Creation of High-Energy Phonons from Low-Energy Phonons in Liquid Helium

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We present a theory for the remarkable observation that high-energy phonons are created from a moving cloud of low-energy phonons in liquid ^{4}He. This phenomenon depends on the loss of high-energy (h) phonons from a moving phonon cloud. The h phonons are created from the low-energy (l) phonons by four-phonon processes and escape from the back of the l-phonon cloud because they have a lower velocity than the l phonons. This theory accounts for the high efficiency of the conversion process; a major part of the energy of the low-energy phonons can be converted to high-energy phonons in a time of order 10 μs. [S0031-9007(99)08478-1]

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It is a remarkable fact that high-energy phonons can be created in liquid ^{4}He from low-energy phonons [1,2]. Moreover, the process is very effective as a major part of the energy originally in the low-energy phonons (l phonons) can be converted to high-energy phonons (h phonons) in a time $\sim$10 μs. Consider a thin film heater and a bolometer detector separated by $\sim$15 mm and immersed in liquid ^{4}He at $T = 60$ mK and zero pressure. A single short pulse (10^{-7} s, 10 mW/mm^2) into the heater gives rise to two phonon pulses at the detector, as shown in Fig. 1a. The faster pulse is due to l phonons, $\varepsilon \approx 1$ K ($\varepsilon = \hbar\omega/k$) and the slower one is due to h phonons, $\varepsilon \geq 10$ K. These phonons have different group velocities (238 and $\leq$190 m/s respectively), and as they propagate ballistically (the l phonons are quasiballistic [3]) they have different times of flight. The phonons separate into two groups because there is a critical phonon energy ε_c ($=$10 K) above which the h phonons are very stable and below which the l phonons thermalize to energies around 1 K. This follows from the various scattering processes that are allowed by the anomalous shape of the dispersion curve [4] shown in Fig. 1b.

We know from the fact that the h phonons quantum evaporate (QE) ^{4}He atoms from the free surface of liquid ^{4}He that these phonons have energy $\varepsilon \sim 10$ K [5,6]. This energy agrees with values of the critical energy for the four-phonon process (4pp), ε_c, derived from the dispersion curve [7] and directly measured using superconducting tunnel junction detectors [8]. The first evidence that h phonons are produced in the liquid, up to 4 mm in front of the heater, came from times of flight in QE studies which were faster than expected for an h phonon propagating all the way from the heater to the free liquid surface [6,9,10]. The difference in time could be explained by assuming that the phonon signal propagated for up to $\sim$4 mm at the higher l phonon velocity and thereafter with the h phonon velocity. This strongly suggested that the l phonons were creating the h phonons [1,2]. The latest study [11], which varied the length of the path in the liquid, showed that the h phonons are created in the liquid in front of the heater with a probability that decreases with distance from the heater.

It was realized that to create a phonon with 10 K energy from phonons with energy around 2 K required several 3pp scatterings to take the energy to near 8 K, but as 3pp cannot take the energy above ε_c, the final step has to be a 4pp [1,2]. However, the l phonons interact so quickly that they are always in thermal equilibrium so there is a

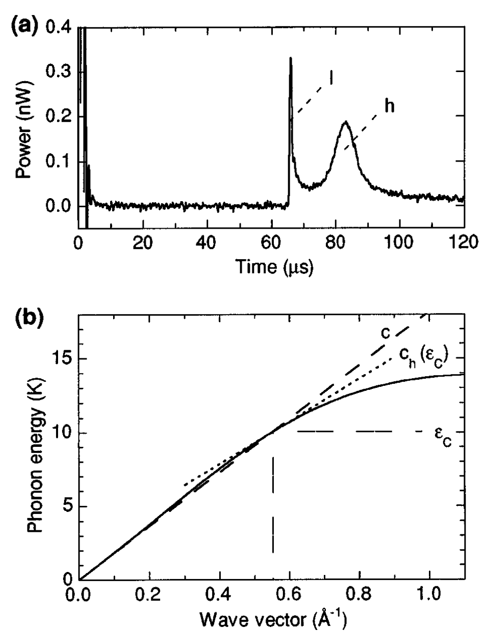

FIG. 1. (a) The measured phonon signal in liquid ^{4}He showing the l and h phonons produced by a single heater pulse: pulse duration $= 10^{-7}$ s, heater power $= 10$ mW/mm^2, propagation length $= 15.7$ mm, $T_{\rm amb} = 60$ mK, pressure $= 0$. (b) The dispersion curve for liquid ^{4}He at zero pressure. The two tangents show the different group velocities for low-energy (dashed line) and 10 K (dotted line) phonons.

