiency on the order of 1% [4,5,9]. To our knowledge, no classical electrodynamical description can account for all our experimental results. However, all the characteristics of the detected radiation can be described in terms of laser systems.

Figure 1 is a sketch of our samples. The array is made of underdamped $\mathrm{Nb/Al/AlO}_x\mathrm{/Nb}$ Josephson tunnel junctions arranged in a square geometry. The spacing between the junctions is 13 μm. The junction area is 16 $\mu\mathrm{m}^2$ and their critical current density is about 1 $\mathrm{kA/cm}^2$. The array is capacitively coupled to a two-junction detector circuit [10]. The distance between the array and the detector is about 100 μm (see Fig. 1).

Figure 2(a) shows the current-voltage (IV) characteristic of a ten column by ten row array (the rows are

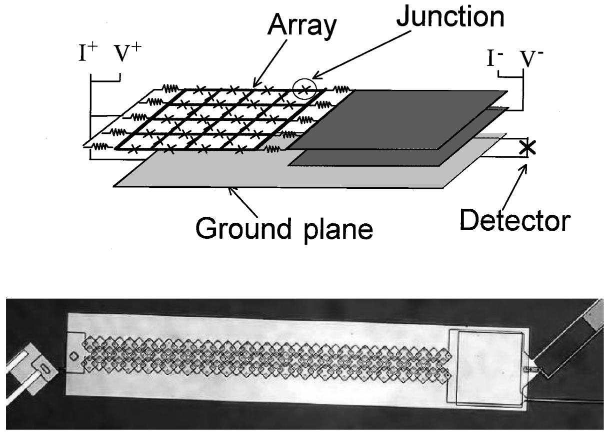

FIG. 1. Sketch of a typical sample (top). The square array, to the left, is capacitively coupled to a detector circuit, on the right. A ground plane underlies the entire structure. Picture of a 3×36 array (bottom). The length of the array is about 470 μm.

perpendicular to the direction of the bias current), obtained by sweeping the bias current many times within the range of values shown in the plot. The curve is very hysteretic and shows several branches spaced by about 2.7 mV, the gap voltage of a single junction [11,12]. The first branch corresponds to a state where only a single row of junctions in the array is switched to the gap voltage, while all the other rows are in the zero-voltage state (the supercurrent branch). Each higher voltage branch indicates the switching of an additional row to the gap voltage; eventually all ten rows switch. Because of the hysteresis of the *IV* characteristic, it is possible to control the number of junctions which are at a nonzero voltage.

When we apply an external magnetic field of about 40 Oe in the plane of the array [13], sharp steps appear in the low-voltage region of the *IV* curve [see Fig. 2(b)]. Their voltage spacing is much smaller (400 μV in this sample) and very easily distinguished from the spacing of the gap-voltage branches. Constant-voltage steps in the *IV* characteristic of a Josephson junction indicate the presence of high-frequency radiation, with the frequency ν related to the voltage spacing of the steps by Eq. (1). When caused by external radiation, they are called Shapiro steps [14]. The steps in Fig. 2(b) are self-induced (occurring in the absence of external radiation). They are due to the supercurrent oscillations in junctions locking to a resonance in the array structure (at 200 GHz in this array). We will refer to these steps as self-induced resonant steps (SIRS).

Detailed measurements and analysis of *IV* curves show that the number of steps corresponds to the number of rows biased in this resonant state. For example, the first SIRS corresponds to a single row of the array biased on the resonant state, while the tenth SIRS corresponds to all ten rows in the array biased on this state. The hysteresis of the *IV* curve allows control of the number of *active* junctions. (In the following, we will refer to the junctions biased on the SIRS as *active* junctions.)

