PHYSICAL REVIEW LETTERS

VOLUME 81 | 3 AUGUST 1998 | NUMBER 5

Collapse and Bose-Einstein Condensation in a Trapped Bose Gas with Negative Scattering Length

Yu. Kagan,[1] A. E. Muryshev,[1] and G. V. Shlyapnikov[1,2]
[1]Russian Research Center Kurchatov Institute, Kurchatov Square, 123182 Moscow, Russia
[2]FOM Institute for Atomic and Molecular Physics, Kruislaan 407, 1098 SJ Amsterdam, The Netherlands
(Received 31 December 1997)

Evolution and collapse of a trapped Bose condensate with negative scattering length are predetermined by 3-body recombination of Bose-condensed atoms and by feeding of the condensate from the nonequilibrium thermal cloud. The collapse, starting once the number of condensate atoms reaches the critical value, ceases and turns to expansion when the density becomes so high that the recombination losses dominate over attractive interparticle interaction. As a result, we obtain a sequence of collapses, each of them followed by dynamic oscillations of the condensate. In every collapse the 3-body recombination burns only a part of the Bose-condensed atoms. [S0031-9007(98)06775-1]

PACS numbers: 03.75.Fi

After the discovery of Bose-Einstein condensation (BEC) in trapped clouds of alkali atoms [1–3], one of the central questions in the field of Bose-condensed gases concerns the influence of interparticle interaction on the character of BEC. In this respect the Rice experiments with ^{7}Li [2,4] attract a special interest, since a weakly interacting gas ($n|a|^3 \ll 1$, where n is the gas density, and a the scattering length) of ^{7}Li is characterized by attractive interaction between atoms ($a < 0$). As known [5], in spatially homogeneous Bose condensates with $a < 0$ the negative sign and nonlinear density dependence of the energy of interparticle interaction predetermine an absolute instability of the homogeneous density distribution, associated with the appearance of local collapses. A strong rise of density in the course of the collapse enhances intrinsic inelastic processes and leads to decay of the condensate. In a trapped gas the picture drastically changes. As has been revealed in [6,7], the discrete structure of the trap levels provides the existence of a metastable Bose condensate with $a < 0$, if the level spacing $\hbar\omega$ exceeds the interparticle interaction $n_0|\tilde{U}|$ ($\tilde{U} = 4\pi\hbar^2 a/m$, m is the atom mass, n_0 the condensate density, and ω the trap frequency). In terms of the number of Bose-condensed atoms N_0 this condition can be written as

$$N_0 < N_{0c} \sim l_0/|a|, \qquad (1)$$

where $l_0 = (\hbar/m\omega)^{1/2}$ is the amplitude of zero point oscillations in the trap.

The existence of a condensate in a trapped gas with $a < 0$ has a clear physical nature. In an ideal trapped gas the BEC occurs in the ground state of the trapping potential. For attractive interparticle interaction the transfer of a condensate particle to excited states decreases the interaction energy by $\sim n_0|\tilde{U}|$. But the energy of interaction with the trapping field increases by $\sim\hbar\omega$, and under condition (1) the change of the total energy is positive. In other words, there is a gap between the condensate and the lowest excited states. With increasing N_0 to the critical value N_{0c}, the gap disappears, and there will be an instability corresponding to the appearance of excitations with zero energy. As found in [6], for $N_0 < N_{0c}$ the nonlinear Schrödinger equation for the condensate wave function Ψ_0 has a stationary solution which becomes unstable at $N_0 \geq N_{0c}$.

However, the analysis performed in [7] shows that the picture is more complicated. Actually, for $N_0 < N_{0c}$ there are two global states with the same N_0 and total energy E. In the first of them Ψ_0 is almost a Gaussian and is localized in a spatial region of the size $\sim l_0$. The other one is a nonstationary collapsing state localized in a much smaller spatial region. The two states are separated by a large energy barrier, and the transition amplitude is exponentially small, with the exponent depending on $(N_{0c} - N_0)$. From statistical considerations it is clear that in the course of accumulation of particles in the lowest trap level they turn out to be in the Gaussian state. There are peculiar fluctuations in this state, leading to the formation of "small dense clusters" of atoms. But the formation probability is again exponentially small (see [7]). Thus, for $N_0 < N_{0c}$ the condensate will be formed in this metastable state which, however, does not decay on a time scale characteristic for the experiment.

