VOLUME 82, NUMBER 8 PHYSICAL REVIEW LETTERS 22 FEBRUARY 1999

New Measurement of the Anomalous Magnetic Moment of the Positive Muon

R. M. Carey,[1] W. Earle,[1] E. Efstathiadis,[1] M. F. Hare,[1] E. S. Hazen,[1] B. J. Hughes,[1] F. Krienen,[1] J. P. Miller,[1] V. Monich,[1] J. Ouyang,[1] O. Rind,[1] B. L. Roberts,[1] L. R. Sulak,[1] A. Trofimov,[1] G. Varner,[1] W. A. Worstell,[1] E. Benedict,[1] I. Logashenko,[1,3] J. Benante,[2] H. N. Brown,[2] G. Bunce,[2] J. Cullen,[2] G. T. Danby,[2] J. Geller,[2] H. Hseuh,[2] J. W. Jackson,[2] L. Jia,[2] S. Kochis,[2] R. Larsen,[2] Y. Y. Lee,[2] M. Mapes,[2] W. Meng,[2] W. M. Morse,[2] C. Pai,[2] C. Pearson,[2] I. Polk,[2] R. Prigl,[2] S. Rankowitz,[2] J. Sandberg,[2] Y. K. Semertzidis,[2] R. Shutt,[2] L. Snydstrup,[2] A. Soukas,[2] A. Stillman,[2] T. Tallerico,[2] M. Tanaka,[2,*] F. Toldo,[2] D. von Lintig,[2] D. Warburton,[2] K. Woodle,[2] A. Chertovskikh,[3] V. P. Druzhinin,[3] G. V. Fedotovich,[3] D. N. Grigorev,[3] V. B. Golubev,[3] B. I. Khazin,[3] A. Maksimov,[3] Yu. Merzliakov,[3,*] N. Ryskulov,[3] S. Serednyakov,[3] Yu. M. Shatunov,[3] E. Solodov,[3] Y. Orlov,[4] D. Winn,[5] A. Grossmann,[6] J. Gerhaeuser,[6] K. Jungmann,[6] P. von Walter,[6] G. zu Putlitz,[6] B. Bunker,[7] W. Deninger,[7] P. T. Debevec,[7] D. W. Hertzog,[7] T. D. Jones,[7] C. Polly,[7] S. Sedykh,[7] D. Urner,[7] U. Haeberlen,[8] K. Endo,[9] H. Hirabayashi,[9] S. Kurokawa,[9] A. Yamamoto,[9] M. A. Green,[10] P. Cushman,[11] J. Kindem,[11] L. Duong,[11] S. Giron,[11] R. McNabb,[11] D. Miller,[11] C. Timmermans,[11] D. Zimmerman,[11] Y. Mizumachi,[12] M. Iwasaki,[13] H. E. Ahn,[14] H. Deng,[14] S. K. Dhawan,[14] A. Disco,[14] F. J. M. Farley,[14] X. Fei,[14] M. Grosse-Perdekamp,[14] V. W. Hughes,[14] D. Kawall,[14] S. I. Redin,[14] and A. Steinmetz[14]

[1]*Department of Physics, Boston University, Boston, Massachusetts 02215*
[2]*Brookhaven National Laboratory, Upton, New York 11973*
[3]*Budker Institute of Nuclear Physics, Novosibirsk, Russia*
[4]*Newman Laboratory, Cornell University, Ithaca, New York 14853*
[5]*Fairfield University, Fairfield, Connecticut 06430*
[6]*Physikalisches Institut der Universität Heidelberg, 69120 Heidelberg, Germany*
[7]*Physics Department, University of Illinois at Urbana-Champaign, Urbana-Champaign, Illinois 61801*
[8]*MPI für Medizinische Forschung, 69120 Heidelberg, Germany*
[9]*KEK, High Energy Accelerator Research Organization, Tsukuba, Ibaraki 305-0801, Japan*
[10]*Lawrence Berkeley National Laboratory, Berkeley, California 94720*
[11]*Department of Physics, University of Minnesota, Minneapolis, Minnesota 55455*
[12]*Science University of Tokyo, Tokyo, Japan*
[13]*Tokyo Institute of Technology, Tokyo, Japan*
[14]*Department of Physics, Yale University, New Haven, Connecticut 06511*

