"Superconductor-Insulator Transition" in a Single Josephson Junction

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VI curves of resistively shunted single Josephson junctions with different capacitances and tunneling resistances are found to display a crossover between two types of *VI* curves: one without and another with a resistance bump (negative second derivative) at zero bias. The crossover corresponds to *the dissipative phase transition* (superconductor-insulator transition) at which macroscopic quantum tunneling delocalizes the Josephson phase and destroys superconductivity. Our measured phase diagram does not agree with the diagram predicted by the original theory, but does coincide with a theory that takes into account the accuracy of voltage measurements and thermal fluctuations. [S0031-9007(98)08351-3]

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A Josephson junction is a unique physical object on which one can test a great variety of important physical concepts of modern physics, such as macroscopic quantum tunneling of the phase, quantum mechanical coherence, Coulomb blockade, etc. An important place in this list is occupied by the so called *dissipative phase transition* (DPT), predicted for various systems [1–3]. The physical origin of this transition is the suppression of macroscopic quantum tunneling of the phase by the interaction with a dissipative quantum-mechanical environment, described by the Caldeira-Leggett model. Macroscopic quantum tunneling destroys superconductivity of a junction, whereas suppression of tunneling restores Josephson current. Hence, this transition is often called a superconductor-insulator transition (SIT).

The detection of DPT in a *single* Josephson junction is of principal importance since it is the simplest system where this transition is expected, without any risk of being masked by other physical processes, as is possible in more complicated systems such as regular or random Josephson junction arrays. Some evidence of DPT (SIT) in a single Josephson junction has already been reported [4], but only for the case of weak Josephson coupling. It has not been enough to trace the whole phase diagram, including the range of strong Josephson coupling where the theoretical predictions are especially intriguing.

In this Letter, we present results of our measurements on $R = dV/dI$ vs I curves, for a variety of single small isolated Josephson junctions, shunted and unshunted, with different values of capacitance C and normal state tunneling resistance R_T. We have detected a crossover between two types of *RI* curves with an essentially different behavior at small currents. Relating this crossover with the DPT, we were able to map out the *whole* phase diagram for a Josephson junction. The position of the observed phase boundary does not agree with that expected from the original theory. However, the theory, revised to take into account a finite accuracy of our voltage measurements (viz., the minimum voltage that we are able to detect), explains well the observed phase diagram. We also argue that the real signature of DPT is a modification of *VI* curves as observed in our experiment: the SIT, traditionally defined as the change of sign of thermoresistance dR/dT, is not necessarily identical to the DPT. The measured phase diagram provides the first observation of DPT for a single Josephson junction in the whole interval of the Josephson coupling.

Our sample consists of a shunted superconducting Al-AlO_x-Al tunnel junction (area 150×150 nm^2). Its resistance $R_T = 3.4$–21 kΩ was determined by reducing the shunt resistance $R_s = 4$–75 kΩ off from the normal state resistance measured at 0.1 K. The shunted junction was connected to four measurement leads via 20 μm long thin film Cr resistors $R_L \approx 100$ kΩ. This ensures a well-defined resistive environment governed by the shunt (see Fig. 1). The value of R_s was deduced using the length of the shunt and the measured resistivity of the Cr sections in the leads. The circuits, both shunted and unshunted, were fabricated using electron beam lithography and triple-angle evaporation. The Cr resistors and shunt (10–15 nm thick, 100 nm wide) were evaporated at right angle of incidence. When exposing the chrome metal sections in *e*-beam writing, an accurately tuned electron dose ensured that the Al replicas were evaporated on the side of the resist and thus removed during lift-off. Within

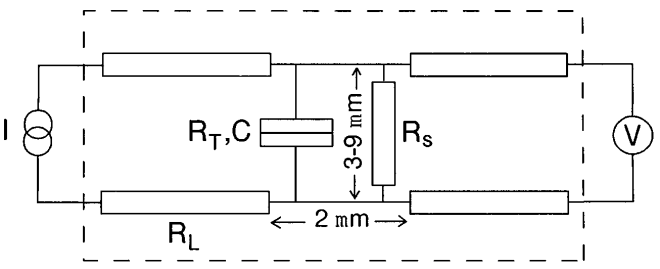

FIG. 1. Schematic diagram of shunted tunnel junction in a resistive environment.

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5%, no change was observed in R_s when B and T were swept over 0 . . . 2 T and 0.1 . . . 4 K, respectively. On the dilution refrigerator, the samples were mounted inside a tight copper enclosure, and the measurement leads were filtered using 0.5 m of Thermocoax cable.