The problem of a metastable Bose condensate in a trapped gas with $a < 0$ was also discussed in [8–11]. The appearance of a Bose condensate with the number of particles $N_0 < N_{0c}$ was found in the Rice experiments with trapped ^{7}Li [4], where $N_{0c} \sim 1000$.

The present paper is aimed at the analysis of the formation and evolution of a trapped condensate with $a < 0$ in the presence of the Knudsen above-condensate cloud. Assuming the conditions which in the absence of collapse would provide the number of condensate particles $N_0 \gg N_{0c}$, we show that the key features of the condensate evolution are predetermined by the particle flux from the above-condensate cloud to the condensate and by 3-body recombination of Bose-condensed atoms. Once the number of condensate particles reaches the critical value N_{0c}, the Bose-condensed cloud undergoes a collapse. However, we find that the compression reaches its maximum and turns to expansion when the density of the collapsing condensate becomes so high that the recombination losses dominate over attractive interparticle interaction. The recombination losses "burn" the condensate to $N_0 < N_{0c}$, but the flux from the above-condensate cloud again increases N_0 and a new collapse occurs, etc. As a result, we obtain a sequence of collapses, each of them followed by dynamic oscillations of the condensate. It is important that the recombination in the course of the collapse does not burn the condensate completely, and N_0 always remains finite.

We consider a Bose gas with $a < 0$ and total number of particles $N \gg N_{0c}$ in an isotropic harmonic potential $V(r) = m\omega^2 r^2/2$. The BEC transition temperature is determined by the relation $T_c = 1.05\hbar\omega N^{1/3}$ [12] and greatly exceeds the interparticle interaction $n|\tilde{U}|$. Therefore, at temperatures $T \gg n|\tilde{U}|$ $(T < T_c)$ the equilibrium number of condensate particles can be found in the ideal gas approach: $\bar{N}_0 = N[1 - (T/T_c)^3]$. The equilibrium BEC requires $\bar{N}_0$ smaller than N_{0c}, which immediately leads to the inequality $\Delta T = T_c - T \ll T_c$.

For $\bar{N}_0 > N_{0c}$ the equilibrium BEC is impossible. Below we show that in this case there will be a strongly nonequilibrium evolving Bose-condensed phase. We discuss two limiting cases. The first of them assumes that the conditions, which in the absence of interparticle interaction would lead to the equilibrium BEC with $\bar{N}_0 \gg N_{0c}$, are created abruptly. In this case, once the condensate is already present in the system, the flux of particles from the nonequilibrium above-condensate cloud to the condensate is induced by the condensate interaction with the above-condensate atoms and is given by

$$dN_0/dt = \gamma' N_0; \qquad \gamma' \approx \gamma_0[1 - N_*/N(t)], \tag{2}$$

where N_* is the total number of particles corresponding to $\bar{N}_0 = N_{0c}$. The parameter γ_0 is of order the frequency of elastic collisions and, hence, much smaller than ω. The number of Bose-condensed atoms and, hence, the recombination losses per each collapse cannot significantly exceed N_{0c}. Therefore, if initially $N(t=0) - N_* \gg N_{0c}$, the condensate evolution predominantly proceeds with practically constant γ', except for the final stage, where $N(t) - N_*$ is already comparable with N_{0c}.

