Scaling of the Conductivity with Temperature and Uniaxial Stress in Si:B at the Metal-Insulator Transition

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Using uniaxial stress S to tune Si:B through the metal-insulator transition at a critical value S_c, we find the dc conductivity at low temperatures shows an excellent fit to the scaling form $\sigma(S,T) = AT^x f[(S - S_c)/T^y]$ on both sides of the transition. The scaling functions yield reliable determinations of the temperature dependence of the conductivity in the metallic and insulating phases in the critical region. [S0031-9007(98)08115-0]

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The metal-insulator transition in doped semiconductors and amorphous metal-semiconductor mixtures is a continuous quantum phase transition which occurs in the limit of zero temperature. A scaling approach similar to that used for continuous phase transitions driven by temperature, within which the properties of the system do not depend on microscopic details and are controlled by a diverging length scale, suggests that the conductivity in the vicinity of the transition (critical) point can be described by a scaling form,

$$\sigma(t,T) = \sigma_c(T) f[(t - t_c)/T^y], \qquad (1)$$

where t is the control parameter (such as dopant concentration, magnetic field, or uniaxial stress) that drives the transition at the critical value $t = t_c$, and

$$\sigma_c(T) = AT^x \qquad (2)$$

is the low-temperature limit of the conductivity at the critical point. By identifying the diverging time scale τ at the transition with $\hbar/kT$, and assuming conventional dynamical scaling $\tau \propto \xi^z$, where $\xi(t) \propto (t - t_c)^{-\nu}$ is the diverging correlation length scale as the transition is approached, one easily obtains $y = 1/z\nu$, and $x = \mu/z\nu$, where μ is the critical exponent characterizing the onset of metallic conductivity at zero temperature:

$$\sigma(t, T \rightarrow 0) \propto (t - t_c)^{\mu}. \qquad (3)$$

The applicability of this scaling formulation [1] to the metal-insulator transition was first demonstrated for noninteracting electrons by Wegner [2] and by Abrahams *et al.* [3], and subsequently extended to incorporate electron-electron interactions by many authors [4–7]. Within this theoretical framework, the values of the critical exponents μ, ν, and z are determined by the symmetry of the effective field theory, and depend on the presence or absence of symmetry-breaking fields, such as magnetic field, spin-orbit interactions, or magnetic impurities, which determine the universality class of the system [8,9].

The problem of metal-insulator transitions has a venerable history [10]. Data for amorphous metal-semiconductor mixtures [11], magnetic semiconductors [12], and heavily compensated persistent photoconductors [13] all suggest an exponent $\mu \approx 1$. In contrast, although the continuous nature of the transition was first demonstrated in uncompensated doped Si and despite considerable effort over more than two decades, a consensus regarding the critical behavior in uncompensated doped semiconductors has yet to emerge [14]. There continues to be debate concerning a number of fundamental issues. Thus, for example, (i) the value of the critical conductivity exponent μ has been variously cited as equal to $1/2$ and 1; (ii) the breadth of the critical regime, and thus the range of the critical parameter t one can safely use to determine the exponent is not known; and (iii) the form of $\sigma_c(T)$ in the critical region very near the transition has been claimed to be $\propto T^{1/2}$ and $\propto T^{1/3}$ [15].

The procedure generally used to determine the critical exponent μ entails measuring the conductivity to as low a temperature as possible to obtain a single extrapolated $T = 0$ value to be used in the fit to Eq. (3). In contrast, full scaling with temperature, Eq. (1), obviates the need for potentially unreliable extrapolations to $T = 0$ and yields a determination of μ based on all data taken at all temperatures and values of the control parameter in the critical regime. Inspection of published data for amorphous metal-semiconductor mixtures such as NbSi [11] and the persistent photoconductor $Al_{0.3}Ga_{0.7}As$ [13] indicates that the conductivity of these systems obeys Eq. (1) in the metallic phase [16,17]. On the other hand, the conductivity of Si:P measured by Paalanen *et al.* [18] to temperatures below 5 mK does not obey scaling, while approximate temperature scaling has been reported by Stupp *et al.* [19] at somewhat higher temperatures on the metallic side very near the transition in the same system [20].