Two types of observed *RI* curves are shown in Fig. 2. In the "superconductor" type (Fig. 2a), the resistance has its minimum at zero bias and increases monotonically up to subgap resistance (in parallel with R_s) given by the maxima in the figures. In the "blockade" type (Fig. 2b), a higher resistance "bump" appears at small currents, i.e., the resistance is maximum at zero bias. The width of this feature becomes more pronounced with decreasing Josephson coupling [5]. In both cases, the resistance returns smoothly to its normal state value after the subgap maximum.

In order to determine the phase diagram, we plot in Fig. 3 the character of our samples ("superconductor"/"insulator") on the coordinate plane $(R_q/R, E_J/E_C)$ which are the intrinsic parameters of DPT [2]. The DPT boundary is to separate open and solid symbols in Fig. 3. Here, $R_q = \frac{h}{(2e)^2} = 6.5\ \mathrm{k}\Omega$ is the quantum resistance, and the resistance $R = R_s R_{\mathrm{qp}}/(R_s + R_{\mathrm{qp}})$ characterizes the total Ohmic dissipation, R_{qp} being the quasiparticle resistance. The Josephson energy E_J was calculated from R_T using the Ambegaokar-Baratoff relation while the Coulomb energy $E_C = e^2/2C$ was estimated from normal state *VI* curves: the junction capacitance was obtained from the offset at large bias voltages using the formula $V_{\mathrm{offset}} = (e/2C)[R_s/(R_s + R_T)]$ that comes from a simple balance of currents through the junction and the shunt. As seen from the summary of junction parameters in Table I, the ratio E_J/E_C falls between 0.85 and 14.1.

Zero-bias resistance R_0 is displayed as a function of temperature in Fig. 4. Near T_c, all of the samples have a pronounced peak which has also been observed by Shimazu *et al.* [6]. At lower temperatures, the resistance first decreases and then starts to increase again (Fig. 4b). This reentrant behavior is not observed in samples with large E_J/E_C which stay superconducting all the way down to lowest temperatures (Fig. 4a).

Let us discuss the phase diagram for DPT expected from theory. In the classical limit at zero temperature, the Josephson phase φ is trapped in some well of the "tilted washboard" potential $U(\varphi) = -E_J \cos\varphi - (\hbar/2e)I\varphi$. This localized-phase state corresponds to a superconducting state. In fact, this localization is never perfect: (i) at finite temperatures, the phase can hop from one well to another via thermal activation; (ii) at very low temperatures, the phase is able to escape from a well via macroscopic quantum tunneling which is an exponential function of the barrier height $\propto E_J$ [7]. When we say that the junction is a superconductor, we mean that (i) its resistance is essentially smaller than the normal junction resistance, and (ii) its resistance increases with temperature such as in a metal because enhanced thermal fluctuations produce an increased phase slip rate $d\varphi/dt$.

Because of quantum-mechanical tunneling, the bound states in different wells form an energy band such as in a solid [8]. The band energy is a periodic function of the quasicharge Q (an analog of quasimomentum in a solid) with the period $2e$. If $E_J/E_C \gg 1$, that corresponds to the "tight binding" limit in the solid-state theory, then $E(Q) = E(Q + 2e) = \Delta[1 - \cos(\pi Q/e)]$. Here, the band half-width is given by [8]

$$\Delta = \frac{16}{\sqrt{8\pi}} \left(\frac{E_J}{2E_C}\right)^{1/4} \hbar\omega_p \exp\left[-\left(\frac{8E_J}{E_C}\right)^{1/2}\right], \quad (1)$$

where $\omega_p = \sqrt{8E_J E_C}/\hbar$ is the plasma frequency. For a small quasicharge $Q \ll 2e$ one may use the "effective-mass" approximation $E(Q) = Q^2/2C^*$, where the

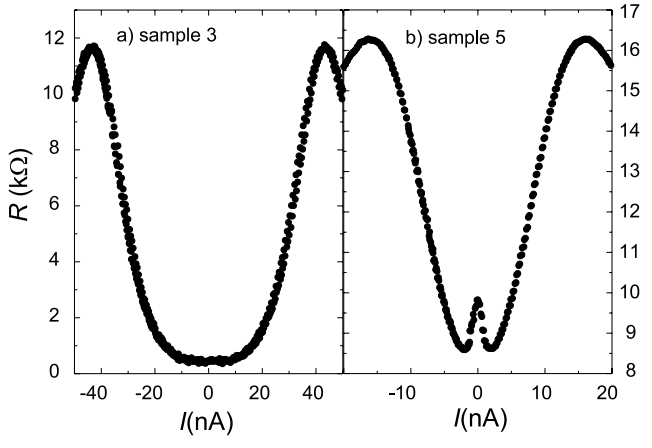

FIG. 2. Resistance vs current for two samples showing different behavior: (a) sample 3 with $R_T = 3.7\ \mathrm{k}\Omega$ and $R_s = 11\ \mathrm{k}\Omega$; (b) sample 5 with $R_T = 12.4\ \mathrm{k}\Omega$ and $R_s = 22\ \mathrm{k}\Omega$.

