

## Coulomb Blockade without Tunnel Junctions

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Tunnel junctions are not needed to provide single electron effects in a metallic island. Eventually the tunnel junction may be replaced by an arbitrary scatterer. To formulate this in exact terms, we derive and analyze the effective action that describes an arbitrary scatterer. It is important that even a *diffusive* scatterer provides a sufficient isolation for single electron effects to persist. We also consider the fluctuations of the effective charging energy. [S0031-9007(99)08403-3]

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It is well known that electric charge of an isolated piece of conducting material can take only discrete values corresponding to the integer number of electrons in there. This property persists if this isolated piece, the island, is connected to electron reservoir by means of a resistive tunnel junction. It is the recognition of this mere fact that lead to an outburst of the entire field of single electron phenomena [1].

The single electron effects are best visible provided the conductivity of the tunnel junction is much smaller than the conductance quantum  $G_Q \equiv e^2/2\pi\hbar$ . The ground state energy as a function of induced charge  $q$  is given by minimization of Coulomb energy,  $E_C(n + q/e)^2$ , with respect to discrete charge  $n$ . The result is periodic in  $q$  with a period  $e$  [1]. The analysis of the reverse case,  $G \gg G_Q$ , requires advanced theoretical methods [2–4]. Despite the partial controversy in results, all authors agree that in this case the ground state energy retains the periodic  $q$  dependence, that manifests the Coulomb blockade. The effective charging energy,  $\tilde{E}_C$ , that is, the  $q$ -dependent part of the ground state energy, is suppressed by a factor of  $\exp(-G/2G_Q)$  in comparison with  $E_C$ .

Still the analysis has been restricted to tunnel junctions. The next step has been made in [5,6] where Coulomb

blockade has been studied in the situation where the isolation is provided by a quantum point contact with almost perfect transparency. It has been shown that the charge quantization survives. Albeit the charging energy is strongly suppressed vanishing to zero at perfect transmission.

In this paper, we construct a general theory of Coulomb blockade that can embrace tunnel junctions, quantum point contacts, diffusive conductors, and eventually any type of scattering.

The results are as follows. Charging energy vanishes only for perfect point contacts. For a very wide class of conductors that have conductivity  $G \gg G_Q$ , the charging energy is exponentially suppressed,  $\ln(\tilde{E}_C/E_C) \propto -\alpha G/G_Q$ ,  $\alpha$  being a dimensionless coefficient depending on the type of the conductor. For disordered conductors, for instance, diffusive ones, the charging energy strongly fluctuates. This happens even if the fluctuations of the conductance are small.

The most equivalent mathematical framework to describe the charging effects in full has been reviewed by Schön and Zaikin in [7]. The partition function of the system is presented in the form of the path integral over the field  $\varphi(\tau)$  ( $\beta \equiv \hbar/T$ ),

$$Z = \int \prod_{\tau} d\varphi(\tau) \exp \left[ -\mathcal{L}_{sc}[\varphi(\tau)] + \int_0^{\beta} d\tau \left( -\frac{\dot{\varphi}(\tau)^2}{2E_C} - i \frac{q\dot{\varphi}(\tau)}{e} \right) \right]. \quad (1)$$

The form of the last term presumes that the partition function can be presented as a sum over topological sectors that are labeled by an integer  $W = \int_0^{\beta} d\tau \dot{\varphi}(\tau)/2\pi$  winding number of  $\varphi(\tau)$ ,

$$Z = \sum_W Z_W \exp(-i2\pi Wq/e). \quad (2)$$

Here  $Z_W$  does not depend on  $q$ .