In this paper we report conductivity measurements obtained using uniaxial stress to tune through the metal-insulator transition in Si:B. As demonstrated by Paalanen

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et al. [18] in their classic experiments in Si:P, uniaxial stress allows very fine control in a single sample, and gives precise relative determinations of the critical parameter t. For a closely spaced set of stresses S near the critical stress S_c, we demonstrate that the conductivity of Si:B obeys scaling, Eq. (1), on both sides of the metal-insulator transition. We show that the conductivity in the critical regime is consistent with $\sigma_c \propto T^{1/2}$. This is the same temperature dependence as has been calculated [21] and observed [18] in the perturbative region on the metallic side (weakly disordered metal). On the insulating side, we find that the conductivity crosses over to $\sigma \propto \exp[-(T^*/T)^{1/2}]$, the Efros-Shklovskii (ES) form expected for variable-range hopping in the presence of Coulomb interactions [22], with a prefactor $\propto T^{1/2}$ corresponding to the temperature dependence of the critical curve.

A bar-shaped $8.0 \times 1.25 \times 0.3$ mm^3 sample of Si:B was cut with its long dimension along the [001] direction. The dopant concentration, determined from the ratio of the resistivities [23] at 300 and 4.2 K, was 4.84×10^{18} cm^{-3}. Electrical contact was made along four thin boron-implanted strips. Uniaxial compression was applied to the sample along the long [001] direction using a pressure cell described elsewhere [23]. Four-terminal measurements were taken at 13 Hz (equivalent to dc) for different fixed values of uniaxial stress at temperatures between 0.05 and 0.76 K. Measurements were restricted to the linear region of the I-V curves.

Si:B is considerably more sensitive to stress than Si:P [24]. This is because the acceptor state in Si:B has a fourfold degeneracy in the unstressed cubic phase, which is lifted by uniaxial stress into two doublets, each retaining only the Kramers degeneracy. By contrast, the sixfold valley degeneracy (on top of the required Kramers or spin degeneracy) of an effective mass donor in Si has already been removed (even in zero stress) by the central-cell correction of the phosphorus dopants [25]. A consequence of the additional degeneracy in Si:B is that uniaxial compressive stress drives metallic Si:B into the insulating phase, unlike Si:P where (relatively larger) stresses drive an insulating sample through the transition into the metallic phase. This is in qualitative agreement with predictions for effective mass donor systems which take into account mass anisotropy [26] as well as degeneracy in the presence of electron correlation [27].

The conductivity of Si:B measured at 4.2 K was found to be a linear function of stress. Since the conductivity at 4.2 K is also approximately linear with dopant concentration for concentrations near the transition, it follows that $(S - S_c) \propto (n - n_c)$, and either may be used as the control parameter $(t - t_c)$ in the scaling equations (1) and (3). It has generally been assumed that the same value of the critical conductivity exponent μ should then be obtained by varying stress or dopant concentration.

The conductivity as a function of temperature is shown on a log-log scale in Fig. 1 for twenty selected values of the uniaxial stress. The upper curves bend upward as T is

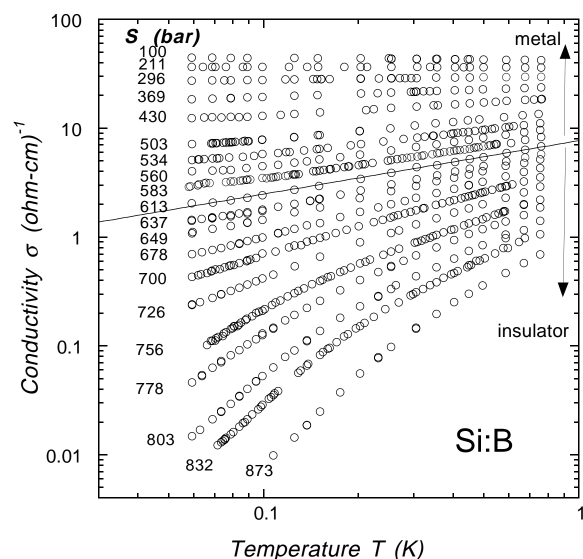

FIG. 1. Conductivity versus temperature on a log-log scale for different values of stress.

decreased, tending toward finite (metallic) conductivities at $T = 0$, while the lower curves are concave downward, indicating that they are in the insulating phase. The critical curve at $S = S_c$ is a straight line heading toward $\sigma = 0$ at $T = 0$, and follows a power law.

Fits to S-T scaling, Eq. (1), were carried out for different choices of S_c ranging between 560 and 726 bars. Figure 2 shows a scaled plot of σ/σ_c vs $(\Delta S/S_c)/T^{0.31}$ (where $\Delta S = S - S_c$) for the best choice, $S_c = 613$ bars, yielding exponents $x = 0.5$ and $y = 0.31$. The conductivity at the critical point thus exhibits a square root dependence on T. A dependence $\propto T^{1/3}$, claimed for several other systems at the critical point [15] is inconsistent with our data in stressed Si:B. In terms of the standard exponents, one obtains $\mu = 1.6$ and $z\nu = 3.2$. If we assume (as is generally done) [28] that $\mu = \nu$, it follows that the dynamical exponent $z = 2$.

