

Effect of Ferromagnetic Spin Correlations on Superconductivity in Ferromagnetic Metals

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We study the renormalization of quasiparticle properties in weak ferromagnetic metals due to spin fluctuations, away from the quantum critical point for small magnetic moment. We explain the origin of the s -wave superconducting instability in the ferromagnetic phase and find that the vertex corrections are small and that Migdal's theorem is satisfied away from the quantum critical point. [S0031-9007(98)08130-7]

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More than 30 years ago, Doniach [1] and Berk and Schrieffer [2] showed that, in the paramagnetic phase, the phonon-induced, s -wave superconductivity in exchange-enhanced transition metals is suppressed by ferromagnetic spin fluctuations, in the neighborhood of the Curie temperature. At the same time a theory of superconductivity coexisting with long-range ferromagnetic order was developed by Larkin and Ovchinnikov [3], and by Fulde and Ferrell [4] for magnetic-impurity-induced ferromagnetism in metals. Without experimental evidence for the coexistence of superconductivity and ferromagnetism, this theory has been of academic interest only. It is generally accepted that ferromagnetism suppresses superconductivity and the apparent contradiction between the above two pictures has not been clarified.

On the experimental side, recent advances have allowed for the investigation of the quantum critical region in correlation-induced, weak ferromagnetic metals [5] as well as in some heavy-fermion compounds [6]. When hydrostatic pressure is applied on a transition metal compound such as MnSi or ZrZn₂, the Curie temperature can be driven down to zero at a critical pressure. In the neighborhood of this critical pressure the paramagnetic-ferromagnetic phase transition is driven by quantum critical fluctuations. So far, experiments have failed to find superconductivity in the paramagnetic phase of these compounds and as we argue below, the physics close to the phase transition is not well understood.

These experiments have motivated us to study the *ferromagnetic* regime relatively close to the critical point which is described as a highly correlated but weakly ferromagnetic metal. In this investigation we extend the Doniach-Berk-Schrieffer (DBS) theory into the *ferromagnetic phase* of the transition metal compounds. We develop a microscopic theory of the ferromagnetic state, based on the interactions mediated by spin fluctuations between the fermions, and explain the microscopic origin of the unexpected s -wave superconductivity recently predicted by us [7] on the basis of phenomenological considerations. In doing so, we find that, in contrast to the paramagnetic case, ferromagnetic fluctuations *enhance* pairing correlations—resolving the longstanding dilemma referred to in the opening paragraph.

Our starting point is the Stoner state which is a Hartree-Fock solution of some Hamiltonian, below the mean-field ferromagnetic instability [8]. This state is a product of two Slater determinants with an electron mass possibly renormalized by the band structure. At this level of approximation the correlations do not renormalize the mass. Although the Stoner state has nonzero magnetization, it is known that the Hartree-Fock approximation overestimates the exchange. However, we will assume that fluctuations about this mean-field saddle point do not completely destroy the ferromagnetic order. We cannot overemphasize that this starting point is *not* perturbatively connected to the paramagnetic Fermi-liquid state. The next step is to include the correlations which produce weakly interacting quasiparticles with renormalized mass m^* near the Fermi surface. The renormalization in the neighborhood of the two Fermi surfaces is described by the single particle Green's function

$$G_{\sigma}(\vec{p}, \omega) = \frac{z}{\omega - v_F(|\vec{p}| - p_{\sigma}) + i\delta_{\sigma}[\vec{p}]} + G_{\text{inc}} \quad (1)$$

which is diagonal with the quantization axis parallel to the z axis. Here \vec{p} is the three-dimensional momentum of the particle, p_{σ} ($\sigma = \uparrow, \downarrow$) is the Fermi momentum of the spin- σ electrons, v_F is the Fermi velocity, and $\delta_{\sigma}[\vec{p}] \equiv \delta \times \text{sign}(|\vec{p}| - p_{\sigma})$, with δ an infinitesimal real number. The quasiparticle properties are hidden in the quasiparticle residue $z < 1$ and the effective mass. In principle the Fermi velocity and the quasiparticle residue also depend on the spin index, but in the neighborhood of the phase transition, $p_{\uparrow} - p_{\downarrow} \ll p_{\uparrow}, p_{\downarrow}$ and they are equal. In the case of weak ferromagnetic metals the incoherent part of the Green's function describing the physics away from the Fermi surface is a smooth function of \vec{p} and ω which renormalizes the properties at the Fermi surface and introduces no new physics. The Green's function describes a system of quasiparticles with spontaneous magnetization given by Dzyaloshinskii's theorem [9]