The action  $\mathcal{L}_{sc}$  describes tunnel junction. It can be evaluated by using the tunneling Hamiltonian method that gives

$$\mathcal{L}_{sc} = \frac{G_T}{2\beta^2 G_Q} \int_0^{\beta} d\tau \int_0^{\beta} d\tau' \frac{\sin^2\{[\varphi(\tau) - \varphi(\tau')]/2\}}{\sin^2[\pi(\tau - \tau')/\beta]}. \quad (3)$$

This form shows that the tunnel junction is quite different from a linear resistor which is described by a form

$$\mathcal{L}_{sc} = \frac{G}{8\beta^2 G_Q} \int_0^{\beta} d\tau \int_0^{\beta} d\tau' \frac{[\varphi(\tau) - \varphi(\tau')]^2}{\sin^2[\pi(\tau - \tau')/\beta]}, \quad (4)$$

which is bilinear in  $\varphi$ . It has been frequently assumed that a coherent diffusive conductor can be described by (4) and consequently exhibits no charging effects. We show below that it is not so. However, the relation (4) holds for an arbitrary conductor in the limit of small  $\varphi$ . It is worth noting that in the limit of  $G \gg G_Q$  the typical fluctuations of the phase are small indeed,  $\delta\varphi^2 \simeq G_Q/G \ll 1$ .

We sketch the derivation of  $\mathcal{L}_{sc}$  for an arbitrary conductor. Our basic assumptions are (i) no inelastic scattering occurs in the conductor; (ii) the conductor is sufficiently short,  $E_C \tau_{trav}/\hbar \ll 1$ ,  $\tau_{trav}$  being typical traversal time through the conductor; (iii) the island is sufficiently large, so that  $E_C$  greatly exceeds the average level spacing in the island. Two first assumptions allow us to characterize the conductor by an elastic scattering matrix disregarding retardation effects and possible energy dependence of the scattering matrix. The third assumption allows us to disregard coherence between the electrons transmitted to and

coming from the island, so that it can be regarded as an electron reservoir. These assumptions are flexible enough to treat various scatterers including those where the elastic scattering is determined by many-body effects. For instance, if the scatterer consists of two resistive tunnel junctions with an island in between, transmission may be strongly suppressed by Coulomb blockade effect. Such a compound scatterer fits the conditions (i) and (ii) provided the elastic cotunneling dominates the transport at energies of the order of  $E_C$ .

These assumptions correspond to a fermionic action of the following form

$$-\mathcal{L} = \int_0^\beta d\tau \left\{ \sum_{m=1}^M \int_{-\infty}^0 dx [\chi_m^\dagger(x, \tau) (\partial_\tau + i v_m \partial_x) \chi_m(x, \tau) + \psi_m^\dagger(x, \tau) (\partial_\tau - i v_m \partial_x) \psi_m(x, \tau)] \right. \\ \left. + \sum_{n=1}^N \int_0^\infty dx [\chi_n^\dagger(x, \tau) (\partial_\tau - i v_n \partial_x) \chi_n(x, \tau) + \psi_n^\dagger(x, \tau) (\partial_\tau + i v_n \partial_x) \psi_n(x, \tau)] + E_C [Q(\tau) - q/e]^2 \right\}. \quad (5)$$

Here the island is on the right ( $x > 0$ ),  $n$  and  $m$  label transport channels in the island and the reservoir, respectively.  $\chi$  stands for fermion fields coming to the scatterer, and  $\psi$  stands for outgoing modes;  $v_n$  are velocities in the channels. The scatterer is completely characterized by the scattering matrix  $\hat{S}_{kl}$  that sets a boundary condition for  $\psi$  and  $\chi$

$$\psi_k(0) = \sum_l S_{kl} \chi_l(0). \quad (6)$$

Here  $k, l$  label modes on both sides of the scatterer. The charge in the island is given by

$$Q(\tau) = \sum_{n=1}^M \int_0^\infty dx [\chi_n^\dagger(x, \tau) \chi_n(x, \tau) + \psi_n^\dagger(x, \tau) \psi_n(x, \tau)]. \quad (7)$$