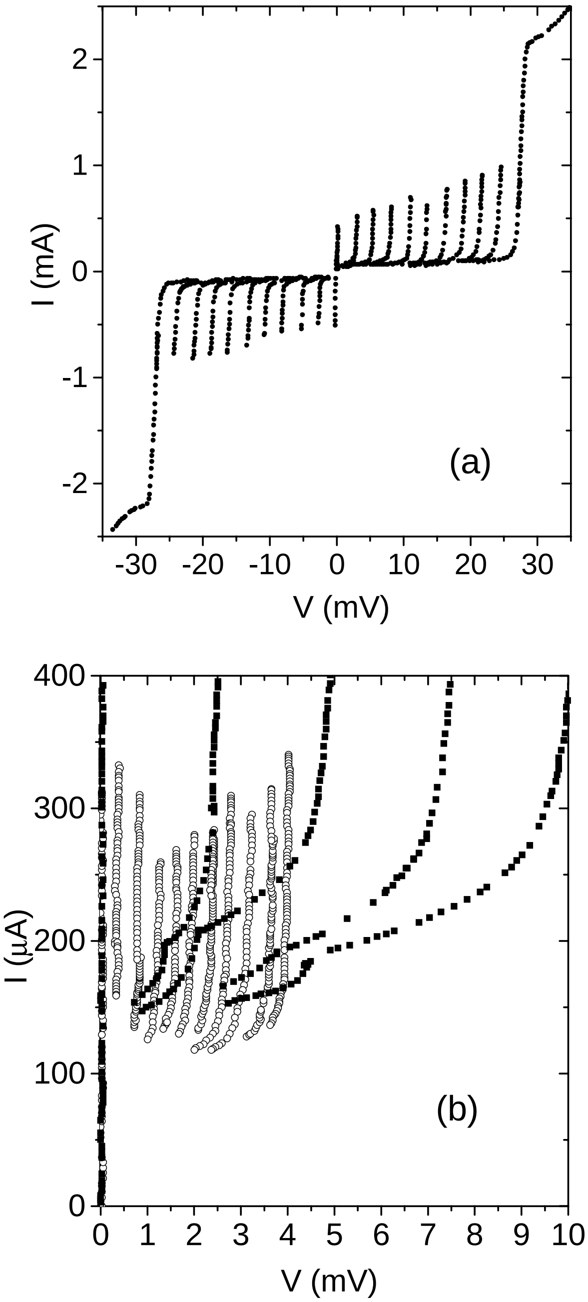

FIG. 2. (a) Current-voltage characteristic of a 10×10 array. Each branch corresponds to the switching of an additional row to the gap voltage. (b) Enlargement of the low-voltage region (solid squares). In the presence of an external magnetic field in the plane of the array, $H = 40$ Oe, sharp resonant steps appear (empty circles).

Figure 1 shows a 3×36 array, with the ground plane above (rather than below) the array. This array showed resonant steps at 306 μV, corresponding to a Josephson frequency of about 148 GHz. This resonance is related to the size of the array cell and to the distance between the array and the ground plane and is given to within 10% by

$(1/2\pi)\sqrt{1/LC}$, where C is the junction capacitance and L is the inductance of the transmission line connecting adjacent junctions. Similar resonances were previously found in Josephson interferometers [15]. The resonance shifts to lower frequencies when this distance is increased and disappears when the ground plane is removed. The frequency of the resonance is not affected by temperature or magnetic field. We also varied the length of the array in the range 300 μm to 3 mm and found no measurable change in the frequency for arrays fabricated on the same chip. For arrays fabricated on different chips the spread in emission frequency was within 10% [16].

Figure 3 shows the detected ac power as a function of the input dc power for the array in Fig. 1. In the case of identical junctions in the array, the input dc power is proportional to the number of junctions which are in the same non-zero-voltage dynamical state, since each junction absorbs the same power from the dc supply.

When biasing the array on the steps, no detectable radiation was coupled into the detector junction up to the 14th SIRS, i.e., up to 14 active rows. Note, however, that the SIRS indicate the presence of radiation in the array. Above this threshold, the ac power increases as the square of the dc power, i.e., as the square of the number of active junctions. The maximum detected power in this array is about 0.05 μW, about 17% of the input dc power.

We performed similar measurements on a longer array, 3×230, about 3 mm long. In this array the resonant frequency was 160 GHz. The result is shown in Fig. 4(a). No output power is measured below 61 active rows (although, again, SIRS were present in the array IV characteristic). Above this threshold, the data are scattered in a wide region of the plot for the following reason. The current-voltage characteristic of such a big array is very complicated and covers a wide voltage range. At a given bias point, there are three possible stable states for each junction: (1) the zero-voltage branch, (2) SIRS, and (3) the gap-voltage branch. For such a big array, the complexity of the measured IV curve does not allow us to distinguish whether *all* the non-zero-voltage junctions are biased on the SIRS or a few of them are on the gap-voltage branch. We made the assumption that, for a given dc bias, the data corresponding to the highest emitted ac power [marked with squares in Fig. 4(a)] correspond to all the non-zero-voltage junctions synchronized on the resonant state. For these points, the dc power is proportional to the number of active junctions.

