Multimode Analysis of the Hollow Plasma Channel Wakefield Accelerator

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The hollow plasma channel is analyzed as an accelerating structure. The excitation of the channel by an ultrarelativistic beam is analyzed. Coupling to the fundamental and all higher-order azimuthal modes of the excited electromagnetic fields is derived. Implications of this work for plasma-based accelerators, including beam loading and beam breakup, are discussed. Small initial transverse displacements of the beam are shown to couple to deflecting modes in the channel. The asymptotic growth rate of the resultant beam breakup instability is analyzed and a method for reducing the growth is proposed. [S0031-9007(99)08413-6]

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The reach of high-energy physics is limited by its instruments, accelerators, and passive conducting structures attaining large accelerating fields by resonant excitation [1]. In conventional accelerators, the size of these accelerating fields is limited by breakdown. For two decades, plasma-based accelerators [2] have been investigated as a means of overcoming this breakdown constraint. Two schemes of plasma excitation have been the focus of much of the work: the laser wakefield accelerator and the plasma wakefield accelerator. In the laser wakefield accelerator, a short intense laser pulse excites a plasma wave through radiation pressure (the ponderomotive force). In the plasma wakefield accelerator, the plasma wave is excited by the self-fields of an intense relativistic particle beam.

For the laser wakefield accelerator one of the most severe limitations is the weakening of the laser pulse intensity due to diffraction. To overcome this limitation, the use of a preformed plasma channel to provide optical guiding has been proposed [3,4]. A parabolic profile was first studied [3], and subsequently a hollow plasma channel [4,5]. In a hollow plasma channel, the transverse profile of the driver is decoupled from the transverse profile of the accelerating mode. Therefore, for a relativistic driver, the accelerating gradient is uniform and the focusing fields are linear [4]. In addition, the accelerating mode of the hollow plasma channel is fully electromagnetic, unlike the electrostatic fields excited in a homogeneous plasma. These properties make it well suited as a structure for both particle beam wakefield accelerators as well as laser driven wakefield accelerators.

Preformed channel creation is currently being explored experimentally [6–9]. Since the original demonstration of the guiding of a low intensity laser in a plasma channel by Milchberg and co-workers [6], several groups are examining methods of channel formation [6–9] and the guiding of high intensity lasers [6–8]. Methods of forming a plasma channel include inverse bremsstrahlung heating of the plasma by a precursor laser pulse resulting in hydrodynamic expansion and channel formation [6] and ionization of a preformed capillary tube [7].

In this Letter, we characterize an externally formed hollow plasma channel as an accelerating structure, independent of the structure excitation mechanism (laser or particle beam). Our results allow us for the first time to set down the basic scalings for the plasma channel accelerator, including current limiting higher-order mode couplings such as beam breakup (BBU) instabilities [10].

We consider an equilibrium plasma density $n_e(r) = n_o H(r - b)$, where H is the step function, b is the channel radius, and n_o is the number density of the plasma. The drive pulse duration is taken to be much shorter than the response time of the (assumed motionless) ions. The mode structure can be derived from Maxwell's equations and the linearized fluid equations for a cold collisionless plasma. Only transverse modes (i.e., $\nabla \cdot \vec{E} = 0$) exist in the channel, and since there are no linear surface currents, the continuity of $\nabla \times \vec{E}$ requires the mode in the plasma to also be transverse. The equation for the plasma wake electric field behind the drive pulse is

$$(c^2\nabla^2 - \partial_t^2 - \omega_p^2)\vec{E} = 0\,, \tag{1}$$

where $\omega_p^2(r) = 4\pi n_e e^2/m_e$ is the electron plasma frequency, with m_e the electron rest mass and $-e$ the electron charge. We assume the drive pulse is nonevolving and propagates axially at near the speed of light c such that we may make the "frozen-field" approximation: axial variation at a fixed position is small and the modes are functions of the comoving coordinate $\tau = t - z/c$. The fields are decomposed into discrete azimuthal modes with mode index m, and a Fourier transform in τ is made such that solutions are of the form $\exp[-i\omega_m\tau + im\theta]$. The boundary conditions across the channel wall are continuity of the electric and magnetic field components $\epsilon\vec{E}\cdot\hat{r}$, $\vec{E}\times\hat{r}$, and $\vec{B}$, where $\epsilon = 1 - \omega_p^2/\omega_m^2$.