To proceed, we perform a Hubbard-Stratanovitch trans-

form on interaction term introducing a new variable  $\varphi(\tau)$ ,

$$E_C [Q(\tau) - q/e]^2 \rightarrow -\frac{\dot{\varphi}(\tau)^2}{2E_C} + iQ(\tau)\dot{\varphi}(\tau) - i\frac{q\dot{\varphi}(\tau)}{e}. \quad (8)$$

The resulting action is quadratic in fermions so that they can be integrated out and the action can be represented as a functional of  $\varphi(\tau)$ . We do this calculating Green functions of the fermions in the presence of field  $\varphi$  and scattering potential. Special attention shall be given to the fact that in one dimension the Green function  $G(x, \tau, x', \tau'; [\varphi(\tau)])$  is not continuous at coinciding arguments, so that the problem shall be regularized by letting scattering occur in a small but finite region of space. The result does not depend on a regularization procedure, so we use the one that makes for an easy calculation. The following action for fermion fields  $a_k^\dagger, a_k$ ,

$$-\mathcal{L} = \int_0^\beta dt \int_{-\infty}^\infty dz \left\{ \sum_{k=1}^{N+M} a_k^\dagger(z, \tau) [\partial_\tau + i v_k \partial_z + i \dot{\varphi}(\tau) \sigma_k] a_k(z, \tau) + \sum_{k,l}^{M+N} a_k^\dagger(z, \tau) \tilde{\delta}(z) \sqrt{v_k} H_{kl} \sqrt{v_l} a_l(z, \tau) \right\}, \quad (9)$$

where  $\sigma_k = 1$  for the channels in the island and  $\sigma_k = 0$  otherwise,  $z = \pm(2\sigma_k - 1)x$  for incoming (outgoing) modes,  $\tilde{\delta}(z)$  is a smooth approximation of delta function, and proves to be a proper regularization if  $\hat{S} = \exp(i\hat{H})$ . [We reverse coordinates in the channels in such a way that all outgoing (incoming) modes go to (come from)  $\pm\infty$ .] The action (9) can be integrated over fermions and the answer can be presented as a formal series in  $\hat{H}$ :

$$-\mathcal{L}_{sc} = \sum_{m=1}^\infty \text{Tr}[(\sqrt{\hat{v}} \hat{H} \sqrt{\hat{v}} \hat{G}^{(0)})^m], \quad (10)$$

where operator multiplication and trace function includes summation over channels, integration over  $\tau$ , and integration over  $z$  with weight factor  $\tilde{\delta}(z)$ . Here we introduce  $\hat{G}^{(0)}$ , Green function in the absence of  $\hat{H}$ . To comply with assumption (ii) we assume that the spread of  $\tilde{\delta}(z)$  is small in comparison with  $v_n/\varepsilon$ ,  $\varepsilon$  being a typical Matsubara frequency involved. In this case,

$$\hat{G}_{kl}^{(0)}(z, \tau, z', \tau'; [\varphi(\tau)]) = \delta_{kl} \frac{i}{v_k} [\theta^+(\tau - \tau') \theta(z - z') - \theta^-(\tau - \tau') \theta(z' - z)] (e^{i[\varphi(\tau) - \varphi(\tau')]} \sigma_k + 1 - \sigma_k), \quad (11)$$

where Fourier components of  $\theta^{\pm}$  are  $\theta(\pm\varepsilon)$ . The key step is to resum the series and to present the action as a series

in  $\Delta G = G_0(x, \tau, x', \tau'; [\varphi(\tau) = 0]) - G_0(x, \tau, x', \tau'; [\varphi(\tau)])$ . The  $\Delta G$  is continuous at  $x = x'$  and does not depend on  $x, x'$  within the spread of  $\tilde{\delta}(z)$ . This allows one to integrate over  $z$  in Eq. (10).