(Received 12 October 1998)

The muon anomalous magnetic moment has been measured in a new experiment at Brookhaven. Polarized muons were stored in a superferric ring, and the angular frequency difference, ω_a, between the spin precession and orbital frequencies was determined by measuring the time distribution of high-energy decay positrons. The ratio R of ω_a to the Larmor precession frequency of free protons, ω_p, in the storage-ring magnetic field was measured. We find $R = 3.707\,220(48) \times 10^{-3}$. With $\mu_\mu/\mu_p = 3.183\,345\,47(47)$ this gives $a_{\mu^+} = 1\,165\,925(15) \times 10^{-9}$ (± 13 ppm), in good agreement with the previous CERN measurements for μ^+ and μ^- and of approximately the same precision. [S0031-9007(99)08503-8]

PACS numbers: 13.40.Em, 14.60.Ef

We report on a new measurement of the anomalous g value a_μ of the μ^+ from the Brookhaven Alternating Gradient Synchrotron (AGS) experiment E821. This result comes from data collected during 1997 in our first run. The anomalous g value is related to the gyromagnetic ratio by $a_\mu = \frac{(g-2)}{2}$.

The theoretical value of a_μ has been calculated to high precision and the comparison with experiment provides an important test of the standard model (SM) [1]. Indeed the agreement of a_μ(expt) from previous CERN experiments [2,3] with a_μ(SM) has confirmed μ-e universality and the expected modification of the photon propagator associated with virtual hadrons. A higher precision value for a_μ(expt) will test the electroweak contribution to a_μ and also speculative theories beyond the standard model which might contribute to a_μ. Indeed, the high sensitivity of a_μ to contributions from models such as supersymmetry or compositeness is a central reason for the interest in a high-precision measurement [1].

Our experiment has been designed to achieve a fractional error of 0.35 parts per 10^6 (ppm), i.e., an error in a_μ of 4×10^{-10}, which would be an improvement of a factor of 20 compared to previous measurements [2,3]. The general method is the same, but it incorporates several major new features and advances which are described below.

For polarized muons moving in a uniform magnetic field $\vec{B}$, which is perpendicular to the muon spin direction and to the plane of the orbit, and with an electric quadrupole field $\vec{E}$ for vertical focusing [2], the angular frequency difference, ω_a, between the spin precession frequency ω_s and the cyclotron frequency ω_c, is given by

$$\vec{\omega}_a = -\frac{e}{mc}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta} \times \vec{E}\right]. \quad (1)$$

The dependence of ω_a on the electric field is eliminated by storing muons with the "magic" $\gamma = 29.3$, which corresponds to a muon momentum $p = 3.09$ GeV/c. Hence, measurement of ω_a and of B determines a_μ. At the magic gamma, the muon lifetime in the storage ring is $\gamma\tau = 64.4$ μs, the $(g - 2)$ precession period is 4.37 μs, and, for the central orbit radius of 7.11 m, the cyclotron period is 149 ns.

The storage ring magnet is a superferric 700-ton, 14-m-diameter circular "C" magnet, with the opening facing inwards toward the ring center. The field is excited by three 14-m-diameter superconducting coils which carry 5.2 kA from a low voltage power supply to produce the 1.45-T magnetic field [4]. The short term field stability over several AGS cycles was better than 0.1 ppm, and the long term instability of up to 100 ppm was primarily due to thermal expansion in the magnet yoke.

A number of features are available for shimming the magnet. These include iron wedges in the air gap between the yoke and pole pieces (each of which covers 10° in azimuth), edge shims, and current loops on the pole pieces running 360° around the storage ring with a radial spacing of 0.25 cm. The current loops were used only for studies in the 1997 run. For the data reported here, the field averaged over azimuth around the ring had a uniformity over the 9-cm-diameter storage region of 25 ppm.