The conductivity in the insulating phase, normalized to the critical conductivity, is shown in Fig. 3 on a semilogarithmic scale as a function of $(T^*/T)^{1/2}$, where $T^* \propto (\Delta S)^{1/y}$. Given that $\sigma = \sigma_c$ at $T^* = 0$ so that the curve must go through the upper left corner, the straight line of Fig. 3 shows that for $T^*/T > 10$ the conductivity obeys the exponentially activated hopping form [22] expected in the presence of a gap in the density of states due to electron interactions, with a temperature-dependent prefactor given by the critical curve, namely,

$$\sigma(T) \propto T^{1/2} \exp[-(T^*/T)^{1/2}]. \tag{4}$$

We have thus demonstrated that the ES hopping form expected in the insulating phase is also included in the scaled conductivity near the transition. Deviations are evident for $T^*/T < 10$. In this regime hopping energies are comparable to or larger than the energy width of the Coulomb gap, and a crossover has been suggested and observed to Mott variable range hopping with an exponent

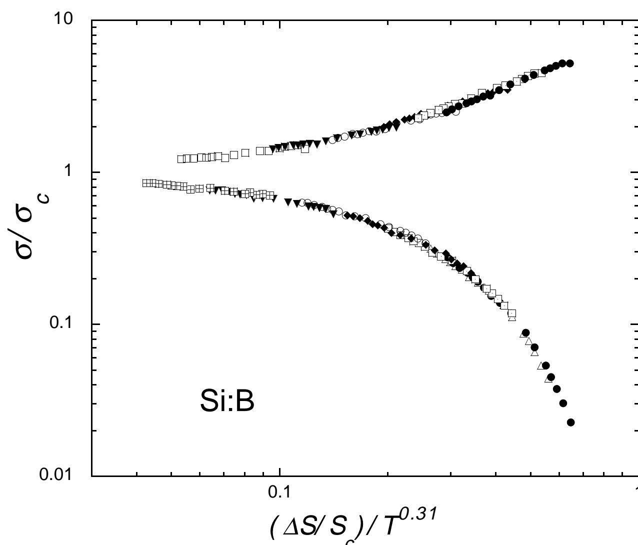

FIG. 2. σ/σ_c versus the scaling variable $(\Delta S/S_c)/T^y$ on a log-log scale, with $y = 1/(\nu z) = 0.31$.

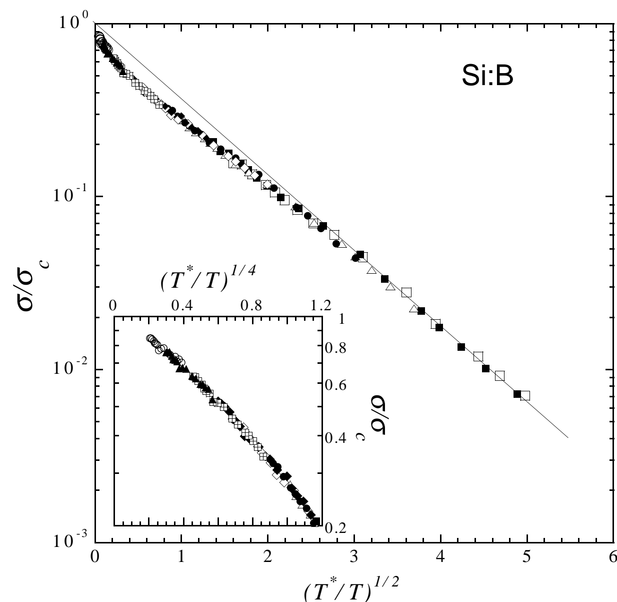

FIG. 3. For the insulating phase, σ/σ_c versus $(T^*/T)^{1/2}$ on a semilogarithmic scale. Plotted as a function of $(T^*/T)^{1/4}$ in the inset, the continuous curvature of the data relative to a straight line passing through the upper left corner indicates that the conductivity is inconsistent with Mott variable-range hopping.

1/4 rather than 1/2. The inset shows the normalized conductivity as a function of $(T^*/T)^{1/4}$. Noting again that the curve must pass through the top left corner, the data are clearly inconsistent with a straight line, demonstrating that Mott hopping is not observed to be part of the scaled conductivity in uncompensated Si:B.