$$m_0 = \frac{1}{12\pi^2} (p_{\uparrow}^3 - p_{\downarrow}^3) = \frac{n_{\uparrow} - n_{\downarrow}}{2}, \quad (2)$$

where m_0 is the uniform, static magnetization, and n_σ is the density of spin- σ particles. For weak ferromagnets the magnetization m_0 is proportional to the exchange splitting $\Delta = p_\uparrow - p_\downarrow$. Here we have assumed that all particles are in an eigenstate of the z component of the spin operator and for definiteness we will assume that $p_\uparrow > p_\downarrow$. The low-energy excitations of the system described by this Green's function are quasiparticle excitations in the neighborhood of the two Fermi surfaces as well as collective spin excitations. The spontaneously broken SU(2) symmetry guarantees the existence of a massless Goldstone mode [10] described by the propagator

$$D_G(\vec{q}, \omega) = -\frac{\Delta N(0)v_F}{2} \frac{\omega_s(\vec{q})}{(\omega + i\delta)^2 - \omega_s^2(\vec{q})}, \quad (3)$$

$N(0)$ is the density of states at the Fermi surface. In the case of a ferromagnetic metal the magnetization is a conserved quantity and the spin-wave dispersion is $\omega_s(\vec{q}) = D|\vec{q}|^2$ where $D = v_F\Delta/p_F^2$ is the spin stiffness. The longitudinal response of the system is described by the propagator [11]

$$D_l(\vec{q}, \omega) = -\frac{N(0)p_F^2}{2} \frac{1}{\xi^{-2} + |\vec{q}|^2 - i\pi p_F^2 \omega / 2v_F |\vec{q}|}, \quad (4)$$

where $\xi \sim m_0^{-1}$ is the correlation length. The interaction of the quasiparticles with these collective spin excitations can be described by the interaction [12]

$$H_{\text{sf}} = g_0 \sum_{\vec{k}\vec{q}\alpha\beta} c_{\vec{k}\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{\vec{k}+\vec{q}\beta} \vec{S}_{-\vec{q}}, \quad (5)$$

where g_0 is the bare momentum-independent coupling constant, $c_{\vec{k}\alpha}^\dagger$ and $c_{\vec{k}\alpha}$ are the anticommuting, quasiparticle creation and annihilation operators respectively, $\vec{\sigma}_{\alpha\beta}$ are the Pauli matrices, and $\vec{S}_{-\vec{q}} = (\sum_{\vec{p}\gamma\delta} c_{\vec{p}\gamma}^\dagger \vec{\sigma}_{\gamma\delta} c_{\vec{p}-\vec{q}\delta})$ is the three component spin fluctuation field. The vector field $\vec{S}_{-\vec{q}}$ is the average magnetization at a particular wave vector. In the ferromagnetic phase this average is different from zero, while in the paramagnetic phase it is strictly zero. Nevertheless, it has been used to describe magnetically enhanced paramagnetic metals, although it can be mathematically justified only in the ferromagnetic phase.

Recently we have shown [7], ignoring the vertex corrections, that the self-energy leading to the exact Green's function, Eq. (2) is local and leads to logarithmic dependence of the quasiparticle residue on the magnetization. When the magnetization approaches the quantum critical point the quasiparticle residue vanishes and the Fermi-liquid theory breaks down. At finite temperatures in the neighborhood of the Curie temperature the spin fluctuations lead to a non-Fermi-liquid specific heat $C/T \sim \ln T$ consistent with recent experiments on MnSi and ZrZn₂ as well as on some of the heavy-fermion compounds [5,6].

