Ultrasound Propagation in Externally Stressed Granular Media

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Experimental observations of pulsed ultrasonic transmission through granular glass beads under oedometric loading are presented. We observe in the transmitted signals the *coexistence* of a coherent ballistic pulse traveling through an "effective contact medium" and a specklelike multiply scattered signal. The relative amplitudes of these signals strongly depend on the ratios of the bead size to the wavelength and to the detector size. Experimental data support recent descriptions of the inhomogeneous stress field within granular media. [S0031-9007(99)08548-8]

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Granular media exhibit interesting and unusual properties, different from those of either solids or liquids [1,2]. Photoelastic visualizations of 2D and 3D static confined granular systems under external stress have provided evidence of strong inhomogeneities in the distribution of contact forces between grains [3,4], giving rise to strong force chains extending over a scale much larger than the size of an individual grain [3–5]. Characterizations of these stress distributions are of importance to the understanding of the structure and mechanical properties of granular media, in particular, in view of improving handling technologies [1,2].

Owing to their efficient penetration inside elastic media, sound waves offer a very useful, and sometimes unique, technique for investigation of real (optically opaque or strongly light diffusing) 3D granular assemblies. The measurement of their velocity and attenuation can provide a sensitive probe of both the structure and mechanical properties of the material through which they propagate. The description of sound propagation in granular materials is thus a problem of fundamental interest. Recently, Liu and Nagel carried out a set of experiments involving sound propagation in a glass bead system under gravity [6]. In these experiments, sound pulses were generated via a loudspeaker and detected by a small microphone of size comparable with the grain diameter d. Typical responses in the time domain at low vibration amplitudes consisted of a sharp rising edge followed by a few strong spikes and a decaying tail. On the basis of the large difference between the time-of-flight velocity ($\approx$300 m/s) and the group velocity ($\approx$60 m/s) determined from the slope of the phase spectrum of the total detected signal, these authors claimed that there is an ambiguity about the determination of sound velocity in such a system. Also, they found that sound propagation is very sensitive to changes in the packing configuration produced by thermal expansion of a single grain or by high amplitude vibrational excitation. They interpreted this as being due to sound propagating within the granular medium predominantly along strong force chains.

In soil mechanics and geophysics, however, the effective medium approach (a long wavelength description) has been commonly used to describe sound propagation in granular media, and the determination of the (compressional or shear) sound velocity does not seem problematic [7,8]. This observation leads to a fundamental question: Is it possible to reconcile these two descriptions of sound propagation in a granular medium? In this Letter, we address this question by presenting new results pertaining to the transmission of low amplitude ultrasonic pulses through noncohesive glass bead packings confined in a vessel under external load. Special emphasis will be put on the nature of sound propagation for different ratios of the wavelength to the size of the glass beads.

Experiments.—A schematic diagram of the experimental setup is shown in the inset of Fig. 1a. Our samples consist of random packings of glass beads, contained in a Plexiglas cylinder of height 35 mm, inner diameter 30 mm, with the top and bottom surfaces made of close fitting pistons. The beads are poured into the container, being vibrated horizontally. The compactivity thus obtained is found to be of the order of 0.63, and is reproducible, for a given bead sample, to within less than 1%. Once the cell is filled, a normal load corresponding to apparent pressures P ranging from 0.03 to 3 MPa is applied to the upper piston using a jackscrew arrangement, while the lower piston is held fixed (oedometric loading). At such load levels, the contribution of gravity to the stresses transmitted by the Hertzian contacts is negligible.

To optimize the ultrasonic excitation and detection, piezoelectric transducers, essentially sensitive to compressional vibrations along their axes, are put in direct contact with the glass beads at the top and bottom of the container (Fig. 1a). The acoustic source consists of a 12 mm diameter uniformly vibrating transducer. The excitation is realized by using one-cycle pulses of 2 μs duration with broadband spectrum (20 kHz–1 MHz) centered at 500 kHz. The vibration amplitude of the source is measured to be 10 nm in air by means of an optical interferometer. As for the detector, centered on the axis of the source, two different transducers are used: a large one, of diameter 12 mm equal to that of the emitter, and a small one, of diameter 2 mm. The signal-to-noise ratio is improved by repetitive averaging of the detected signals