thermal population of phonons right up to ε_c. This means that we do not have to be concerned with the details of the 3pp up-scattering processes. However, we must consider how some of these phonons are scattered by 4pp to $\varepsilon > \varepsilon_c$. Also we shall see that these h phonons have a very long lifetime against decay to $\varepsilon < \varepsilon_c$, even within the l phonon cloud, which is a consequence of all the l phonons having almost the same direction. This leads to a nonthermal distribution of h phonons, the consequences of which will be discussed in another place [12]. Finally, we see that the h phonons are left trailing behind the l phonons as they travel at different velocities.

We treat the low- and high-energy phonons as two nearly separate systems. This is possible because the dispersion curve imposes quite different relaxation times on the two groups. Phonons with $0 < \varepsilon < \varepsilon_{3c}$ (=8.5 K) interact via 3pp because of the anomalous dispersion. Phonons with $\varepsilon > \varepsilon_c$ (=10 K) can interact only via 4pp because the normal dispersion in this energy range prevents any 3pp. For $\varepsilon_{3c} < \varepsilon < \varepsilon_c$ multiphonon interactions are allowed. The 3pp scattering rate (τ_3^{-1}) [13] is approximately 2 orders of magnitude faster than that for 4pp [14,15], so the low-energy phonons can be taken to be in thermal equilibrium within their group, at a temperature T. We include the phonons with $\varepsilon_{3c} < \varepsilon < \varepsilon_c$ in this group. Although there is no detailed theory of their scattering time it is expected that their relaxation times are more than an order of magnitude smaller than the corresponding times for 4pp [13]. The liquid ^{4}He in which they propagate is taken to be at zero temperature.

The two groups of phonons are shown schematically in Fig. 2a. The arrows indicate creation of h phonons from l phonons, and the decay of h phonons to l phonons. Also there is loss of h phonons from the interacting cloud of phonons because h phonons have a lower group velocity than l phonons, i.e., we use the words *creation* and *decay* for h phonons within the cloud of l phonons, and *loss* for transmission of h phonons out of the cloud of l phonons.

The sequence of events is shown schematically in Fig. 2b. After the heater has injected a pulse of energy in the liquid the l phonon cloud rapidly equilibriates to a Bose-Einstein (B-E) distribution of phonons, at temperature T, in a time τ_3 which is much shorter than the pulse duration t_p. These phonons occupy a fairly narrow cone, typically 10° half angle [2], in momentum space because the l phonon velocities are approximately in the same direction; see Fig. 3a. The cloud of length $L = ct_p$ is composed almost entirely of l phonons as any h phonons that were initially in the cloud are rapidly lost by dispersion in a time $\sim 5t_p$. So the cloud is not in equilibrium as the Bose-Einstein distribution has lost its tail for $\varepsilon \geq \varepsilon_c$. As the system attempts to equilibriate, the l phonons give their energy to the h phonons on a time scale $\tau_{4u}^{(0)}(T)$; see Fig. 2a.

At time t_1 the cloud has moved a distance ct_1 and has created some h phonons, a fraction of which are trailing behind the cloud with velocity c_h. We call this the h cloud.