The field was monitored by 366 fixed NMR probes placed above and below the beam vacuum chamber [5]. Periodically the field in the storage region was mapped in 1-cm steps by an NMR trolley, with 17 NMR probes, which operates in vacuum inside the beam vacuum chamber. In 1997, the relative monitoring of the field was done at the 0.4-ppm level, and the absolute calibration relative to the free proton [6] was good to 0.5-ppm.

The magnetic field which enters in Eq. (1) is the average field seen by the muon distribution. The use of pion injection (discussed below) to store muons in the ring fills the phase space of the storage ring uniformly. This was verified for the radial distribution by measuring the positron time spectrum and taking the Fourier transform, thereby obtaining the distribution of rotation frequencies in the ring. This frequency distribution matched that expected if the radial phase space was uniformly filled. The vertical distribution of muons in the storage ring was studied by using current loops to add a small radial magnetic field, which moved the beam vertically thus changing the efficiency for muon storage in the ring. It was found that the center of the distribution was about 1 mm above the center of the storage volume, in agreement with a beam dynamics calculation which used the measured radial magnetic field.

The AGS operated at 24 GeV/c and provided eight proton bunches per 2.6-s cycle, each with $\sim 5 \times 10^{12}$ protons and a σ of 27 ns. The proton bunches were individually kicked out of the AGS at intervals of 33 ms and directed onto a nickel target of one interaction length. A 3.1-GeV/c beam of positive secondaries with 10^8 particles per bunch was transported along a 116-m beam line to a hole in the back of the yoke of the muon storage ring, at which point approximately 60% of the beam was π^+.

A superconducting inflector magnet [7], 1.7 m in length placed between the hole in the back of the yoke and the edge of the muon storage region, substantially cancels the 1.45-T storage-ring field and delivers the beam approximately parallel to the central orbit but 77 mm farther out in radius.

The positive beam, with momentum 0.5% higher than the central-orbit (magic) muon momentum, exits the inflector magnet into the storage ring. About 25 ppm of the π^+ produce decay muons which are captured into stable orbits in the ring. The polarization of these stored muons is about 97%. However, most of the beam interacts with material outside of the storage region within one turn, producing a large background (flash) for the detectors. The electric quadrupoles, which provide the vertical focusing, are initially powered asymmetrically to scrape the muon beam on a set of circular collimators and thus reduce muon losses during the measurement time. About 1000 muons were stored per proton bunch.

Positrons from the in-flight decay $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$ are detected with Pb-scintillating fiber calorimeters placed symmetrically at 24 positions around the inside of the storage ring [8]. The decay positron time spectrum is [2,3]

$$N_0 e^{-t/\gamma\tau}\{1 + A(E)\cos[\omega_a t + \phi(E)]\}. \quad (2)$$

The normalization constant N_0 and the parity violating asymmetry parameter $A(E)$ depend on the energy threshold placed on the positrons. The fractional statistical error on ω_a is proportional to $A^{-1}N_e^{-1/2}$, where N_e is the number of decay positrons detected above some energy threshold. For an energy threshold of 1.8 GeV we measure A to be 0.4, equal to its theoretical value [3], which we attribute to the good calorimeter energy resolution ($\sigma/E = 10\%$ at 1 GeV) and a scalloped vacuum chamber which minimizes preshowering before the positrons reach the calorimeters.

The photomultiplier tubes of the calorimeter were gated off before injection, and when gated on, they recovered to 90% pulse height in $\leq$400 ns and reached full operating gain in several μs. The flash following injection induced background which varied around the ring, and it was necessary to set individually the time after injection when each calorimeter was gated on, which varied from 12 to 120 μs after injection. Twenty μs after injection the flash was observed to fall approximately as t^{-x} $(1.2 < x < 2.0)$. Data were accumulated for 8.8 muon lifetimes following injection.