Solving Eq. (1) yields the linearly excited mode frequencies of the hollow plasma channel,

$$\omega_m = \omega_p \Omega_m = \omega_p \left[\frac{(1 + \delta_{m0})(m + 1)K_{m+1}(R)}{2(m + 1)K_{m+1}(R) + RK_m(R)} \right]^{1/2}, \quad (2)$$

where $R = \omega_p b/c$ is the normalized channel radius, K_m are mth-order modified Bessel functions of the second kind, and $\delta_{m0} = 1$ for $m = 0$ and zero otherwise. For the linear analysis to be valid, surface plasma density perturbations should be small compared to the channel radius. This implies that a particle drive beam must satisfy $(r_o \omega_p/c)N_b \ll R^2$, where $r_o = e^2/m_e c^2$ is the classical electron radius and N_b is the number of electrons per bunch.

As the beam travels through the structure, it excites modes, and the modes in turn influence the beam propagation. Higher-order moments of the drive pulse, due to drive pulse shape or misalignment, will excite higher-order modes in addition to the fundamental (accelerating) mode. These higher-order modes can cause BBU instabilities, limiting the beam current. This interaction of the beam with the accelerator environment can be quantified by a calculation of the loss factors. The loss factor per unit length κ relates the accelerating gradient to the energy stored per unit length in the structure U by $\kappa = E_z^2/4U$. It is a purely geometrical factor of the structure independent of the excitation mechanism [11]. Since the loss factor is independent of the means of energy deposition, it is a figure of merit to compare accelerating structures.

The conserved electromagnetic energy density, from the Poynting equation, averaged over plasma wake phase is

$$u_{\text{field}} = \frac{1}{16\pi} [(E_r - B_\theta)^2 + (E_\theta + B_r)^2 + E_z^2 + B_z^2]. \quad (3)$$

Using the fluid equations, the energy density stored in the plasma fluid motion can be expressed in terms of the fields as

$$u_{\text{fluid}} = \frac{1}{16\pi} \Omega_m^{-2} [E_r^2 + E_\theta^2 + E_z^2]. \quad (4)$$

Performing the integral over the transverse coordinates of the field and fluid energy densities Eqs. (3) and (4) yields the total energy per unit length stored in the structure due to the excitation of the mth mode,

$$U_m = \int_0^\infty d^2\vec{r}_\perp (u_{\text{field}} + u_{\text{fluid}}) = \frac{c^2}{\omega_p^2} G_m^2 (1 + \delta_{m0}) \frac{R^{2m+1} K_{m+1}(R)}{8\Omega_m^2 K_m(R)}. \quad (5)$$

Here G_m are constants determined by the excitation mechanism, and $G_m R^m$ is the peak axial electric field of the mth mode at the channel radius. The energy stored in the fundamental mode U_0 is a lower bound on the amount of energy per unit length that must be deposited in the structure to produce a desired accelerating gradient G_0 in the channel.

Using Eq. (5), the loss factor per unit length for the mth mode is

$$\kappa_m = \frac{\omega_p^2}{c^2} \left[\frac{K_m(R)}{RK_{m+1}(R)} \right] \left[1 + \frac{RK_m(R)}{2(m + 1)K_{m+1}(R)} \right]^{-1}, \quad (6)$$

where the axial electric fields of the higher-order modes have been evaluated at the channel radius. For comparison, the fundamental mode of a scaled disk-loaded Stanford Linear Accelerator Center (SLAC) structure [1] has a loss factor of $\kappa_0 \approx 2.1 \times 10^3 \lambda^{-2}[\text{cm}]\ \text{V}/(\text{pC m})$, while the fundamental mode loss factor in a hollow plasma channel is $\kappa_0 = 3.6 \times 10^3 \lambda_0^{-2}[\text{cm}][K_0(R)/RK_1(R)]\ \text{V}/(\text{pC m})$, where $\lambda_0 = \Omega_0^{-1} 2\pi c/\omega_p$ is the accelerating wavelength. For $R = 1$, $\kappa_0 = 2.5 \times 10^3 \lambda_0^{-2}[\text{cm}]\ \text{V}/(\text{pC m})$, somewhat higher than the conventional resonantly excited conducting structure, which implies stronger beam loading and smaller stored energy per unit length for a given gradient.