Then the action can be reduced to the trace of logarithm of an operator,

$$-\mathcal{L}_{sc} = \text{Tr}_{n,\tau} \ln [1 - (1 - \hat{r})\hat{\theta}^+ \exp(-i\varphi)\hat{\theta}^- \exp(i\varphi) - (1 - \hat{r}^+)\hat{\theta}^- \exp(-i\varphi)\hat{\theta}^+ \exp(i\varphi)], \quad (12)$$

where operator multiplication involves summation over channel indices inside the island and integration over  $\tau$ . The operator  $\hat{r}$  is the reflection matrix for the island channels.

Expression (12) can be explicitly evaluated in two limits:  $\hat{r} \rightarrow 1$  and  $\varphi \rightarrow 0$ . There, we successfully reproduce Eq. (3) for tunnel junctions and Eq. (4) for an arbitrary scatterer in the linear regime. In general, even an evaluation of Eq. (12) at specific realization of  $\varphi(\tau)$  presents a complicated problem.

Albeit a very important part of analysis of the action (12) can be done exactly. We are able to find the minima of (12) in each topological sector, and thus give a quantitative estimate of effective Coulomb energy in the limit  $G \gg G_Q$ . We consider the configurations of  $\varphi(\tau)$  of the following form:

$$\exp(i\varphi) = \prod_{i=1}^N \frac{u - z_i}{1 - uz_i^*}. \quad (13)$$

Here  $u \equiv \exp(i2\pi\tau/\beta)$ ,  $z_i$  are complex parameters.  $z_i$  can be viewed as coordinates of  $N$  (anti)solitons in the plane of complex  $u$ . If  $|z_i| \rightarrow 1$ , these configurations correspond to sets of Korshunov's solitons [8]. We are interested in configurations where all solitons are of the same sign. In this case either  $|z_i| < 1$  for all  $i$  or  $|z_i| > 1$  so that  $\exp(i\varphi)$  is an analytical function of  $u$  either within or beyond the unitary circle. The winding number  $W = \pm N$ . Using the methods of analytical function theory we show that these configurations indeed minimize the action in the corresponding topological sector. The minimum does not depend on  $z_i$  and equals

$$-\mathcal{L}_W = \frac{1}{2} \ln \det(\hat{r}\hat{r}^+) |W| + \frac{1}{2} \ln \det(\hat{r}/\hat{r}^+) W. \quad (14)$$

The second term is imaginary and can be viewed as a trivial shift of induced charge  $q$ :  $q \rightarrow q + i \ln \det(\hat{r}/\hat{r}^+)$ . The first term is of importance since it describes the suppression of statistical weight of topological sectors with  $W \neq 0$  in comparison with the trivial sector. It has been shown in [2,3] that the suppression of these statistical weights leads to suppression of effective charging energy. This allows us to write down a simple formula for effective

charging energy

$$\tilde{E}_C \propto E_C \prod_n R_n^{1/2}, \quad (15)$$

where  $R_n$  are eigenvalues of the reflection matrix  $\hat{r}\hat{r}^+$ . This formula is valid provided the suppression is big. A similar relation has been obtained by Flensburg [6] in a much more restrictive framework. In the limit of almost perfect transmission,  $R \rightarrow 0$ , we reproduce the results of Matveev [5].

Recent theoretical advances allow us to characterize  $R_n$  of a scatterer/conductor of virtually any type (see [9] for a review). This makes the relation (15) easy to use for concrete examples. From now on, we will concentrate on diffusive conductor in the limit  $G \gg G_Q$ . It is a disordered conductor, so that it is characterized by distribution of  $R_n$ , or transmissions  $T_n = 1 - R_n$ . It has been shown in [10] that the transmission distribution of a diffusive conductor depends only on its conductance,  $\rho(T) = G/(2G_Q T \sqrt{1-T})$ . We average the logarithm of (15) with this distribution to obtain

$$\tilde{E}_C/E_C \propto \exp\left(-\frac{\pi^2 G}{8G_Q}\right). \quad (16)$$

This is the main result of this work. The diffusive scatterer of the same resistance as a tunnel junction suppresses Coulomb blockade much more efficiently. To give some numbers, let us choose  $1/G = 4\text{kohm}$ . In this case, the suppression factor is about 25 for a tunnel junction and almost 3000 for a diffusive conductor.