The calorimeter pulses were continuously sampled by custom 400-MHz waveform digitizers (WFDs), which provided both timing and energy information for the positrons. Both the NMR and WFD clocks were phase locked to the same LORAN-C frequency signal [9]. The waveforms were zero suppressed and stored in memory in the WFD until the end of the AGS cycle. Between AGS acceleration cycles the WFD data were written to tape for off-line analysis, as were the calorimeter calibration data and the magnetic field data.

A laser/LED (light-emitting diode) calibration system was used to monitor calorimeter time and gain shifts during the data-collection period. Early-to-late timing shifts over the first 200 μs were, on average, less than 20 ps. Phototube gain shifts were less than 1.0%.

For the off-line analysis, the detector response (waveform shape) to positrons was determined from our data for each calorimeter. These shapes were then fit to all pulses in the data to determine a time, an amplitude, and a width parameter for each pulse.

In addition to positron pulses, the data contained narrow pulses (1 or 2 WFD channels or $\leq$8-ns total width), which are probably due to γ rays from neutron capture producing a background near the photomultiplier tubes. This background could be distinguished from positron pulses on the basis of pulse shape and was reduced to a negligible level off-line.

Time histograms were formed for each detector, which were further divided into different running periods depending on the time when the phototubes were gated on. This made for 39 independent sets of data which were analyzed separately and were in agreement ($\chi^2/\nu = 46/39$). On average, the start time for fitting the data to obtain ω_a was 75 μs, with the earliest being 22 μs. This time was determined by taking the earliest time for which the eight-parameter function [see Eq. (3)] adequately described the data as determined by a χ^2 criterion and stability of the fit parameters. In total we obtained 11.8×10^6 positrons with energy greater than 1.8 GeV during a data-collection time of 160 h.

Because of an unfortunate misadjustment of the minimum digitizing time of the WFD during data taking with high AGS proton intensity, a distortion of the positron time spectrum was produced. This could be accounted for with an eight-parameter function which contained a time-dependent efficiency and decay asymmetry. The data were fit by minimizing χ^2 to

$$N_0 \frac{e^{-(t/\gamma\tau)}}{(t - t_0)^{\alpha}} [1 + (A_1 t + A_2)\cos(\omega_a t + \phi)] + B, \tag{3}$$

where $t_0 = 5$ μs after injection and B is a constant background term. The parameter α is necessary since the flash introduces a changing pedestal which, when combined with the WFD setup problem, resulted in a time-dependent energy threshold. Since the observed asymmetry parameter changed with time, we needed to include a time-dependent part, $A_1 t$, as well as the constant term A_2. An example of an eight-parameter fit to one set of the data is given in Fig. 1.

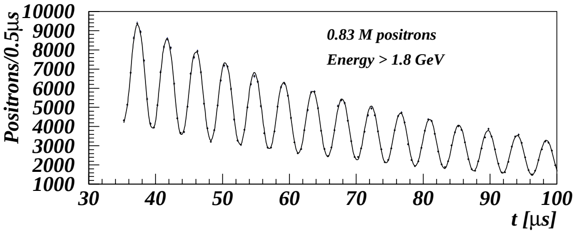

FIG. 1. A portion of an eight-parameter fit to one of the data sets [see Eq. (3)]. The fit extended to 440 μs. The solid line is the fit, and the data points are given with their error bars. The χ^2/d.o.f. was 1.028 for 803 d.o.f.

It was necessary to make two corrections to the frequency obtained from the fitting. For muons with the magic momentum, ω_a is not affected by the electric field. For the ensemble of muons in our storage ring there is a small electric field correction of +0.69 ppm to ω_a, since $\delta p/p = \pm 0.5\%$ for our ring. There is also a pitch correction of +0.4 ppm because of the vertical betatron oscillations [2,3]. Systematic errors are listed in Table I.