We will assume that the spherical symmetry of the Bose-condensed cloud, characteristic for $N_0 < N_{0c}$, is retained when N_0 reaches N_{0c} and the cloud collapses. A strong rise of density in the collapsing condensate enhances intrinsic inelastic processes. The most important will be the recombination in 3-body interatomic collisions. This process occurs at interparticle distances of order the characteristic radius of interaction between atoms. The kinetic energy of produced (fast) atoms and vibrationally excited molecules is determined by the binding energy of the molecule and greatly exceeds all other characteristic energies in the gas. Therefore, the recombination has a local character, and the recombinational decrease of the local value of the condensate density is described by the relation $\dot{n}_0(\mathbf{r},t) = -\alpha_r n_0^3(\mathbf{r},t)$, where α_r is the recombination rate constant. It is important that the recombination simply leads to the loss of atoms and does not break the coherence between the particles remaining in the condensate. Hence, the recombinational decrease of the density is equivalent to the extra term $-(\alpha_r/2)|\Psi_0|^4\Psi_0$ in the time derivative of the condensate wave function, and we can account for the 3-body recombination by explicitly including this term in the time-dependent nonlinear Schrödinger equation for Ψ_0. The same arguments apply to the feeding of the condensate from the above-condensate cloud. Then, in the dimensionless form the equation for Ψ_0 reads

$$i\frac{\partial\tilde{\Psi}_0}{\partial\tau} = -\Delta_\rho\tilde{\Psi}_0 + \rho^2\tilde{\Psi}_0 - |\tilde{\Psi}_0|^2\tilde{\Psi}_0 - i\xi|\tilde{\Psi}_0|^4\tilde{\Psi}_0 + i\gamma\tilde{\Psi}_0. \tag{3}$$

Here $\rho = r/l_0$, $\tau = \omega t/2$ are the dimensionless coordinate and time variables, and $\tilde{\Psi}_0 = \Psi_0/\tilde{n}^{1/2}$, where the characteristic density $\tilde{n} = (8\pi l_0^2|a|)^{-1} \approx N_{0c}/8\pi l_0^3$. The recombination losses in the condensate and its feeding by the particle flux from the above-condensate cloud are described by the last two terms in Eq. (3), with $\xi = \alpha_r\tilde{n}^2/\omega$ and $\gamma = \gamma'/\omega$. For any realistic parameters we have $\xi \ll 1$. The parameter γ is also small: As mentioned above, for the Knudsen thermal cloud one has $\gamma' \ll \omega$.

For $N_0 > N_{0c}$ Eq. (3) does not have stationary or quasistationary solutions even at $\xi = \gamma = 0$. Once the number of particles in the condensate exceeds the critical value N_{0c}, the Bose-condensed cloud starts to collapse. First it undergoes a purely dynamic compression determined by the nonlinear interaction term $-|\tilde{\Psi}_0|^2\tilde{\Psi}_0$. The compression is accelerating with increasing $\tilde{\Psi}_0$, the compression time scale being $\tau \sim 1/|\tilde{\Psi}_0^2|$. The total compression time is determined by a slow initial stage and turns out to be $t \sim \omega^{-1}$ $(\tau \sim 1)$. From Eq. (3) one can see that the compression is constrained by the recombination losses and ceases when the condensate density reaches $n_0 \sim n_* = \tilde{U}/\hbar\alpha_r$, i.e.,

$$|\tilde{\Psi}_0|^2 \sim |\tilde{\Psi}_{0*}|^2 \approx \xi^{-1} \gg 1\,. \tag{4}$$

The 3-body recombination accompanied by the particle losses predominantly occurs at maximum densities $n \sim n_*$. When the number of condensate particles becomes smaller than N_{0c}, the collapse turns to expansion and the 3-body recombination strongly decreases. The characteristic time interval, where the recombination losses are important, is $t_* \sim (\alpha_r n_*^2)^{-1}$ $(\tau_* \sim |\tilde{\Psi}_{0*}|^{-2} \sim \xi)$. The total particle losses in the collapse are $\Delta N_0 \sim N_{0c}\alpha_r n_*^2 t_*$. We see that ΔN is independent of ξ (and γ), or at least weakly depends on its value. This is confirmed by numerical calculations in a wide range of ξ. They show that the fraction of lost Bose-condensed atoms is approximately one-half, although the internal structure of the collapse depends on the value of ξ.