Weak ferromagnetic metals are very interesting because the gapless Goldstone mode coexists with the longitudinal excitations which are gaped. The longitudinal

spin-fluctuation propagator Eq. (4) is similar to the model susceptibility [peaked at the $\vec{Q} = (\pi, \pi)$ nesting vector] used in the theory of antiferromagnetic metals. However, in our case the expression for the susceptibility is rigorous, following from the poles of the four-point vertex at small momentum transfer [11].

The first vertex corrections to the three-point vertex in the weak ferromagnetic metal, Fig. 1, are

$$\Lambda_{\uparrow\uparrow}^{(1)}(p, p+k) = \Lambda_{\uparrow\uparrow}^{(1)} + \Lambda_{\uparrow\uparrow G}^{(1)}, \quad (6)$$

$$\Lambda_{\uparrow\downarrow}^{(1)}(p, p+k) = \Lambda_{\uparrow\downarrow}^{(1)} + \Lambda_{\uparrow\downarrow G}^{(1)}, \quad (7)$$

where

$$\begin{aligned} \Lambda_{\uparrow\uparrow}^{(1)} &= ig_0^2 \int dq G_\uparrow(q) D_l(q-p) G_\uparrow(q+k), \\ \Lambda_{\uparrow\uparrow G}^{(1)} &= ig_0^2 \int dq G_\uparrow(q) D_G(q-p) G_\downarrow(q+k), \\ \Lambda_{\uparrow\downarrow}^{(1)} &= ig_0^2 \int dq G_\downarrow(q) D_l(q-p) G_\uparrow(q+k), \\ \Lambda_{\uparrow\downarrow G}^{(1)} &= ig_0^2 \int dq G_\downarrow(q) D_G(q-p) G_\downarrow(q+k), \\ dq &\equiv \frac{d^4 q}{(2\pi)^4}. \end{aligned} \quad (8)$$

Here we have assumed an expansion of the full vertex

$$z\Lambda_{\alpha\beta}(p, p+k) = 1 + \Lambda_{\alpha\beta}^{(1)} + \dots, \quad (9)$$

and $p, q,$ and k are four-vectors.

It is important to distinguish the order of the small momentum-transfer and energy-transfer limits. In the limit which defines the Fermi-liquid parameters through the four-point vertex, the Ward identity

$$\begin{aligned} \lim_{\omega \rightarrow 0} \lim_{\vec{q} \rightarrow 0} \Lambda_{\alpha\beta}(p, p+q) &= \left(1 - \frac{\partial \Sigma}{\partial \omega}\right) \\ \delta_{\alpha\beta} &= \frac{1}{z} \delta_{\alpha\beta}, \end{aligned} \quad (10)$$

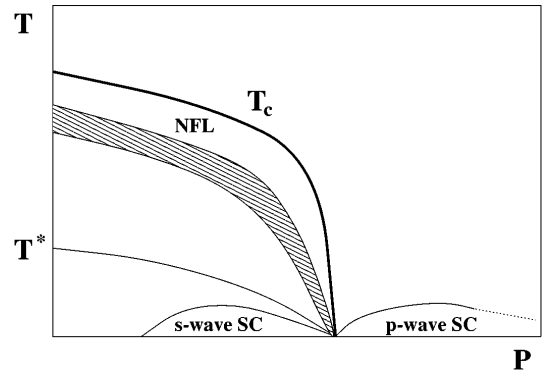


FIG. 1. The phase diagram of a weak ferromagnetic metal. The non-Fermi-liquid crossover region and the scale set by the temperature $T^* = T_c^2/\epsilon_F$ are explained in the text.

shows that the vertex is proportional to the inverse quasiparticle residue. The effective pairing potential in principle can be constructed from the three-point vertex with the requirement that the triplet scattering amplitude is zero. In second order perturbation theory, however, the momentum independence of the self-energy and the vanishing of the triplet scattering amplitude are incompatible, and so far we have not been able to construct a pairing potential with the above properties. Nevertheless, one can see that the singlet scattering amplitude is attractive leading to a pairing instability in the singlet channel [7]. Physically, the Pauli exclusion principle keeps quasiparticles with the same spin apart, leading to a negative charge depletion between them. This charge distribution attracts another quasiparticle with the opposite spin leading to the singlet pairing.