This experiment measures the frequency ratio $R = \omega_a/\omega_p$, where ω_p is the free proton NMR frequency in our magnetic field. Including the pitch and electric field corrections we obtain $R = 3.707\,220(47)(11) \times 10^{-3}$, where the first error is statistical and the second systematic. Adding these two errors in quadrature gives a 13-ppm relative error. We obtain a_{μ^+} from

$$a_{\mu^+} = \frac{R}{\lambda - R} = 1\,165\,925(15) \times 10^{-9} \tag{4}$$

in which $\lambda = \mu_\mu/\mu_p = 3.183\,345\,47(47)$ [10]. This new result is in good agreement with the mean of the CERN measurements for a_{μ^+} and a_{μ^-} [3,10] of $a_\mu = 1\,165\,923(8.4) \times 10^{-9}$ ($\pm$7.2 ppm). Assuming *CPT* symmetry, the weighted mean of the three measurements gives a new world average of $a_\mu = 1\,165\,923.5(7.3) \times 10^{-9}$ ($\pm$6.3 ppm).

The theoretical value of a_μ in the standard model has its dominant contribution from quantum electrodynamics, but the strong and weak interactions contribute as well [1]. The QED contribution is [11] $a_\mu(\text{QED}) =$

TABLE I. Systematic errors in ppm. In several cases the number reported is an upper limit.

Systematic effect	ϵ (ppm)
1. Magnetic field B	1.0
2. Muon distribution and $\langle B \rangle$	0.9
3. WFD time dependent efficiency	1.5
4. Nuon losses	0.2
5. Timing shifts	0.1
6. Radial E field, pitch correction	0.05
7. Fitting start time	2.0
8. BInning effects	0.2
Total systematic error	2.9

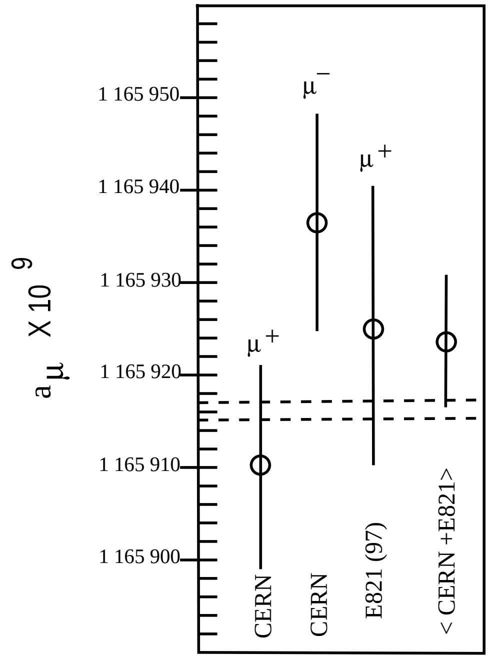

FIG. 2. The three measurements of the muon anomalous magnetic moment and their weighted average. The 1σ region allowed by the standard model (see text) is indicated by the dashed lines.

$1\,165\,847.06(2) \times 10^{-9}$. The hadronic contribution and uncertainty are dominated by the single vacuum polarization loop with hadrons present, which is determined from a dispersion relation using data from e^+e^- annihilation to hadrons and from hadronic τ decay [12]. A contribution from higher order hadronic vacuum polarization [13] and light-by-light scattering must be included. Two recent independent calculations [14,15] of the hadronic light-by-light scattering contribution are in good agreement. The total hadronic contribution is [12,13,15] $a_\mu(\text{had}) = 67.71(77) \times 10^{-9}$ (58.07 ± 0.66) ppm. The electroweak contribution is [11,16] $a_\mu(\text{weak}) = 1.51(4) \times 10^{-9}$ (1.30 ± 0.03) ppm. The standard model value is $a_\mu(\text{SM}) = 1\,165\,916.28(77) \times 10^{-9}$ (± 0.66 ppm). In Fig. 2 the three precise measurements of a_μ and their average are shown, along with the standard model prediction. The current experimental and theoretical values agree.