To further appreciate the implications of Eq. (6), consider a numerical example where, for simplicity, only the fundamental mode, with a wavelength $\lambda_0 = \Omega_0^{-1} 2\pi c/\omega_p \approx 146\ \mu\text{m}$, is excited. For a channel radius $b \approx 20\ \mu\text{m}$, $R \approx 1$ and the loss factor is $\kappa_0 \approx 12\ \text{MV}/(\text{pC m})$. If a 10 GV/m gradient is desired, the energy stored in the structure is $U = G_0^2/4\kappa_0 \approx 2\ \text{J/m}$. Assuming the drive pulse is fed once per meter, one sees that the drive pulse energy must exceed 2 J, accounting for losses due to coupling to the accelerating mode.

The energy stored in the plasma structure U_m is equal to the energy deposited by the driver,

$$U_m = \frac{1}{c} \int d^3\vec{r}\, \vec{J}_{\text{ext}} \cdot \vec{E}_m, \quad (7)$$

where $\vec{J}_{\text{ext}}$ is the current due to the driver. For an ultrarelativistic charge q at a radius a (with $a < b$), the total energy deposited in the plasma structure is

$$U_{\text{total}} = \sum_m U_m = \sum_m \kappa_m (a/b)^{2m} q^2. \quad (8)$$

This is the total energy loss in the sense that, unlike a conventional structure, there are no other synchronous modes supported by the structure. Furthermore, the relation $\beta^2\epsilon < 1$ will always be satisfied for the plasma-vacuum structure since $\epsilon = 1 - \omega_p^2(r)/\omega^2 \le 1$, and therefore no energy will be lost radially in the hollow plasma channel. A channel with a finite plasma width, in contrast, will have losses. This has been analyzed for an electromagnetic laser pulse guided by a finite plasma width channel [12], where $\omega \gg \omega_p$. The radial emission from the wakefield itself, with $\omega \sim \omega_p$, in a channel with a finite plasma width has not been investigated.

The above results can be used to model particle beam dynamics in a hollow plasma channel. The longitudinal and transverse forces on an ultrarelativistic beam due to its interaction with the plasma can be calculated from the convolution of the charge distribution of the beam with the wakefields $\vec{W} = \vec{E} + \hat{z} \times \vec{B}$ produced by all proceeding charges. The wakefields can be determined by solving Eq. (1) for the excited fields and using Eq. (7) to determine the amplitude excited for a given source. The wakefields excited in the channel by an ultrarelativistic point charge q, passing through the channel at radius a (with $a < b$) and azimuthal angle $\theta = 0$ are

$$\vec{W}_{\|} = -q \sum_m \hat{W}_{\|m}(\tau) r^m a^m \cos(m\theta)\hat{z}\,, \tag{9}$$

$$\vec{W}_{\perp} = q \sum_m \hat{W}_{\perp m}(\tau) r^{m-1} a^m [\hat{r}\cos(m\theta) - \hat{\theta}\sin(m\theta)]\,, \tag{10}$$

where

$$\hat{W}_{\|m}(\tau) = \frac{2\kappa_m}{b^{2m}} \cos[\Omega_m \omega_p \tau]\,, \tag{11}$$

$$\hat{W}_{\perp m}(\tau) = \frac{2m\kappa_m c}{b^{2m}\Omega_m \omega_p} \sin[\Omega_m \omega_p \tau]\,. \tag{12}$$

Here Ω_m and κ_m are given by Eqs. (2) and (6), respectively. Note that if the charge is near the axis of the channel ($a \ll b$) then the longitudinal wakefield is dominated by the $m = 0$ mode and the transverse wakefield by the $m = 1$ mode.