In Fig. 4(b), we plot the ac power corresponding to the points selected in Fig. 4(a) as a function of the square of the dc power. The plot shows that the ac power again grows as the square of the number of active junctions, up to a maximum detected power of 0.122 μW, about 5% of

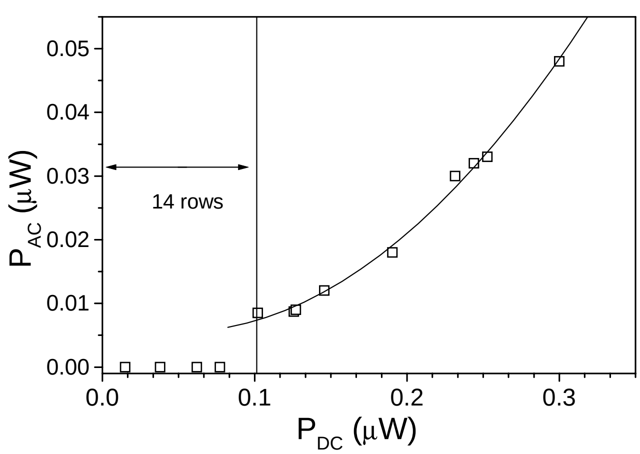

FIG. 3. Detected ac power as a function of the dc input power for a 3×36 array (open squares). If all the non-zero-voltage junctions are on the same dynamical state (resonant step, in this case), the input dc power is proportional to the number of these junctions. No output power is measured below 14 active rows. The solid line is the best quadratic fit for the data above 14 rows. For the units used in the plot, $P_{\mathrm{ac}} = 0.007 - 0.07P_{\mathrm{dc}} + 0.7P_{\mathrm{dc}}^2$.

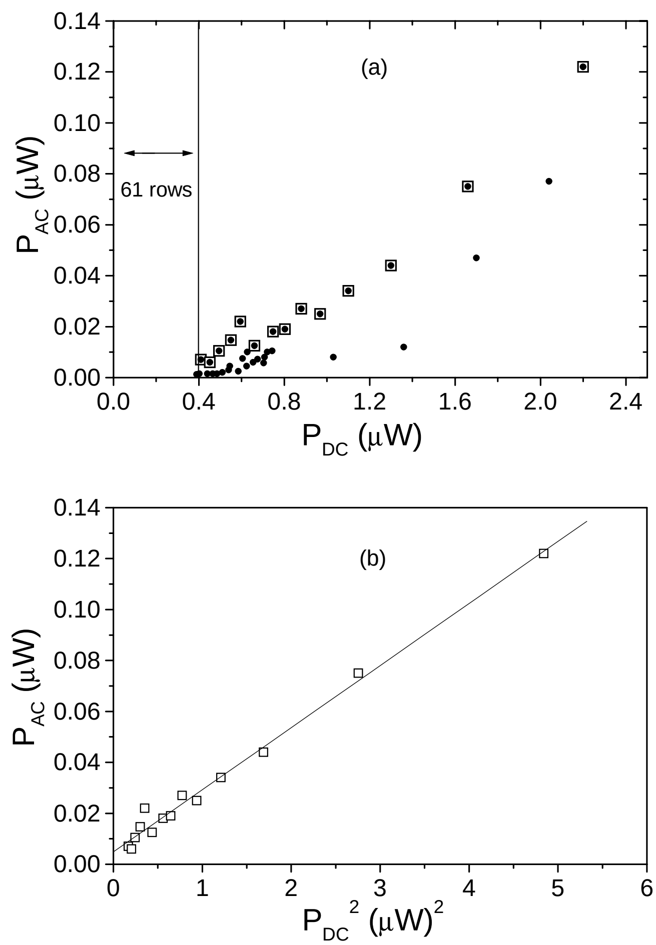

FIG. 4. (a) Detected ac power as a function of the dc input power for a 3×230 array (solid circles). No output power is measured below 61 active rows. The selected data, corresponding to highest emitted ac power for a given dc bias, are marked with open squares. (b) Detected ac power as a function of the square of the dc input power corresponding to the data points selected in (a).

the corresponding input dc power. This maximum power occurs at an array bias voltage of 77 mV, corresponding approximately to the 230th resonant step.