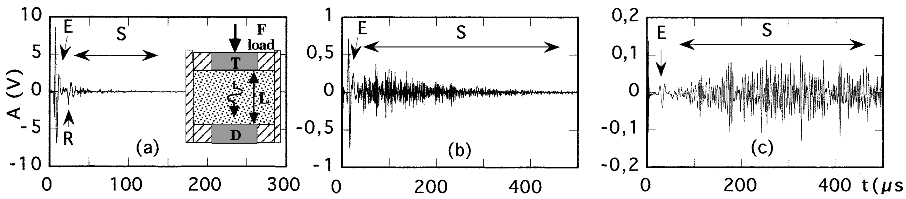

FIG. 1. Ultrasonic signals measured by a 12-mm-diam transducer in glass bead packings of different sizes under external normal stress $P = 0.75$ MPa at (a) $d = 0.2-0.3$ mm, (b) $d = 0.4-0.8$ mm, (c) $d = 1.5 \pm 0.15$ mm. E and R correspond, respectively, to the coherent ballistic pulse and its echo reflected from the bottom and top surfaces. S is associated with multiply scattered sound waves. The inset of (a) shows a schematic diagram of the apparatus: T and D are, respectively, the ultrasonic emitter and detector. Note the different time scale in (a).

using a digital oscilloscope and sent to a microcomputer for processing.

Figure 1 illustrates ultrasonic signals transmitted through packings of beads of three different sizes (obtained from Centraver): (a) Polydisperse $d = 0.2-0.3$ mm; (b) polydisperse $d = 0.4-0.8$ mm; (c) $d = 1.5 \pm 0.15$ mm, under an external load $P = 0.75$ MPa. They are detected with the large transducer, placed at the same reduced distance $L/d \approx 18$ away from the source. To ascertain that the ultrasound propagates from one grain to its neighbors only through their mutual contacts and not via air, we have checked that no ultrasonic signal is detected at vanishing external load. This test, together with the sound velocity measurements described below (Fig. 4), ensures that sound transmission through the interstitial fluid, i.e., Biot's slow wave [9], is not involved in our experiments.

We first investigate the features common to all of the signals. Let us, for example, focus on the packing of intermediate size beads ($d = 0.4-0.8$ mm). As seen in Fig. 1b, the detected ultrasonic signal is basically composed of two parts: (i) an early well-defined short pulse, which we label E, (ii) closely followed by an irregular signal, S, which spreads over a time interval of hundreds of μs. We determine a time of flight associated with the arrival of the E pulse front, measured at an amplitude of 5% of the peak-to-peak one, from which we obtain a velocity, $V_{\text{eff}} = 1070 \pm 30$ m/s. By performing a separate spectral analysis of E and S, we find (Fig. 2) that E carries a rather narrow band of low frequencies, while S has a broadband strongly irregular high frequency spectrum.

On the other hand, we have investigated the effect of detector size. Figure 3a shows the signal detected by the small (2 mm wide) transducer on a packing of the same beads under the same load as in Fig. 1b. It is clearly seen that reducing the detector size leads to a considerable enhancement of the amplitude of S relative to E. Note that the irregular temporal fluctuations associated with S remain stable over the duration (typically <1 min) of an experimental run. We have ascertained this by checking the reproducibility of the signal as well as its stability against the number (50 to 100) of repetitive averaging. That is, no "aging effect" is observed on this time scale. However, a weak evolution of the signals is identifiable on much longer time scales. A systematic study of this effect is in progress.

A fundamental difference between E and S lies in their sensitivity to changes in packing configurations. This appears when comparing a first signal measured under a static load P with that detected after performing a "loading cycle," i.e., complete unloading, then reloading to the same P level. As illustrated in Fig. 3, S is highly nonreproducible, i.e., configuration sensitive. However, E exhibits a reversible behavior. More precisely, we characterize its degree of reproducibility by adjusting the reloading level to the value P' which yields the best superposition between the two E signals. We find that $\Delta P/P = (P' - P)/P$ is always no more than a few percent—for example, for the 0.4–0.8 mm bead packings at $P \approx 0.75$ MPa, $\Delta P/P \leq 4\%$.

From these experimental results, we can reasonably infer that E is a self-averaging signal, which thus probes sound propagation in an equivalent effective medium. This we confirm by determining its group velocity V_g from the phase spectrum of E alone, as windowed out of the total signal. This analysis is performed without deconvoluting the excitation pulse: Indeed, the known electric input pulse shape does not give any direct access to the excitation energy really injected through the transducer-granular medium inhomogeneous contact. The frequency dependence of the phase is linear, indicating

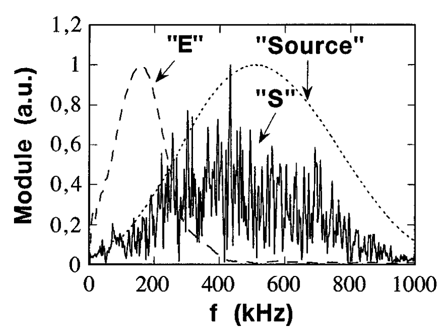

FIG. 2. Spectra of the E and S signals windowed from the total temporal response. The spectrum of the injected pulse (source) is given for comparison.