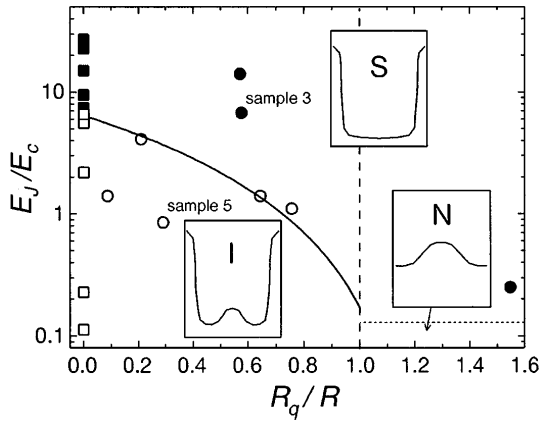

FIG. 3. Phase diagram of shunted Josephson junction. The phase boundary lies between insulatorlike (open symbols, I) and superconductorlike (solid symbols, S) samples. Unshunted samples (squares) are collected at $R_q/R = 0$. The solid line is the theoretical phase boundary calculated using Eq. (3) with $\omega_p = 2 \times 10^{11}\ 1/\mathrm{s}$ and $\tau_s = 2 \times 10^{-9}$ s. The dotted line is the transition line for strong dissipation to a state with no supercurrent (N) found by Yagi *et al.* [4]. For details, see text.

TABLE I. Measured shunted junctions. R_T is deduced from the slope of the normal state *IV* curve at high bias voltage. The effect of parallel shunt R_s is subtracted. C is calculated from the high bias offset voltage using $V_{\text{offset}} = (e/2C)[R_s/(R_s + R_T)]$. The value of R_s is estimated from the known wire resistivity.

Sample	R_T (kΩ)	C (fF)	R_s (kΩ)	E_J/E_c
1	9.7	1.8	75	1.4
2	4.5	2.5	31	4.0
3	3.7	3.4	11	6.8
4	3.4	6.6	11	14.1
5	12.4	1.5	22	0.85
6	8.1	1.7	10	1.4
7	5.9	2.2	8.6	1.1
8	21	0.8	4.2	0.25

effective capacitance (an analog of the effective mass) $C^* = e^2/\pi^2\Delta$ can exceed the geometric capacitance C essentially.

The band theory predicts Ohm's law $V = RI$ at small current bias $I \ll e/RC^*$. This corresponds to the quasicharge $Q = C^*V = IRC^*$. However, with increasing current bias the quasicharge approaches the Brillouin-zone boundary ($Q = \pm e$). Then another regime of phase motion sets in [8]: The phase performs Bloch oscillations, $\varphi = 2eE(Q)/\hbar I$, with $Q = It$ leading to the period $2e/I$. In this regime dissipation is suppressed, corresponding to a decreasing resistance V/I.

Thus, at small current bias the *RI* curve must have a bump of width e/RC^* (a voltage of e/C^*), and the Josephson junction *always* behaves as a normal junction with Ohmic resistance R. At larger currents $I \gg e/RC^*$, however, the junction has a tendency to become superconducting again. This behavior is a direct outcome of the band picture for the phase motion, as was shown in Ref. [8]. It was obtained also using more rigorous path-integral methods [23]. Therefore, a blockade bump in the *RI* curve of a Josephson junction is a clear manifestation of phase delocalization and the band picture.

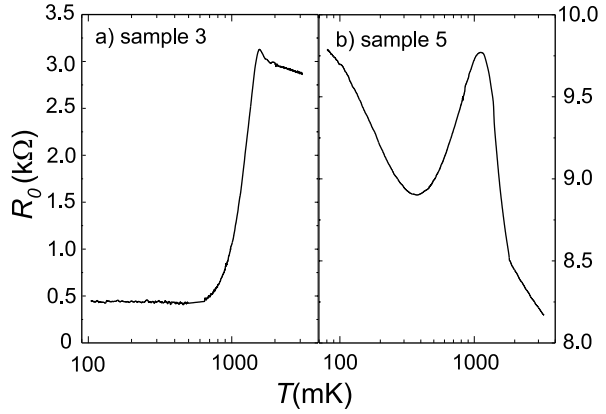

FIG. 4. Zero-bias resistance R_0 as a function of temperature for samples 3 and 5.