The recombination-induced turn of the collapse to expansion causes dynamic oscillations of the condensate: Because of the presence of the confining potential the expansion is followed by compression. These oscillations, with the period depending on ω, resemble the condensate oscillations under variations of the trapping field (see [13]). We will perform the analysis, relying on Eq. (3) and, hence, omitting the influence of the above-condensate cloud on the condensate oscillations.

For revealing a qualitative picture we present the results of numerical calculation of Eq. (3) with $\xi = 10^{-3}$, $\gamma = 10^{-1}$. Figure 1 shows the time dependence of the number of Bose-condensed atoms, $N_0(t)$. The time $t = 0$ is chosen such that $N_0(0) = 0.75N_{0c}$ and the Bose-condensed cloud is still stable with respect to collapse. The feeding of the condensate from the above-condensate cloud increases N_0 and, once N_0 becomes higher than N_{0c}, the collapse occurs. The 3-body recombination in the course of the collapse burns approximately half of the Bose-condensed atoms. Then, on a time scale $\sim\gamma^{-1}$ the particle flux from the above-condensate cloud increases the number of condensate atoms to $N_0 > N_{0c}$, and a new collapse occurs. It is accompanied by approximately the same particle losses as those in the previous collapse.

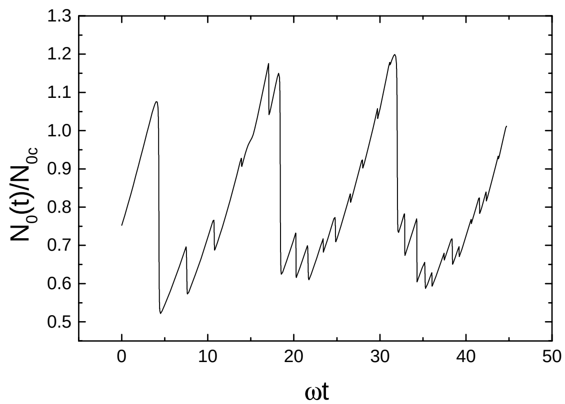

FIG. 1. The ratio $N_0(t)/N_{0c}$ versus ωt for $\xi = 10^{-3}$ and $\gamma = 10^{-1}$. The time $t = 0$ is selected such that $N_0(0) = 0.75N_{0c}$.

The described oscillatory evolution of the condensate continues at larger times. The fine structure of the curve $N_0(t)$, demonstrating moderate particle losses in the time intervals between the collapses, originates from the compression in the course of oscillations of the condensate.

The existence of a sequence of collapses, due to feeding of the condensate from the above-condensate cloud, was addressed in [14] and has been discussed after submission of the present paper in [15]. However, the quasistationary approximation for the 3-body recombination, used in [15], does not adequately describe the influence of this process on the dynamics of collapse and, in particular, leads to the loss of all atoms in the collapsing condensate.

The structure of the condensate oscillations is clearly seen in Fig. 2, where we present the spatial distribution of the condensate density, $n_0(r,t)r^2$, at various times t. For $t = t_1$, where the compression did not yet reach its maximum, the density n_0 at small r strongly increases compared to the initial distribution. But already after a short time $(t_2 - t_1) \ll \omega^{-1}$ the Bose-condensed cloud passes

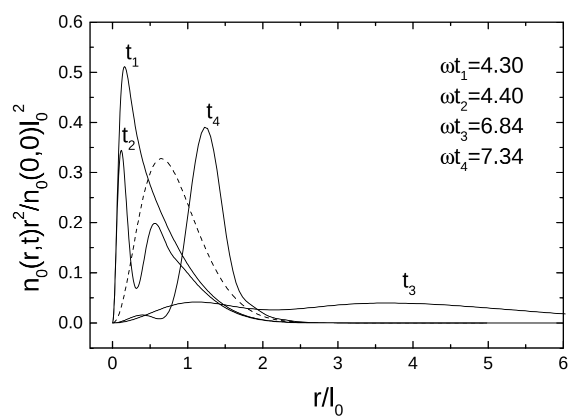

FIG. 2. The condensate density profile for various times t. The dashed curve corresponds to $t = 0$ $[N_0(0) = 0.75N_{0c}]$.

through the point of maximum compression, and both the density and the number of condensate particles decrease due to recombination losses. Then the condensate starts to expand. A strong expansion of the condensate occurs at times of order ω^{-1} ($t = t_3$). The expansion is followed by compression, with a comparatively large increase of the density ($t = t_4$).