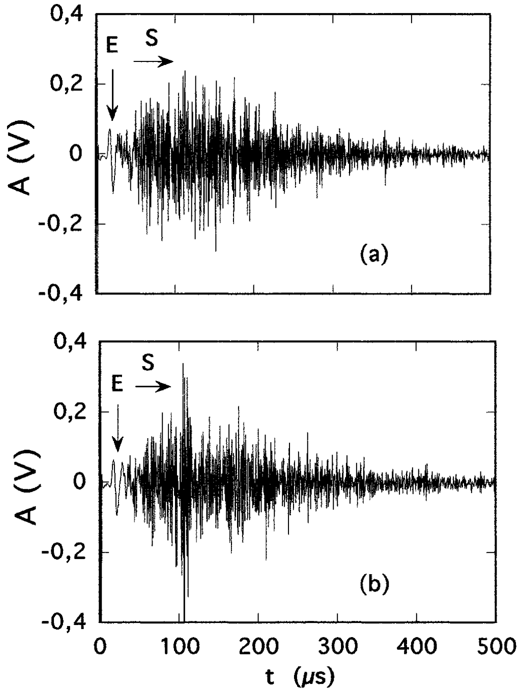

FIG. 3. Ultrasonic signals through the same bead packing as Fig. 1b detected by a smaller transducer under the same normal stress $P = 0.75$ MPa: (a) First loading; (b) reloading.

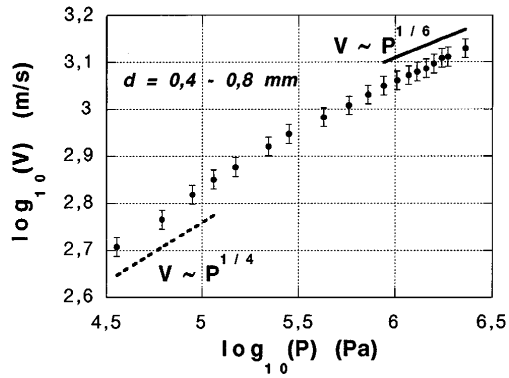

FIG. 4. Sound velocity V (data points) of the coherent E wave in the bead packing, $d = 0.4-0.8$ mm versus the applied stress P.

that E corresponds to nondispersive wave propagation. Moreover, we find that, for all of our experiments, V_g coincides with the time-of-flight velocity to within 20% at most. That is, the velocity of the E pulse unambiguously defines a sound velocity V_{eff} in our granular medium. This is confirmed by the agreement between the P dependence of V_{eff}, plotted in Fig. 4 for the 0.4–0.8 mm beads, with that found in previous works [7,8]. Namely, $V_{\mathrm{eff}}(P)$ follows a quasi power law, whose exponent slowly decreases from its low P value, $\approx 1/4$, to the Mindlin-Hertz value, $1/6$, at high load. We can also define an effective acoustic wavelength associated with the E pulse: $\lambda_{\mathrm{eff}} = V_{\mathrm{eff}}/\nu$ (with ν the central frequency of E) of an order of several mm in our experiments.

Let us finally consider the effect of bead size. As illustrated in Fig. 1, the evolution of the signals transmitted through packings of beads of various sizes with the same reduced thickness L/d exhibits systematic trends. As d increases, (i) the amplitude of the E pulse decreases rapidly, with its spectrum shifting towards low frequencies, and (ii) the amplitude of S relative to E increases; moreover, the spectrum of the $d = 1.5$ mm sample clearly exhibits strong filtering out of high frequencies.

Discussions.—For the frequencies used in our experiments, which lie much below the acoustic resonances ($\nu_{\mathrm{res}} \approx V_{\mathrm{glass}}/d$) of individual glass beads, we can model the acoustic properties of the granular medium as those of a random network of point masses (the beads) connected by bonds—the frictional Hertz contacts between neighboring beads. Mindlin has shown that, when such a contact is submitted to a static shear force fs lower than the static friction threshold ($f_s < \mu f_n$, with f_n the normal load), its elastic shear and compressive stiffness are comparable, of order $d(PE^{*2})^{1/3}$, with E^*, the elastic modulus of glass [10]. We know from experiments and simulations that, under static loading, the normal and shear loads of individual bead contacts exhibit a random distribution. Hence, the topological disorder of the granular medium induces space fluctuations of both density and local elastic stiffness.