The Ward identity which we mentioned earlier shows that the effective pairing is enhanced for small magnetizations since $z^{-1} \sim \ln m_0$ and this enhancement is due to the longitudinal collective mode.

In the physical limit where energy is conserved, the corresponding Ward identity is

$$\lim_{\vec{q} \rightarrow 0} \lim_{\omega \rightarrow 0} \Lambda_{\alpha\beta}(p, p + q) = \frac{v_F}{z} \frac{dp_\alpha}{d\mu} \sigma_{\alpha\beta}^z, \quad (11)$$

$$G_\sigma(\vec{p} + \vec{q}, \epsilon + \omega) G_{\sigma'}(\vec{p} + \vec{q} + \vec{k}, \epsilon + \omega) = \frac{z}{v_F} \frac{G_\sigma(\vec{p} + \vec{q}, \epsilon + \omega) - G_{\sigma'}(\vec{p} + \vec{q} + \vec{k}, \epsilon + \omega)}{|\vec{p} + \vec{q}| - |\vec{p} + \vec{q} + \vec{k}| - (p_\sigma - p_{\sigma'})}, \quad (15)$$

it is not difficult to obtain the expansion

$$z \Lambda_{\sigma\sigma';l}(p_\sigma, p_{\sigma'}) = 1 + \frac{g_0^2 N^2(0) z^2}{16 p_\sigma} \ln \frac{\pi^2 p_\sigma^4 + 4\Delta^4}{\pi^2 p_\sigma^4 + 4(\Delta^2 + p_c^2)^2} + \dots \quad (16)$$

and

$$z \Lambda_{\sigma\sigma;G}(p_\sigma, p_\sigma) = 1 + \frac{g_0^2 N^2(0) z^2}{4} \ln \left(1 + \frac{\Delta p_c}{p_F^2} \right) + \dots \quad (17)$$

We have used a momentum cutoff p_c reflecting the different physics at very small distances. Very similar logarithmic behavior can be seen in the vertex expansion of $\Lambda_{\sigma\sigma';l/G}(p_\sigma, p_{\sigma'} + \Delta)$. This implies that the self-energy is weakly momentum dependent close to the phase transition. Therefore a local ferromagnetic Fermi-liquid theory [15] can be used to describe weak ferromagnetic metals in a regime where the magnetization is sufficiently small, but away from criticality (since the fluctuations in the critical regime are beyond the scope of Fermi-liquid theory). This confirms the s -wave pairing instability [7] in the ferromagnetic phase.

The above adiabaticity is a consequence of the smallness of the exchange splitting Δ compared to the Fermi momentum $p_F \equiv (p_\uparrow + p_\downarrow)/2$ and the smallness of the maximum spin wave velocity ω_G compared to the Fermi energy ϵ_F and leads to the validity of Migdal's theorem [16] for weak ferromagnetic metals.

In the DBS theory of spin-fluctuation-enhanced paramagnetic metals it is argued that the sharply peaked static spin susceptibility

where $\sigma_{\alpha\beta}^z$ is the Pauli matrix, v_F^0 is the Fermi velocity of the noninteracting Fermi gas, and there is no summation over repeated indexes.

In calculating the vertex corrections we first set the frequency to zero and then take the limit for the momentum. Because, we are working in the broken symmetry phase, a distinction must be made for vertex corrections involving particles on one of the two Fermi surfaces and vertex corrections involving particles on different Fermi surfaces. In the former case the limit

$$\Lambda_{\sigma\sigma}(|\vec{p}| \rightarrow p_\sigma, |\vec{p}| \rightarrow p_\sigma), \quad (12)$$

while in the latter the limit

$$\Lambda_{\sigma\sigma'}(|\vec{p}| \rightarrow p_\sigma, |\vec{p}| \rightarrow p_{\sigma'} + \Delta) \quad (13)$$

must be taken. In both cases we use the spectral representation for the propagators $D_{l/G}$

$$D_{l/G}(\vec{q}, \omega) = \frac{2}{\pi} \int_0^\infty \frac{z \operatorname{Im} D_{l/G}(\vec{q}, z)}{z^2 - \omega^2 - i\delta}. \quad (14)$$