As already mentioned, the assumption of constant γ relies on the inequality $N(t) - N_* \gg N_{0c}$. When the latter violates because of the recombination losses, the parameter γ decreases with $N(t)$. In order to demonstrate the final stage of the evolution we present in Fig. 3 the dependence $N_0(t)$ calculated self-consistently for the time-dependent γ, with γ' from Eq. (2) and $N(t = 0) - N_*$ equal to $2.5N_{0c}$ and to $2N_{0c}$. One can see that after two collapses the system approaches the equilibrium state, with N_0 smaller than N_{0c} and depending on the value of $N(t = 0) - N_*$. Again, $N_0(t)$ always remains finite.

One of the remarkable features of the collapse is the rise of dynamic energy of the condensate, induced by the recombination losses in the collapsing cloud. For $N_0 < N_{0c}$ the kinetic (K) and potential ($P < 0$) energy of the condensate are of the same order of magnitude, and the total energy $E = (K + P) \sim N_0\hbar\omega$. In the course of the dynamic compression both K and $|P|$ strongly increase, whereas E is conserved. As a result, for a strong compression we have $K, |P| \gg E$ and, hence, $K \approx |P|$. Since $K \propto N_0$, and $|P| \propto N_0^2$, the loss of $\delta N_0 \ll N_0$ particles changes the total energy by an amount

$$\delta E = \frac{\delta N_0}{N_0}(2|P| - K) > 0\,. \qquad (5)$$

In the course of particle losses the relation between K and $|P|$ changes, which can reverse the sign of δE.

The kinetic energy of fast atoms and vibrationally excited molecules, produced in the course of recombination, greatly exceeds the energy $\delta E/\delta N_0$ acquired by the condensate in the recombination event. Therefore, due to the recombination-induced increase of the condensate energy, the fast atoms and molecules simply carry away from the system slightly less energy than in the case of recombination in vacuum. Equation (3) and the above analysis implicitly assume that the mean free path of the fast atoms and molecules is much larger than the sample size, and they escape from the trap without collisions with the gas atoms. The energy transferred to the system is concentrated in macroscopic oscillations of the condensate. In fact, this can be seen already in Fig. 2.

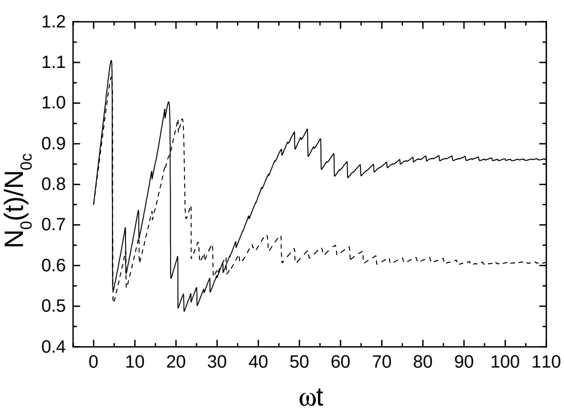

FIG. 3. The ratio $N_0(t)/N_{0c}$ versus ωt for the time-dependent γ [$\xi = 10^{-3}$, $\gamma(0) = 10^{-1}$, $N_0(0) = 0.75N_{0c}$]. The solid curve corresponds to $N(0) - N_* = 2.5N_{0c}$, and the dashed curve to $N(0) - N_* = 2N_{0c}$.

It is worth noting that the recombination-induced increase of the condensate energy can lead to the appearance of short-wave excitations which overcome the trap barrier and carry away a significant part of the condensate dynamic energy. Together with damping of the oscillations of the condensate, caused by its interaction with the thermal cloud, this problem is especially important for smaller values of ξ and requires a separate analysis.