Below we consider the fluctuations of  $\tilde{E}_C$  and the effect of weak localization. To make a qualitative estimation, we note that the fluctuation of  $G$  is of the order of  $G_Q$ . The weak localization correction is of the same scale. Therefore, the fluctuations of an exponential like (16) must be of the order of its average value. The same should hold for the weak localization effect. It is remarkable that quantitative consideration gives even bigger values.

This quantitative treatment can be performed along the lines of [11] and [12]. There are formulas that can be directly applied to the quantity of interest  $\ln(\tilde{E}_C/E_C) = \frac{1}{2} \sum_n \ln R_n$ . It appears that both the fluctuation and the localization effect are dominated by a contribution of the *universal* cooperon-diffusion mode, the one which provides Wigner-Dyson statistics of closely spaced transmission eigenvalues [11]. The contribution of this mode logarithmically diverges at very small  $R$  and shall be cut off at  $R \simeq G_Q/G$ , the average value of transmission spacing. For pure statistical ensembles, the fluctuation is given by

$$\langle\langle \ln^2(\tilde{E}_C/E_C) \rangle\rangle = \frac{N_{dc}}{4} \ln(G/G_Q), \quad (17)$$

where  $N_{dc}$  is the number of massless cooperon and diffusion modes. It ranges from 1 to 8. The weak localization

correction is

$$\langle \ln(\tilde{E}_C/E_C) \rangle_{\text{wl}} = -\frac{N_{\text{wl}}}{4} \ln(G/G_Q), \quad (18)$$

where  $N_{\text{wl}}$  is 2, 0, -1 for symplectic, unitary, and orthogonal ensemble, respectively.

$$\langle \langle \ln[\tilde{E}_C(\eta_H)/E_C] \ln[\tilde{E}_C(\eta'_H)/E_C] \rangle \rangle = -\frac{1}{4} \ln\{ \eta_H^2 - \eta_H'^2 [(\eta_H + \eta'_H)^2 + \eta_{SO}^2]^3 [(\eta_H - \eta'_H)^2 + \eta_{SO}^2]^3 \}. \quad (19)$$

This equation is valid provided  $G/G_Q \ll |\eta_H - \eta'_H|$ ,  $\eta_H, \eta'_H, \eta_{SO} \ll 1$ . It shows that the actual value of  $\tilde{E}_C$  can be changed by an order of magnitude by a fairly small change of magnetic field  $\delta\eta_H \approx G_Q/G$ . At a bigger  $\delta\eta_H$  effective charging energy exhibits power-law correlations.

Magnetic field dependence of the weak localization correction is given by

$$\langle \ln(\tilde{E}_C/E_C) \rangle_{\text{wl}} = \frac{1}{4} [3 \ln(\eta_H^2 + \eta_{SO}^2) - \ln(\eta_H^2)]. \quad (20)$$

To conclude, we have shown that the isolation required for discrete charge effects can be provided by any constriction which is not ideally ballistic. We have discussed suppression of the effective charging energy by a diffusive scatterer and found gigantic fluctuations of this quantity. It might seem surprising that the phase-dependent action of a diffusive conductor is not quadratic in phase since this suggests that the electrodynamic of such a conductor may be potentially nonlinear. We note that a proof of the fact that the action is not a quadratic one is in fact already known. Whereas the action of the form (4) leads to Johnson-Nyquist current noise, it was predicted that a coherent diffusive conductor produces a substantial extra short noise [13]. In [14] the short noise has been related to a nonlinear response diagram.

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Experimentally, the fluctuation and weak localization effect are identified by their magnetic field dependence. Following [11] we introduce dimensionless parameters  $\eta_H, \eta_{SO}$  to characterize magnetic field and spin-orbit interaction. We disregard influence of magnetic field on spin. Correlator of two  $\tilde{E}_C$  taken at different values of magnetic field reads

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