The longitudinal wakefields Eq. (9) produced by a bunch will cause an energy spread $\delta\gamma$ within the bunch, which limits the current. For example, if an energy spread of order 0.1% is required in a plasma structure with $b = 20$ μm, $R = 1$ and an accelerating gradient of $G_0 = 10$ GV/m, then $\delta\gamma/\gamma \approx (1/2)W_{\|}/G_0 \approx 0.001$ for a short bunch, and the beam-induced gradient should be held to $2\kappa_0 q \approx 20$ MV/m. The single-bunch charge is then limited to 0.9 pC or 5×10^6 particles. In principle, the energy spread within a single bunch can be minimized and the charge limits increased by shaping the charge distribution of the bunch [13], although this may be difficult to achieve in practice.

The transverse wakefields Eq. (10) can cause BBU instabilities. The transverse displacement of the beam $X(z, \tau)$ can be expressed as a function of two variables: the propagation distance z and the distance from the head of the beam τ. From the Lorentz force equation, assuming the beam is monoenergetic, the evolution of the transverse displacement of the beam due to the dipole transverse wakefields is

$$\left[\frac{\partial}{\partial z}\gamma(z)\frac{\partial}{\partial z} + \gamma(z)k_\beta^2(z)\right] X(z,\tau) = \int_0^\tau c\,d\tau'\,\frac{I(\tau')}{I_o}\,\hat{W}_{\perp 1}(\tau - \tau')X(z,\tau')\,, \tag{13}$$

where $I(\tau)$ is the beam current, $I_o = mc^3/e \approx 17$ kA is the Alfvén current, and $\hat{W}_{\perp 1}$ is given by Eq. (12) with $m = 1$. The right-hand side of Eq. (13) is the cumulative force due to the transverse dipole wakefields of the proceeding charges in the beam. The transverse focusing force in the channel from a plasma wake (created by a drive pulse) and from any external magnets can be described, in the linear approximation, by the betatron wave number $k_\beta(z)$. This model is valid in the ultrarelativistic limit, where phase slippage between particles in the bunch is small. Equation (13) can be solved in a variety of limits to study the single-bunch BBU instability [10].

We consider the case of a bunch much shorter than the natural periods of the wakefield (i.e., $\tau \ll \omega_p^{-1}$). The growth of the beam displacement can be analyzed following Ref. [14]. In the limit that the growth length of the instability is less than k_β^{-1} and assuming the beam energy grows as $\gamma = \gamma_0 + gz$, where g is a constant accelerating gradient in beam energy, solving Eq. (13) yields $X(z,\tau)/X_0 \approx (\gamma_0/\gamma)^{1/4}(8\pi A)^{-1/2}\exp(A)$. Here X_0 is the initial transverse displacement of the beam, γ_0 is the initial injection beam energy, and the exponent has the form

$$A = 2^{7/4}\left[\frac{I}{I_o}\,\frac{\kappa_1(\omega_p\tau)^2}{g^2R^2}\right]^{1/4}(\gamma^{1/2} - \gamma_0^{1/2})^{1/2}. \tag{14}$$

Asymptotically, $A \rightarrow (z/L_g)^{1/4}$, with the instability growth length,

$$L_g = 2^{-7}\,\frac{I_o}{I}\,\frac{gR^2}{\kappa_1(\omega_p\tau)^2}\,. \tag{15}$$

For illustration, $L_g \approx 5$ mm for a 3 fs beam with a charge of 1 pC traveling through a plasma channel with a plasma wavelength of 125 μm, channel radius of 20 μm, and accelerating gradient of 10 GV/m. This instability growth length can be increased by increasing R, which in turn will lower the loss factor of the structure for fixed plasma density.