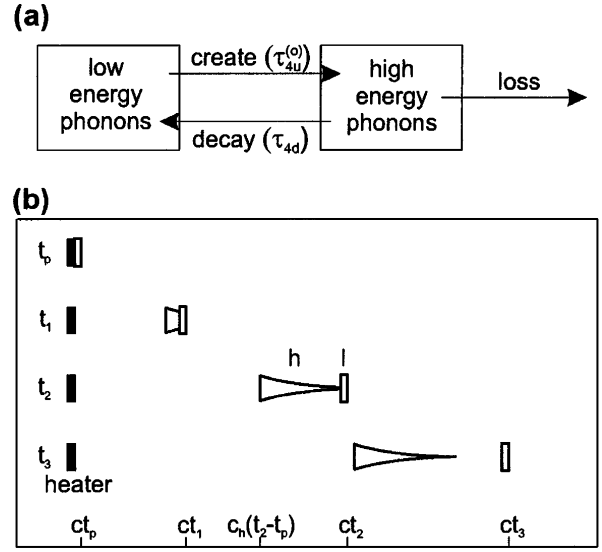

FIG. 2. (a) A schematic representation of the low- and high-energy groups of phonons. The arrows indicate the transitions that *create, decay,* and *lose* high-energy phonons. (b) A schematic diagram of the low- and high-energy phonon clouds at different times in their propagation from the heater to the detector.

This loss of h phonons maintains a deficiency of h phonons in the l cloud which again causes more h phonons to be created by 4pp. At t_2 the h cloud has lengthened because of the continued creation of h phonons and their subsequent loss from the l phonon cloud. As the l phonon cloud loses h phonons it cools, which causes the creation rate of h phonons to drop. At t_3 the two clouds appear to be separate because at some time between t_2 and t_3 the l phonon cloud has cooled so much that it has essentially stopped producing h phonons. The two clouds are now independent and move with velocities c_h and c to the detector.

The kinetic equation for the number density, n_h, of h phonons in the frame moving with the l phonons is

$$\frac{\partial n_h}{\partial t} + \mathbf{u}_h \cdot \nabla n_h = \frac{n_h^{(0)}}{t_{4u}^{(0)}} - \frac{n_h}{t_{4d}}, \tag{1}$$

where $n_h^{(0)}(T)$ is the equilibrium number density of h phonons at temperature T, these are the phonons in the high-energy tail of the B-E distribution (see Fig. 3b), $\mathbf{u}_h = \mathbf{c}_h - \mathbf{c}$, $t_{4u}^{(0)}$ is the 4pp scattering time for the creation of h phonons in equilibrium with l phonons at temperature T, and t_{4d} is the decay time for h phonons within the cloud. We shall see that t_{4d} may be substantially different from $t_{4u}^{(0)}$ in a phonon cloud with a narrow momentum cone.

Multiplying Eq. (1) by ε and integrating over the phase space occupied by the h phonons, we get the equation for the energy density, E_h, of the h phonons:

$$\frac{\partial E_h}{\partial t} + u_c \frac{\partial E_h}{\partial z} = \frac{E_h^{(0)}}{\tau_{4u}^{(0)}} - \frac{E_h}{\tau_{4d}}, \tag{2}$$

where we make a one-dimensional approximation with the z axis chosen to be antiparallel to the propagation direction,