Such an elastic medium thus appears as closely analogous to an amorphous solid, and our observations can naturally be interpreted within the framework used to describe the vibrational properties of glassy solids in terms of an inhomogeneous stress field [11], e.g., irreversible changes of the packing configuration can be paralleled with irreversible flips of two-level states. The excellent reproducibility of the signal from repeated pulses in a single run shows that, at the small vibration amplitudes used here, the triggering of such dissipative events by the sound pulse can be neglected.

We then propose the following qualitative picture. Our sample is contained in a long resonator with reflecting walls. If the sample was homogeneous, our wide acoustic emitter would excite essentially only the lowest mode of this wave guide. In the granular medium, such an injected pulse, as it propagates toward the detector, gets distorted by the disorder. We have seen that the E signal propagates coherently with a well-defined sound velocity V_{eff}. This allows us to define an effective wavelength associated with a frequency ν: $\lambda_{\mathrm{eff}} = V_{\mathrm{eff}}/\nu$. The low frequency part of the pulse, for which $\lambda_{\mathrm{eff}} \gg d$, experiences coherent but attenuated propagation in an average effective medium: This is precisely the E signal. Its coherence is proved by its reproducibility, together with the agreement between the time-of-flight (V_{eff}) and phase-analysis (V_g) determinations of the coherent wave velocity. The scattering of this effective sound wave by the density and stiffness fluctuations results, on one hand, in its attenuation, which is clearly seen, e.g., on the reduced amplitude of the echo visible in Fig. 1a. They are, on the other hand, in their excitation of other modes of the resonator, responsible for the low frequency part of the S signal.

However, most of the S amplitude is due to higher frequency components, such that $\nu \sim V_{\text{eff}}/d$. In this frequency range, ν becomes comparable with a typical cutoff frequency of our discrete system. As illustrated in [12], the modes of the network develop strong space inhomogeneities, up to, finally, complete localization. In the intermediate frequency regime, propagation becomes quasidiffusive [13]. This structure of the spectrum is responsible for the evolution of the signal with increasing bead size. For $d = 0.2-0.3$ mm, most of the injection bandwidth corresponds to the long wavelength regime where the E signal is dominant and scattering effects are small, in agreement with Rayleigh cross-section estimates. As d increases, on one hand, the scattering cross section for given λ increases as $(d/\lambda)^4$, hence the decrease of the amplitude of the unscattered E coherent pulse. On the other hand, a larger fraction on the high ν end of the excitation band falls into the above-mentioned "stop band," thus feeding the S signal only up to below a cutoff frequency which decreases as d increases.

S thus appears as an acoustic speckle signal generated by the superposition of scattered waves—hence its delayed gradual rise, indicative of a quasidiffusive regime. As emphasized in [14], since acoustic detectors measure position integrated amplitudes, the larger the detector is, the more efficient is phase cancellation between the components of S. This accounts for the large relative amplification of S that we observe with the small detector (Fig. 3). This speckle nature of S explains the nature of its variations under unloading-loading cycles. Indeed, as expected, configuration changes result in strong modifications of its detailed time structure. However, in agreement with our strong scattering picture, its global shape, which is determined by the gross statistical characteristics of the mode structure, is only weakly affected by irreversible events.

In summary, our experiments permit one to bridge between two apparently disconnected approaches to acoustic propagation in granular media, namely, the effective medium picture [7,8] and the more recent one [6] which stress mainly configuration sensitive effects. The fact that we have been able to observe, as far as we know, for the first time, the regime of coexistence of an attenuated coherent signal and its specklelike counterpart reconciles both points of view in terms of classical wave propagation in a random medium. In particular, it is clear that the inconsistency found by Liu and Nagel [6] between their time-of-flight and phase-analysis sound velocity determinations stems from the fact that they performed their phase analysis on the entire time signal, while such an analysis can only be meaningful once the S signal has been windowed out. Note that their use of a detector of size comparable with bead diameter would have been quite unfavorable to the extraction of an E signal, while, on the other hand, amplifying the sensitivity to local configuration changes.

We believe that our work points to the considerable interest in acoustic probing as a tool for studying of the mechanical properties of confined granular media. Clearly, before this can be undertaken, one should study in detail the sensitivity of the acoustic response to configurational variations. Already at the present stage, an important semiquantitative conclusion emerges from our results: The E signal is configuration insensitive. Since the coherent signal can only be self-averaging on a scale $\leq \lambda_{\text{eff}}$, this leads us to state that the correlation length of the random density and, more importantly the stress field cannot exceed a fraction of λ_{eff}, i.e., a few bead diameters—a result in qualitative agreement with the numerical results [5] for a 2D system and experimental data [15].

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