For high-energy applications one may prefer not to operate in the weak focusing regime $k_\beta L_g \ll 1$; yet the focusing in the plasma channel due to the accelerating wakefield is weak for relativistic beams. In contrast, if external focusing is applied in the plasma structure, the asymptotic growth of the transverse beam displacement is much reduced. Assuming the external focusing has a dependence on beam energy such that $k_\beta \propto \gamma^{-\alpha}$, Eq. (13) can be solved for the transverse beam displacement of a short bunch,

$$\frac{X(z,\tau)}{X_0} \approx \frac{3^{1/4}}{2^{3/2}\pi^{1/2}}\left(\frac{\gamma_0}{\gamma}\right)^{(1-\alpha)/2}\frac{\exp(A_e)}{A_e^{1/2}} \times \cos\left[\theta - \frac{A_e}{3^{1/2}} + \frac{\pi}{12}\right], \tag{16}$$

with the betatron phase $\theta = g^{-1}\gamma_0^{\alpha} k_0 (1 - \alpha)^{-1}(\gamma^{1-\alpha} - \gamma_0^{1-\alpha})$ and exponent,

$$A_e = \frac{3^{3/2}}{2^{5/3}} \left[\frac{I}{I_o} \frac{\kappa_1(\omega_p \tau)^2}{\alpha g \gamma_0^{\alpha} k_0 R^2} (\gamma^{\alpha} - \gamma_0^{\alpha}) \right]^{1/3}, \qquad (17)$$

where k_0 is the initial betatron wave number at injection. Asymptotically, $A_e \rightarrow (z/L_e)^{\alpha/3}$, with the instability growth length,

$$L_e = \frac{2^{5/\alpha}}{3^{9/2\alpha}} \left(\frac{I_o}{I} \right)^{1/\alpha} \left[\frac{\alpha g^{1-\alpha} \gamma_0^{\alpha} k_0 R^2}{\kappa_1(\omega_p \tau)^2} \right]^{1/\alpha}. \qquad (18)$$

For example, if $\alpha = 1/2$, then the growth rate scales as $L_e \propto (I/I_o)^{-2}(\omega_p \tau)^{-4}$, a more favorable scaling than Eq. (15).

The longitudinal wakefields (beam loading) and transverse wakefields (beam breakup) constrain the charge in a single bunch. Therefore, a high-energy collider must operate with multiple bunches. For multibunch operation, control of BBU requires stagger tuning [15]. Our results indicate a path to stagger tuning for the plasma channel accelerator. The mode frequencies are functions of two independent experimental parameters: the channel radius and the plasma frequency. Therefore these two parameters can be varied such that the higher-order mode frequencies vary over the length of the accelerator while maintaining a constant fundamental (accelerating) mode frequency.

The promise of the hollow plasma channel depends on the ability to form such a structure. A realistic channel will have errors in the mode frequencies and loss factors due to many effects, for example, variation in the plasma density, a finite plasma density in the channel n_c, or finite thickness of the plasma walls. Equation (2) implies that fractional errors in ω_p produce equal fractional errors in ω_m due to the leading ω_p term and a frequency error due to the variation in R. Errors in the channel radius also produce errors in R. From Eq. (2), a straightforward calculation of $(\delta\omega_m/\omega_m)(\delta R/R)^{-1}$ indicates operating at a smaller channel radius increases frequency tolerances to errors in channel radius.

A finite plasma density inside the channel will shift the mode frequencies by $\delta\omega_m/\omega_m \sim n_c/n_o$. Furthermore, an undesirable electrostatic mode may also be excited by the drive pulse in a partially filled channel. This electrostatic mode will be a source of energy loss in the system. One may prefer to operate a plasma accelerator in the linear regime (i.e., $eE/mc\omega_p \ll 1$) to avoid self-trapping of plasma electrons. Self-trapping of electrons from the accelerator viewpoint is one source of "dark current" [16] (electrons emitted from the structure which subsequently must be eliminated) and may set a limit on peak gradient for a multistaged collider. Nonlinear effects, such as self-trapping, become important at a lower field amplitude in the partially filled channel.

The important problem of finite wall thickness leads to a quality factor Q of the plasma structure [5,17]. In our model this figure is infinite. Linear analysis including finite wall thickness [5] and numerical simulations [17] suggests that care needs to be taken for Q to be sufficiently large.

In summary, we have characterized the hollow plasma channel in terms of the fundamental accelerator parameters: mode frequencies and loss factors. The monopole and dipole results provide the limits due to beam loading and single-bunch beam breakup. With these results, one can quantify for the first time the performance of a high-energy machine based on this plasma structure.

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