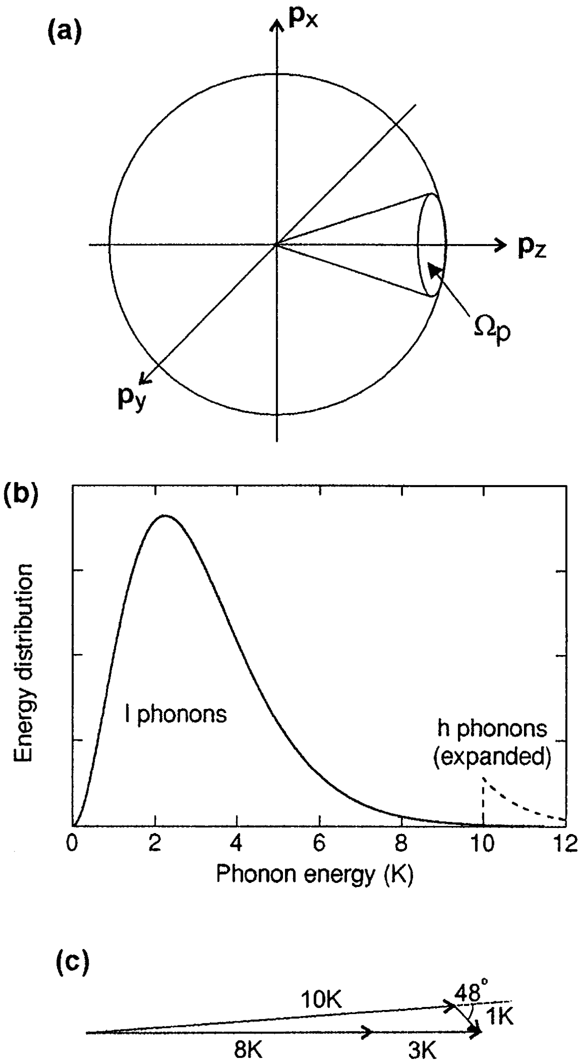

FIG. 3. (a) The solid angle, Ω_p, in momentum space occupied by the low-energy phonons. (b) The energy spectrum of phonons in the low-energy phonon cloud. The energy of the high-energy phonons is shown 30 times the equilibrium phonon energy density. (c) The momenta of the phonons in a typical 4pp. The incoming 8 K and 3 K phonons are from the l phonon cloud. Note the large angle that the created low-energy phonon makes with the created high-energy phonon.

and the average values, u_c ($\approx$ 50 m/s) and $\tau_{4u}^{(0)}$, are taken to be the relative velocity and the 4pp creation time at $\varepsilon = \varepsilon_c$ because of the exponential factor in the distribution function. Also $E_h^{(0)} = \Omega_p k^4 T \varepsilon_c^3 e^{-\varepsilon_c/T}/(2\pi\hbar c)^3$, where Ω_p is the occupied solid angle in momentum space, see Fig. 3a. Also we approximate to a linear dispersion, and terms of next order in $T/\varepsilon_c \ll 1$ have been neglected.

Now $\tau_{4u}^{(0)}$ is found to be $\tau_{4u}^{(0)^{-1}} = f\varepsilon_c^2 T^5$ [15], where $f = 4.4 \times 10^6\ \mathrm{K}^{-7}\,\mathrm{s}^{-1}$ at the saturated vapor pressure.

The energy going to the creation of h phonons comes from the l phonons, so from Eq. (2) we get the rate of change of the energy density of the l phonons:

$$\frac{\partial E_l^{(0)}}{\partial t} = -\frac{E_h^{(0)}}{\tau_{4u}^{(0)}} + \frac{E_h}{\tau_{4d}}, \tag{3}$$

where $E_l^{(0)} = \Omega_p \pi k^4 T^4/120\hbar^3 c^3$, again assuming a linear dispersion.

From Eq. (3) we obtain an equation for the cooling of the l phonon cloud,

$$\frac{\partial T}{\partial t} = -\frac{T}{4E_l^{(0)}}\left(\frac{E_h^{(0)}}{\tau_{4u}^{(0)}} - \frac{E_h}{\tau_{4d}}\right). \tag{4}$$

The set of Eqs. (2) to (4) must be completed by the initial and boundary conditions. The cloud occupies the space $0 < z < L$ and as h phonons move from the point $z = 0$, the front of the l cloud, towards $z = L$, the back of the l cloud, so h phonons are absent at $z = 0$. Hence $E_h(z = 0, t) = 0$. We take the initial conditions to be $T(t = 0) \equiv T_0 = \text{const}$; $E_h(z, t = 0) = 0$.

Now $\tau_{4d} \gg \tau_{4u}^{(0)}$ because once the h phonon has been created in a 4pp it is very unlikely to relax. We can see this with reference to Fig. 3c, where we have drawn the momenta for a typical 4pp, with energy and momentum conservation, 8 K + 3 K $\rightarrow$ 10 K + 1 K. The small phonon, created at the same time as the h phonons (1 K and 10 K, respectively, in this example), can make a large angle to the h phonon; indeed it can be up to 180°. For the decay process which is the time reversal of the creation process, the h phonon must scatter from an l phonon with a similarly large angle. But in the l phonon cloud with a narrow momentum cone there are no such phonons. So τ_{4d} for this process is initially ∞. As the creation of h phonons proceeds, the number of l phonons, at the correct angles to cause h phonon decay, will build up, and so τ_{4d} will decrease. However, we estimate it is always more than an order of magnitude larger than $\tau_{4u}^{(0)}$.