We now examine the behavior of the conductivity on the metallic side. Equation (1) can be rewritten as

$$\sigma(t,T) = (S - S_c)^{\mu} f'[T/(S - S_c)^{z\nu}], \qquad (5)$$

with a different universal function f'. The ratio $\sigma(t,T)/(\Delta S/S_c)^{\mu}$ is shown as a function of $T^{1/2}/(\Delta S/S_c)^{z\nu/2}$ in Fig. 4. The conductivity is everywhere consistent with the form calculated in the weak disorder regime (where perturbative calculations are valid) [21], namely,

$$\sigma(T) = \sigma(0) + BT^{1/2}, \qquad (6)$$

with $\sigma(0) \propto (S - S_c)^{\mu}$.

The critical conductivity exponent $\mu = 1.6$ found in our experiments is considerably larger than other determinations, which range between 0.5 [18,29] and (at most) 1.3 [30] in uncompensated doped semiconductors [14]. We caution that a direct comparison may not be warranted for several reasons. The exponent obtained here is based on experimental results at higher temperatures (50 mK $< T <$ 0.75 K) than, for example, those of Paalanen *et al.* [18], who have emphasized [31] the necessity of going below $T = 50$ mK to obtain reliable extrapolations to $T = 0$. While our analysis uses the potentially more reliable full S-T scaling, a test of its applicability below 50 mK will require measurements to lower temperatures in Si:B. Indeed, confirmation of a breakdown of scaling at very low temperatures would provide strong impetus for further study. Second, it is possible that stress variations result in an inhomogeneous stress distribution and a consequent averaging over a sample consisting of portions that are at different "distances" Δt from the transition. This could "smear" the transition and yield a large value of μ; however, if inhomogeneities were sufficiently serious to cause a measurable increase in μ, they would probably cause measurable deviations from scaling as well. It is important to note also that the temperature dependence of stressed and unstressed samples of comparable conductivities are unambiguously different [23]. This suggests that the question of "universality" of the critical exponent obtained using stress or dopant concentration to tune the transition needs to be examined in more detail.

It has been suggested recently [32] that disordered systems may violate the Chayes *et al.* [33] inequality $\nu \geq 2/d$ (as some uncompensated semiconductors appear to); self-averaging breaks down for such systems. The system would then become inhomogeneous as the critical point is approached, which could imply that this is ultimately a percolation-type transition. The conductivity exponent 1.6 obtained in our experiments is close to that expected for classical percolation in three dimensions; this possibility needs to be examined in more detail, however, and other factors must be ruled out. We note that it is also close to the result for Anderson localization of noninteracting electrons [34]; however, there exists ample evidence of strong electron interactions in both the insulating and metallic phases of Si:B.

To summarize, scaling provides an excellent description of the conductivity near the metal-insulator transition in uniaxially stressed Si:B. Based on data at many values of stress and temperature, the scaling functions yield particularly reliable determinations of the conductivity in the critical region in both the insulating and metallic

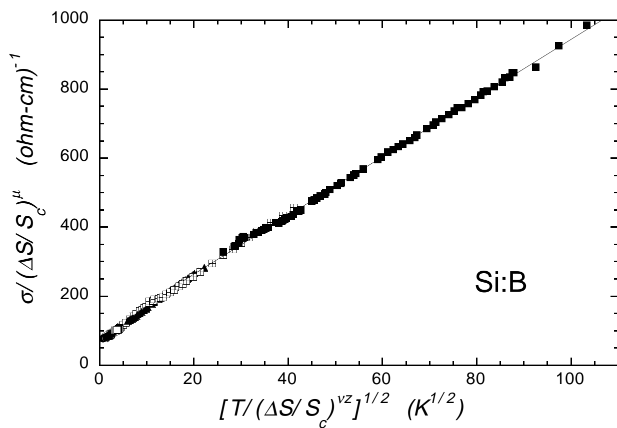

FIG. 4. For the metallic phase, $\sigma/[(\Delta S)/S_c]^{\mu}$ versus $[T/(\Delta S/S_c)^{z\nu}]^{1/2}$.

phases. Although comparison between different systems continues to be problematical, we have shown for the first time that full temperature-stress scaling on both sides gives internally consistent results for the metal-insulator transition in a doped semiconductor.