Using that

$$\chi(0, 0) = \frac{\chi_p}{1 - N(0)V_c} \quad (18)$$

close to the Curie point suppresses the s -wave pairing, because ferromagnetic spin fluctuations act as an effective repulsive force between electrons with opposite spins. Here the χ_p is the Pauli susceptibility and V_c is a pseudopotential. To see why the spin fluctuations have an opposite effect in the ferromagnetic phase it is convenient to write the factor $1 - N(0)V_c$ in the denominator of the spin susceptibility in terms of the Landau Fermi-liquid parameter F_0^a . Then in both paramagnetic and ferromagnetic phases the static spin susceptibility is positive and in the ferromagnetic phase is [7]

$$\chi(0, 0) = \frac{N(0)/2}{|1 + F_0^a|}, \quad (19)$$

while in the paramagnetic phase is

$$\chi(0, 0) = \frac{N(0)}{1 + F_0^a}. \quad (20)$$

In approaching the quantum critical point from the paramagnetic side F_0^a approaches the value -1 from above, while approaching from the ferromagnetic side F_0^a approaches -1 from below. The different sign of $1 - N(0)V_c \sim 1 + F_0^a$ has a dramatic effect on the sign of the spin-fluctuation mediated quasiparticle interaction on the singlet channel which can be seen in the t matrix [2]

$$t(0,0) = V_c + \frac{N^{-1}(0)(F_0^a)^2}{1 + F_0^a}. \quad (21)$$

In the paramagnetic phase the second term is positive, while in the ferromagnetic it is negative leading to s -wave pairing.

What is the physics in the neighborhood of the quantum critical point in weak ferromagnetic metals is still an open question. Another interesting point is that the BCS theory of superconductivity cannot give a quantum critical point, because as the critical temperature approaches zero so does the pairing interaction. Whether a different type of superconductivity exists or a different phase exists in the neighborhood of the quantum critical point on the paramagnetic side of the phase diagram is still an open question.

In Fig. 1 we represent a schematic phase diagram of a "typical" weak ferromagnetic metal. Because the energy scale $T^* = T_c^2/\epsilon_F$ below which the superconducting instability occurs vanishes, the s -wave superconducting state must also vanish at the quantum phase transition. On the paramagnetic side the p -wave superconducting state is expected as predicted by the DBS theory. At finite temperatures close to the ferromagnetic phase transition on the ferromagnetic side, the spin fluctuations renormalize the physical quantities leading to a non-Fermi-liquid specific heat $C \sim T \ln T$, and in Fig. 1 we have shown the crossover between the Fermi-liquid and the non-Fermi-liquid state. We also expect the p -wave superconducting state in the paramagnetic phase to vanish at the quantum critical point. Another possibility is that it remains finite as we go through the phase transition at finite Curie temperature. However, the superconducting transition temperature must be less than the Fermi-liquid scale set by T^* which vanishes at the quantum phase transition, and this implies the vanishing of the p -wave paired state.

The s -wave superconducting state in the ferromagnetic phase is unusual and is a generalization of the Larkin-Ovchinnikov-Fulde-Ferrell (LOFF) state [3,4] studied in the 1960s in metals with magnetic impurities. The difference between this generalized LOFF state and the one originally studied is that the magnetic moment in the former is caused by the quasiparticles which also participate in the pairing, while in the latter the magnetic field is external to the quasiparticle system. Therefore the response of the two systems to an external magnetic field must be quite different. The understanding of how the spin fluctuations are modified by the superfluid density is an interesting question which can shed light on the nature

of this state. Another interesting possibility is that this state has an odd-gap close to half filling induced by the presence of magnon excitations. The details of this state are beyond the scope of the current paper and will be investigated in a future publication.

In conclusion, in this paper we described the physics of a weak ferromagnetic metal from microscopic principles. We have shown that the vertex corrections in the physical limit are small and that the self-energy is local. In the limit of small momentum transfer the vertex function enhances the effective coupling between the quasiparticles in the neighborhood of the quantum phase transition leading to an s -wave superconducting instability.

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