As long as the population of h phonons is well below the maximum number given by dynamic equilibrium without loss, we can neglect the term E_h/τ_{4d} in Eq. (7). This is true if the pulse length L is short enough, i.e., $L \ll u_c \tau_{4d}$ so that all the created h phonons are lost from the l cloud. Substituting expressions for $E_h^{(0)}$, $\tau_{4u}^{(0)}$, and $E_l^{(0)}$ into Eq. (4), and integrating we find an expression for the time dependence of the temperature of the l phonons:

$$Te^{-\varepsilon_c/T} = T_0 e^{-\varepsilon_c/T_0}(1 + t/t_{\mathrm{eff}})^{-1}, \tag{5}$$

where the effective time,

$$t_{\mathrm{eff}} = \frac{4E_l^{(0)}(T_0) T_0 \tau_{4u}^{(0)}(T_0)}{E_h^{(0)}(T_0)\varepsilon_c} \tag{6}$$

has a straightforward physical meaning. At the time $t = t_{\mathrm{eff}}$ the l phonon cloud has already transformed a major part of its energy into h phonons. We see $t_{\mathrm{eff}} \gg \tau_{4u}^{(0)}$, e.g., if $T = 0.8$ K then $t_{\mathrm{eff}}/\tau_{4u}^{(0)} \sim 10^2$.

So the l phonons cool as they propagate and create the trailing cloud of h phonons. As energy is conserved between the h and l phonons, the fraction of energy in the h phonons is given by

$$\Delta(t) = \frac{E_l^{(0)}(T_0) - E_l^{(0)}[T(t)]}{E_l^{(0)}(T_0)} = \frac{1}{E_l^{(0)}(T_0)} \int_0^t \frac{E_h^{(0)}[T(t)]}{\tau_{4u}^{(0)}}\, dt\,. \tag{7}$$

This is an upper limit as we have ignored the term E_h/τ_{4d} in Eq. (3). The integrand in Eq. (7) is the creation rate of the energy density of the h phonons and using the expressions for $E_h^{(0)}$ and $E_l^{(0)}$ we find

$$\frac{E_h^{(0)}}{\tau_{4u}^{(0)}} = \frac{f\Omega_p k^4 \varepsilon_c^5 T^6 e^{-\varepsilon_c/T}}{(2\pi\hbar c)^3}. \tag{8}$$

So we see that as T drops, the creation rate falls and becomes negligible at $T \sim 0.6$ K. At this temperature the h and l phonon clouds separate as indicated in Fig. 2b for some time between t_2 and t_3.

The fraction of energy in the h phonons for three typical starting temperatures is shown in Fig. 4a. It can be seen that a significant part of the energy in the l phonons is rapidly transformed into h phonons. This very effective up-scattering mechanism is due to the h phonons being lost out of the trailing end of the l cloud where they just accumulate without decay.

The pulse shape of the h phonons is proportional to $E_l^{(0)} d\Delta/dt$, which we can get by taking $T(t)$ from Eq. (5) and using Eq. (8). It is also necessary to include dispersion as there will be a spectrum of h phonons, we do this by giving them a B-E spectrum at the temperature of the l phonons at the point they are lost from the l cloud. This spectrum then propagates with the corresponding group velocities to the detector. This is shown in Fig. 4b for a starting temperature of 0.8 K. The time of the peak and the width of the signal are in reasonable agreement with the measured pulse shape shown in Fig. 1a.