The bump on the *RI* curve at small bias looks similar to the bump due to the Coulomb blockade of single-electron tunneling and, moreover, is governed by the same effective Coulomb energy e^2/C^*. On the other hand, in the model which we are discussing here, there is no single-electron tunneling at all if the resistance R is dominated by the shunt resistance R_s (quasiparticle resistance $R_{\text{qp}} \gg R_s$). In fact, we deal with the Coulomb blockade indeed, but it is the Cooper-pair current channel that is blocked [5]. However, in an unshunted junction with $R = R_{\text{qp}}$ the additional Coulomb blockade of single-electron tunneling can increase the zero-bias resistance well above R.

The theory as summarized above would indicate that any Josephson junction must have a blockade bump at zero bias. However, we must take into account an important effect of the environment: suppression of the quantum tunneling between wells by dissipation [1–3]. This decreases the band half-width which now is given by

$$\tilde{\Delta} = \Delta\left(\frac{\Delta}{\hbar\omega_p}\right)^{\alpha/(1-\alpha)}. \qquad (2)$$

Here, $\alpha = R_q/R$ is the dissipation parameter. The renormalized energy $\tilde{\Delta}$ vanishes at $\alpha = 1$ where the band disappears and quantum tunneling becomes impossible [9]. Then, the junction is superconducting down to the lowest current bias. Consequently, the phase line separating the insulator from the superconductor is the $\alpha = 1$ line independently of the energy ratio E_J/E_C (the dashed vertical line on the phase diagram, Fig. 3) [10].

This phase diagram, in which the Josephson junction under weak dissipation remains an insulator even in the limit of $E_J/E_C \to \infty$, is difficult to confirm because the putative, very slow delocalization of phase leads to exceedingly small voltages. Experimentally, the insulator behavior can be observed only if the voltage of the bump, the effective Coulomb gap $e/C^* \sim \tilde{\Delta}/e$, exceeds the minimum voltage V_{min} detectable in our measurements. Therefore, it is reasonable to assume that our measured DPT corresponds not to the condition $\tilde{\Delta} = 0$, but to $\tilde{\Delta} \sim eV_{\text{min}}$. Together with Eqs. (1) and (2) [neglecting an unimportant factor of $(E_J/2E_C)^{1/4}$ in Eq. (1)], the latter condition yields the crossover from the superconductor to the insulator behavior at

$$\frac{E_J}{E_C} = \frac{1}{8}\left(\ln\frac{16}{\sqrt{8\pi}} + (1-\alpha)\ln\omega_p\tau_s\right)^2. \qquad (3)$$

Here, $\tau_s = \hbar/eV_{\text{min}}$ is the phase slip time for the minimum detectable voltage V_{min}. This is the time necessary for a phase change by 2π, i.e., for the phase motion between two wells. In our case, V_{min} is about 0.5 μV which corresponds to $\tau_s \approx 2 \times 10^{-9}$ s. The curve obtained from Eq. (3) using the plasma frequency 2×10^{11} Hz is displayed in Fig. 3. Within our quite large statistical uncertainty, Eq. (3) agrees with the experimental crossover

between superconductor and blockade types of *RI* curves. If we apply the argument by Schön and Zaikin [2] that an insulator state is observable when the phase spreading time $\hbar/\tilde{\Delta}$ is smaller than the observation time τ in our experiment, then the crossover [replacing τ_s by $\tau \sim 1$ s in Eq. (3)] would take place at $E_J/E_C \approx 100$ in contrast to $E_J/E_C \sim 10$ observed in the experiment. Thus, the ability to reveal the blockade bump (insulator behavior) is restricted not by the observation time, but by the accuracy of the voltage measurement.

According to Ref. [8], thermal fluctuations are also able to "wash out" the blockade bump if thermal energy kT is on the order of or larger than e^2/C^*. In this case, the crossover is given by Eq. (3) again, but with τ_s replaced by $\hbar/kT$ which is about 5 times less than τ_s at our minimum temperature of 50 mK. Since the crossover depends logarithmically on τ_s, small uncertainties in τ_s do not shift its position essentially when compared with our experimental uncertainty. In fact, since the numerical factors in the conditions $kT \sim e^2/C^*$ and $V_{\min} \sim e/C^*$ are not known, it is difficult to judge which one of these restrictions is stronger.