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[1] This is often referred to as dynamical scaling; we refer to it as temperature-stress scaling, reserving the term "dynamical" to denote scaling of the finite-frequency conductivity.
[2] F. Wegner, Z. Phys. B **25**, 327 (1976).
[3] E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979).
[4] W. L. McMillan, Phys. Rev. B **24**, 2739 (1981).
[5] A. M. Finkelshtein, Zh. Eksp. Teor. Fiz. **84**, 168 (1983) [Sov. Phys. JETP **57**, 97 (1983)].
[6] C. Castellani, C. DiCastro, P. A. Lee, and M. Ma, Phys. Rev. B **30**, 527 (1984); C. Castellani, G. Kotliar, and P. A. Lee, Phys. Rev. Lett. **59**, 323 (1987).
[7] T. R. Kirkpatrick and D. Belitz, Phys. Rev. **41**, 11 082 (1990).
[8] For excellent reviews, see P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985); D. Belitz and T. Kirkpatrick, Rev. Mod. Phys. **66**, 261 (1994).
[9] Our analysis is in terms of the general scaling form Eq. (1), with exponents x and y; only in the identification of these with exponents ν and z are there additional implicit assumptions concerning the scaling of the diverging time scale with temperature.
[10] N. F. Mott, *Metal-Insulator Transitions* (Taylor and Francis, London, 1974).
[11] D. J. Bishop, E. G. Spencer, and R. C. Dynes, Solid State Electron. **28**, 73 (1985).
[12] S. von Molnar, A. Briggs, J. Flouquet, and G. Remenyi, Phys. Rev. Lett. **51**, 706 (1983).
[13] S. Katsumoto, F. Komori, N. Sano, and S. Kobayashi, J. Phys. Soc. Jpn. **56**, 2259 (1987).
[14] M. P. Sarachik, in *Metal-Insulator Transitions Revisited,* edited by P. P. Edwards and C. N. Rao (Taylor and Francis, London, 1995).
[15] See, for example, M. Maliepaard *et al.,* Phys. Rev. B **39**, 1430 (1989); I. Shlimak *et al.,* Phys. Rev. Lett. **77**, 1103 (1996).
[16] In these materials, a plot of σ vs $T^{1/2}$ yields parallel lines for different n in the metallic phase, so that scaling with temperature and concentration is clearly obeyed.
[17] Scaling has been reported near a Mott-Hubbard transition in $Ni(S,Se)_2$ by A. Husmann *et al.,* Science **274**, 1874 (1996).
[18] M. A. Paalanen, T. F. Rosenbaum, G. A. Thomas, and R. N. Bhatt, Phys. Rev. Lett. **48**, 1284 (1982).
[19] H. Stupp, M. Hornung, M. Lakner, O. Madel, and H. v. Lohneysen, Phys. Rev. Lett. **72**, 2122 (1994).
[20] In amorphous $Si_{1-x}Cr_x$, one-parameter scaling has been proposed in the insulating phase by A. Mobius *et al.,* J. Phys. C **18**, 3337 (1985).
[21] B. L. Altshuler and A. G. Aronov, Sov. Phys. JETP **50**, 968 (1979); Sov. Phys. JETP Lett. **30**, 514 (1979).
[22] B. I. Shklovskii and A. L. Efros, in *Electronic Properties of Doped Semiconductors,* Solid State Series Vol. 45 (Springer-Verlag, Berlin, Heidelberg, 1984).
[23] S. Bogdanovich Thesis, City College of CUNY, 1998.
[24] J. C. Hensel and G. Feher, Phys. Rev. **129**, 1041 (1963); H. Hasegawa, Phys. Rev. **129**, 1029 (1963); J. A. Chroboczek, F. H. Pollak, and H. F. Staunton, Philos. Mag. B **50**, 113 (1984).
[25] R. N. Bhatt, Phys. Rev. B **26**, 1082 (1982).
[26] R. N. Bhatt, Phys. Rev. B **24**, 3630 (1981).
[27] R. N. Bhatt, and S. Sachdev, Phys. Rev. B **34**, 3520 (1986).
[28] Controversy regarding this equality exists only when there is no symmetry-breaking field (pure potential scattering in the absence of a magnetic field), and the renormalization group flows scale to strong coupling.
[29] K. M. Itoh *et al.,* Phys. Rev. Lett. **77**, 4058 (1996).
[30] H. Stupp, M. Hornung, M. Lakner, O. Madel, and H. v. Lohneysen, Phys. Rev. Lett. **71**, 2634 (1993).
[31] G. A. Thomas, M. A. Paalanen, and T. F. Rosenbaum, Phys. Rev. B **27**, 3897 (1983).
[32] F. Pazmandi, R. T. Scalettar, and G. Zimanyi, Phys. Rev. Lett. **79**, 5130 (1997).
[33] J. T. Chayes, L. Chayes, D. S. Fisher, and T. Spencer, Phys. Rev. Lett. **57**, 2999 (1986).
[34] K. Slevin and T. Ohtsuki, Phys. Rev. Lett. **78**, 4083 (1997).