Let us now briefly discuss another limiting case, where the gas temperature is decreasing adiabatically slowly and for $N_0(t) < N_{0c}$ the system is in quasiequilibrium. With decreasing T, the number of Bose-condensed atoms rises and, when it reaches N_{0c}, the collapse occurs. Similarly to the previous case, N_0 drops. Since the total number of particles becomes smaller, the quasiequilibrium is reestablished at lower T_c. As the temperature continues to decrease, N_0 increases and the collapse occurs again, etc. This continues until $N(t) > N_{0c}$. It is important that for $N(t) \gg N_{0c}$ the instantaneous values of T and T_c always remain very close to each other.

Finally, we make a general remark. The collapse as a solution of the nonlinear Schrödinger equation was a subject of extensive analytical and numerical studies. The attention was focused on analyzing the character of the singularity and on finding universal scaling solutions in the absence of dissipation or in the presence of weak dissipative processes (see [16] and references therein). Of particular interest was the search for the so-called strong collapse, which arrived at the concept of "burning point" (small spatial region absorbing particles).

The picture of collapse, described in the present paper, stands beyond this analysis. To an essential extent this is related to the presence of the trapping potential which determines dynamical properties of the system and provides the existence of a peculiar quasistable condensate with a limited number of particles. Another reason is that the collapse occurs in nonequilibrium conditions, and there is a particle exchange between the condensate and the above-condensed cloud. In other words, the condensate is an open system, which predetermines the appearance of a sequence of collapses. In this respect, BEC in ultracold trapped gases with $a < 0$ opens possibilities for observing and studying novel pictures of collapse.

We acknowledge discussions with R. G. Hulet. This work was supported by the Dutch Foundation FOM, by

NWO (Project No. NWO-047-003.036), by INTAS, and by the Russian Foundation for Basic Studies.

[1] M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Science **269**, 198 (1995).
[2] C. C. Bradley, C. A. Sackett, J. J. Tollett, and R. G. Hulet, Phys. Rev. Lett. **75**, 1687 (1995).
[3] K. B. Davis, M.-O. Mewes, M. R. Andrews, N. J. van Druten, D. S. Durfee, D. M. Kurn, and W. Ketterle, Phys. Rev. Lett. **75**, 3969 (1995).
[4] C. C. Bradley, C. A. Sackett, and R. G. Hulet, Phys. Rev. Lett. **78**, 985 (1997).
[5] E. M. Lifshitz and L. P. Pitaevskii, *Statistical Physics, Part 2* (Pergamon Press, Oxford, 1980).
[6] P. A. Ruprecht, M. J. Holland, K. Burnett, and M. Edwards, Phys. Rev. A **51**, 4704 (1995).
[7] Yu. Kagan, G. V. Shlyapnikov, and J. T. M. Walraven, Phys. Rev. Lett. **76**, 2670 (1996).
[8] R. J. Dodd, M. Edwards, C. J. Williams, C. W. Clark, M. J. Holland, P. A. Ruprecht, and K. Burnett, Phys. Rev. A **54**, 661 (1996).
[9] E. V. Shuryak, Phys. Rev. A **54**, 3151 (1996).
[10] M. Houbiers and H. T. C. Stoof, Phys. Rev. A **54**, 5055 (1996).
[11] T. Bergeman, Phys. Rev. A **55**, 3658 (1997).
[12] V. Bagnato, D. E. Pritchard, and D. Kleppner, Phys. Rev. A **35**, 4354 (1987).
[13] Yu. Kagan, E. L. Surkov, and G. V. Shlyapnikov, Phys. Rev. A **54**, R1753 (1996).
[14] C. A. Sackett, C. C. Bradley, M. Welling, and R. G. Hulet, Appl. Phys. B **65**, 433 (1997).
[15] C. A. Sackett, H. T. C. Stoof, and R. G. Hulet, Phys. Rev. Lett. **80**, 2031 (1998).
[16] M. E. Kosmatov, V. E. Shvets, and V. E. Zakharov, Physica (Amsterdam) **52D**, 16 (1991).