In conclusion, the experimental result reported here agrees with the CERN values for a_μ at a similar precision. For our upcoming runs, direct muon injection into the storage ring will be employed, and together with improvements in our knowledge of the magnetic field, the distribution of stored muons, and the detector characteristics, an experimental value for a_μ of much improved statistical and systematic accuracy can be expected. Current high activity at the e^+e^- colliders of Novosibirsk, Beijing, Frascati, and Cornell will lead to a substantially better known value of $a_\mu(\text{had})$ and hence of $a_\mu(\text{SM})$.

We thank N.P. Samios, R. Palmer, R.K. Adair, M. Schwartz, T.L. Trueman, T.B.W. Kirk, D.I. Lowenstein, and the staff of the BNL AGS for the strong support they have given this experiment over 10 years of construction. We thank J.M. Bailey, D.H. Brown, C.W. Heisey, K. Ishida, D. Loomba, W.P. Lysenko, W.B. Sampson, T. Sato, and G.T. Zwart for their contributions to parts of the preparation and running of the experiment. We gratefully acknowledge the contributions of W.W. Williams (deceased) (U. Michigan) in the early phases of this work and in the development of the collaboration. We thank T. Kinoshita, W. Marciano, and A. Czarnecki for helpful discussions on the theory, and L. Barkov for establishing the collaboration between the Budker Institute and E821 both at BNL and on the $R(s)$ measurements at BINP. This work was supported in part by the U.S. Department of Energy, The U.S. National Science Foundation, the German Bundesminister für Bildung und Forschung, The Russian Ministry of Science, and the U.S.-Japan Agreement in High Energy Physics. A. Steinmetz acknowledges support by the Alexander von Humboldt Foundation.

---

*Deceased.

[1] T. Kinoshita and W.J. Marciano, in *Quantum Electrodynamics,* edited by T. Kinoshita (World Scientific, Singapore, 1990), p. 419.
[2] J. Bailey *et al.,* Nucl. Phys. **B150**, 1 (1979).
[3] F.J.M. Farley and E. Picasso, in *Quantum Electrodynamics,* edited by T. Kinoshita (World Scientific, Singapore, 1990), p. 479.
[4] G. Danby *et al.* (to be published); G.T. Danby and J.W. Jackson, in *Proceedings of the 1987 IEEE Particle Accelerator Conference,* edited by E.R. Lindstron and L.S. Taylor (IEEE, New York, 1987), p. 1517.
[5] R. Prigl *et al.,* Nucl. Instrum. Methods Phys. Res., Sect. A **374**, 118 (1996).
[6] X. Fei, V.W. Hughes, and R. Prigl, Nucl. Instrum. Methods Phys. Res., Sect. A **394**, 349 (1997).
[7] F. Krienen, D. Loomba, and W. Meng, Nucl. Instrum. Methods Phys. Res., Sect. A **283**, 5 (1989).
[8] S. Sedykh *et al.,* in *Proceedings of the VII International Conference on Calorimetry in High-Energy Physics (CALOR 97),* edited by E. Cheu, T. Embry, J. Rutherford, and R. Wigmans (World Scientific, Singapore, 1998), p. 269.
[9] LORAN-C User's Handbook, 1992, Superintendent of Documents, U.S. Government Printing Office #050-012-00331-9.
[10] C. Caso *et al.,* Particle Data Group, Eur. Phys. J. C **3**, 1 (1998).
[11] A. Czarnecki, B. Krause, and W. Marciano, Phys. Rev. Lett. **76**, 3267 (1996), and references therein.
[12] M. Davier and A. Höcker, Phys. Lett. B **419**, 419 (1998), and references therein.
[13] B. Krause, Phys. Lett. B **390**, 392 (1997).
[14] J. Bijnens, E. Pallante, and J. Prades, Phys. Rev. Lett. **75**, 1447 (1995); **75**, 3781 (1995); Nucl. Phys. **B474**, 379 (1996).
[15] M. Hayakawa and T. Kinoshita, Phys. Rev. D **57**, 465 (1998), and references therein.
[16] A. Czarnecki, B. Krause, and W.J. Marciano, Phys. Rev. D **52**, R2619 (1995).