All the characteristics of coherent emission from our arrays are analogous to the operation of lasers:

(a) The junctions in the array are strongly coupled to a resonance. This resonance corresponds to a specific propagation mode of the radiation in the cavity formed by the array itself and the ground plane. The oscillating junctions (which are biased at a voltage corresponding to the resonant frequency) are the sources of this radiation field, which in turn produces SIRS in their IV curves [see Fig. 2(b)].

(b) A single Josephson junction is analogous to a single two-level atom [3]: a voltage across the junction naturally creates two energy levels for the Cooper pairs. The transition from the high energy level to the low energy level corresponds to the tunneling of each Cooper pair from the high-voltage electrode to the low-voltage electrode and to the emission of a photon at frequency ν given by Eq. (1).

(c) There is a threshold for the detection of output radiation. A threshold results in a laser when the population inversion is sufficiently high to obtain amplification. In the array, it means that a certain minimum number of active junctions are necessary in order to overcome the dissipation due to losses in the cavity and in the junctions. This threshold increases by increasing the array size, consistent with a laser.

(d) Below threshold, the array emits incoherently, above threshold, it emits coherently (as indicated by the N^2 dependence). In addition, the dc-to-ac power conversion efficiency is 1 order of magnitude higher than other "classic" Josephson junction arrays.

(e) The coherent operation of all the junctions is not affected by increasing the array length up to a few free-space wavelengths. This is consistent with a *local* synchronization mechanism between the incoming radiation at the junction site and the junction emission (such as stimulated emission).

We conclude that our underdamped two-dimensional Josephson junction arrays undergo a transition to a coherent state which is analogous to the transition undergone by lasers. This transition was predicted by Bonifacio *et al.* [17] on the basis of the formal analogy between Josephson junction arrays and free-electron lasers. Our results are the first experimental confirmation of the predictions from Tilley [2] and Rogovin and Scully [3], who theoretically developed the analogies between junctions and coherent atomic systems in the 1970s.

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[1] B. D. Josephson, Phys. Lett. **1**, 251 (1962).
[2] D. R. Tilley, Phys. Lett. **33A**, 205 (1970).
[3] D. Rogovin and M. Scully, Phys. Rep. **25C**, 175 (1976).
[4] A. K. Jain *et al.*, Phys. Rep. **109**, 309 (1984).
[5] S. P. Benz and C. J. Burroughs, Appl. Phys. Lett. **58**, 2162 (1991).
[6] K. Wiesenfeld, S. Benz, and P. A. A. Booi, J. Appl. Phys. **76**, 3835 (1994).
[7] Recent calculations [8] show that the dispersion relation of the transmission line formed by a 1D array and the ground plane are nonlinear. Experimentally, the operation frequency of the array corresponds to the $k = 0$ mode, where k is the wave vector. However, in our arrays, the frequency of the mode is not affected by the boundary conditions of the array (different boundary conditions should correspond to different k's).
[8] T. M. Antonsen and E. Ott (private communication).
[9] A. B. Cawthorne *et al.*, IEEE Trans. Appl. Supercond. **7**, 3403 (1997).
[10] J. Zmuidzinas *et al.*, IEEE Trans. Microwave Theory Tech. **42**, 698 (1994).
[11] K. K. Likharev, in *Dynamics of Josephson Junctions and Circuits* (Gordon and Breach Science Publishers, New York, 1986).
[12] H. S. J. van der Zant *et al.*, Phys. Rev. B **38**, 5154 (1988).
[13] A magnetic field applied in the plane of a Josephson junction changes its critical current I_C, giving a Fraunhofer interference pattern [11]. By tuning I_C, the coupling between a single junction and an external resonator, as well as the size of the self-induced resonant steps, can be optimized [11].
[14] S. Shapiro *et al.*, Rev. Mod. Phys. **36**, 233 (1964).
[15] D. B. Tuckerman and J. H. Magerlein, Appl. Phys. Lett. **37**, 241 (1980).
[16] This spread can be accounted for by the fabrication process not being exactly reproducible: the thicknesses of the SiO_2 layers, determining the distance between the arrays and the ground planes, are slightly different for different chips.
[17] R. Bonifacio *et al.*, Lett. Nuovo Cimento **34**, 520 (1982).