Finally, we want to compare the concepts of *the superconductor-insulator transition* and *the dissipative phase transition*. The common formulation is that superconductor and insulator are specified by the positive and negative sign of dR_0/dT, respectively. Accordingly, one may identify the peak in $R_0(T)$ (see Fig. 4) also as SIT. But the SIT near T_c has nothing to do with DPT predicted theoretically [1–3]: the peak in Fig. 4 corresponds to the temperature at which the normal junction (with the *RI* curve noted by N in Fig. 3) becomes superconducting, i.e., a Josephson junction with a *detectable* critical current (*RI* curve noted by S in Fig. 3). The DPT theory assumes that the critical current is initially finite, but in reality it may be essentially smaller than $eE_J/\hbar$ because fluctuations are especially important at small E_J/E_C. We believe that this discussion is relevant for understanding the data of Yagi *et al.* [4], who observed the SIT for E_J/E_C between 0.1 and 0.2 for strong dissipation $\alpha > 1$ (horizontal dotted line in Fig. 3). These results were considered to be contradictory to the DPT theory which does not predict any transition to the insulating phase at $\alpha > 1$. In fact, there is no disagreement: Yagi *et al.* did not observe DPT where the energy bandwidth is to vanish, but SIT where the critical current disappears. So, our important conclusion is that the concepts of DPT and SIT are not completely identical: both are accompanied by the sign change of dR_0/dT and thus any DPT is SIT, but *not vice versa* [11]. In our case, the zero slope of dR_0/dT occurred at about $R_s = 10\ \mathrm{k}\Omega$, which roughly agrees with the DPT line obtained from the *RI* curves.

In summary, our experiments clearly confirm the existence of the dissipative phase transition in a single Josephson junction, though the observed phase diagram is quite different from that expected originally. The agreement with theory is achieved by taking into account that the position of the measured phase boundary is governed not only by intrinsic junction parameters but also by the accuracy of voltage measurements. Our work is a strong demonstration of quantum effects in a single Josephson junction, especially of the Josephson phase delocalization and the band picture of phase motion.

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[1] A. J. Leggett, S. Chakravarty, A. T. Dorsey, M. P. A. Fisher, A. Garg, and W. Zwerger, Rev. Mod. Phys. **59**, 1 (1987).

[2] G. Schön and A. D. Zaikin, Phys. Rep. **198**, 237 (1990).

[3] U. Weiss, *Quantum Dissipative Systems* (World Scientific, Singapore, 1993).

[4] R. Yagi, S. Kobayashi, and Y. Ootuka, J. Phys. Soc. Jpn. **66**, 3722 (1997).

[5] D. B. Haviland, L. S. Kuzmin, P. Delsing, and T. Claeson, Europhys. Lett. **16**, 103 (1991).

[6] Y. Shimazu, Y. Yamagata, S. Ikehata, and S. Kobayashi, J. Phys. Soc. Jpn. **65**, 89 (1996).

[7] M. Tinkham, in *Single Charge Tunneling*, edited by H. Grabert and M. H. Devoret (Plenum Press, New York, 1992), Sect. 3.4, p. 139.

[8] K. K. Likharev and A. B. Zorin, J. Low Temp. Phys. **59**, 347 (1985).

[9] Because of dissipation, the energy is not conserved during a quantum mechanical transition. The energy loss may be estimated as $\Delta E_d \sim (\hbar/2e)^2(1/R\tau_{tr})$, where $\tau_{tr} \sim (\hbar/2e)\sqrt{C/E_J} \sim 1/\omega_p$ denotes the time of tunneling between two potential wells (the traversal time). It is interesting to note that complete suppression of macroscopic quantum tunneling at $\alpha > 1$ corresponds to the condition that the dissipated energy ΔE_d exceeds the ground-state energy $E_0 \sim \hbar\omega_p$ in the well.

[10] In fact, the phase transition line depends on E_J/E_C at $E_J/E_C \leq 1$ if the Ohmic resistance originates from quasiparticles ($R \sim R_{\mathrm{qp}}$) [2]. However, all of our experimental data for small E_J/E_C were obtained for shunted junctions ($R \sim R_s$), so we ignore this deviation from the vertical transition line.

[11] In the past, the difference between DPT and SIT was ignored, the latter term being used more frequently. In order to emphasize the link with previous studies, the term SIT was used with quotation marks in the title of our Letter although this paper deals mostly with DPT.