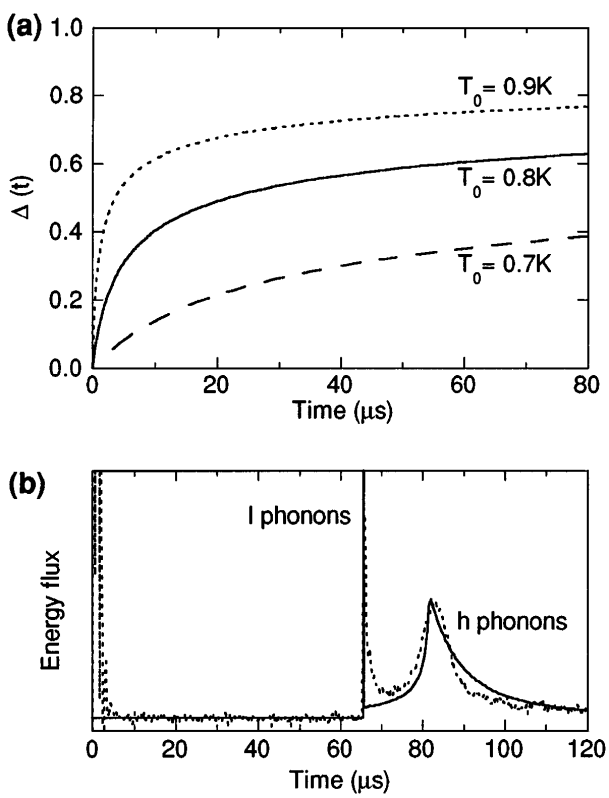

FIG. 4. (a) The fraction of energy, Δ, in the h phonons relative to the total energy, is shown as a function of time for three initial temperatures, which are typical for the experimental conditions. (b) The pulse shape of the h phonons according to the model and including dispersion (solid line). This is to be compared with the measurements in Fig. 1a, shown here as a dotted line.

In conclusion, we have presented a theory for the creation of high-energy phonons from a moving cloud of low-energy phonons which has been observed experimentally. The high-energy phonons are created from the low-energy phonons by 4pp as the system tries to reach equilibrium. They are then lost from the back of the l phonon cloud because the h phonons have a lower velocity than the l phonons. For a short pulse the h phonons are lost from the l cloud as soon as they are created, and once they are lost they do not decay but just propagate ballistically. This theory accounts for the high efficiency of the conversion process and the main characteristics of the measured phonon signals. This phenomenon is an unusual example of energy going from low-energy phonons to high-energy ones.

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[1] M. A. H. Tucker and A. F. G. Wyatt, J. Phys. Condens. Matter **6**, 2813 (1994).
[2] M. A. H. Tucker and A. F. G. Wyatt, J. Phys. Condens. Matter **6**, 2825 (1994).
[3] The l phonons are interacting by 3pp but the scattering angles are small, especially after the first few millimeters by which point the l pulse has cooled, and so the propagation velocity is essentially the sound velocity.
[4] See review by H. J. Maris, Rev. Mod. Phys. **49**, 341 (1977).
[5] M. J. Baird, F. R. Hope, and A. F. G. Wyatt, Nature (London) **304**, 325 (1983).
[6] M. Brown and A. F. G. Wyatt, J. Phys. Condens. Matter **2**, 5025 (1990).
[7] W. G. Stirling, in *Proceedings of the 75th Jubilee Conference on ^{4}He,* edited by J. G. M. Armitage (World Scientific, Singapore, 1983), p. 109.
[8] R. C. Dynes and V. Narayanamurti, Phys. Rev. B **12**, 1720 (1975).
[9] M. A. H. Tucker and A. F. G. Wyatt, Physica (Amsterdam) **165B & 166B**, 493 (1990).
[10] A. C. Forbes and A. F. G. Wyatt, in A. F. G. Wyatt, J. Phys. Condens. Matter **8**, 9249 (1996), Fig. 13.
[11] M. A. H. Tucker and A. F. G. Wyatt, J. Low Temp. Phys. **113**, 621 (1998).
[12] A suprathermal distribution of h phonons is predicted.
[13] T. J. Sluckin and R. M. Bowley, J. Phys. C **7**, 1779 (1974).
[14] M. A. H. Tucker and A. F. G. Wyatt, J. Phys. Condens. Matter **4**, 7745 (1992).
[15] I. N. Adamenko *et al.* (to be published).