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Probing a Singular Potential with Cold Atoms: A Neutral Atom and a Charged Wire

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By charging a thin wire we realize a pure *attractive* $1/r^2$ potential of variable strength for cold neutral atoms. Scattering experiments are performed where the charged wire is placed inside a confined gas of cold lithium atoms. We observe the "falling" of atoms into the $1/r^2$ singularity which so far has been only theoretically discussed in textbooks. [S0031-9007(98)06725-8]

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The fast development of atom cooling techniques [1] in recent years has brought about a variety of schemes for trapping and storing cold atoms such as magnetic traps, dipole traps, or optical lattices. Atomic motion in these *trapping* potentials was studied extensively. In this Letter we describe how to use cold atoms to study *singular* potentials, in particular the cylindrical $1/r^2$ potential. Among all singular potentials the attractive $1/r^2$ potential is special [2], because it lies on the border between $1/r^n$ potentials where the radial motion can be stabilized by angular momentum ($n < 2$) and those where this stabilization is not possible ($n > 2$). For a $1/r^2$ potential no stable orbits exist; instead there is a characteristic class of trajectories for which atoms *fall* into the singularity (Fig. 1) [2].

We have realized a $1/r^2$ potential of variable strength in a scheme where a cold, neutral atom interacts with a straight, thin, charged wire (see Fig. 1) [3,4]. In our experiments we monitor the number of cold lithium atoms in a magnetic-optic trap (MOT) [5] centered on the $1/r^2$ potential of the wire. At extremely low light levels the MOT effectively acts as a box holding a gas of atoms. Atoms that "fall" into singularity are lost from the trap as they hit the wire surface where they are adsorbed or thermally scattered. The corresponding loss rate in the number of atoms is then characteristic for the singularity and its strength (Figs. 2, 3).

In the following, we will first describe the interaction between a neutral atom and a charged wire and then present our experiments and results in more detail.

The electrical field of a wire with line charge q induces a dipole moment $d_i = \alpha E$ in a neutral atom of polarizability α which is then attracted towards the wire [3]. The interaction potential (cylindrical coordinates),

$$V_{\mathrm{pol}}(r) = -\frac{1}{2}\, d_i E = -\frac{1}{2}\, \alpha E^2(r) = -\frac{2\alpha\, q^2}{r^2}, \quad (1)$$

is always *attractive* and has exactly the same radial form ($1/r^2$) as the centripetal potential barrier ($V_L = L_z^2/2Mr^2$) created by angular momentum L_z which is always *repulsive*. The attractive wire potential and the repulsive centripetal potential can be combined in the total

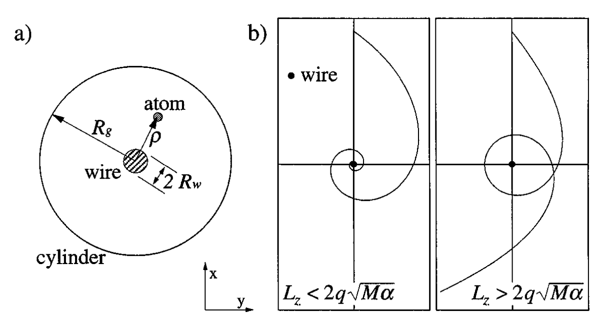

FIG. 1. An atom and a charged wire. (a) Schematic of our geometry. The charged wire is surrounded by a concentric grounded cylinder (in our experiment the walls of the vacuum chamber) in order to define the electrical boundary conditions. (b) Classical trajectories for $L_z < 2q\sqrt{M\alpha}$ (falling in the singularity) and $L_z > 2q\sqrt{M\alpha}$ (scattering).

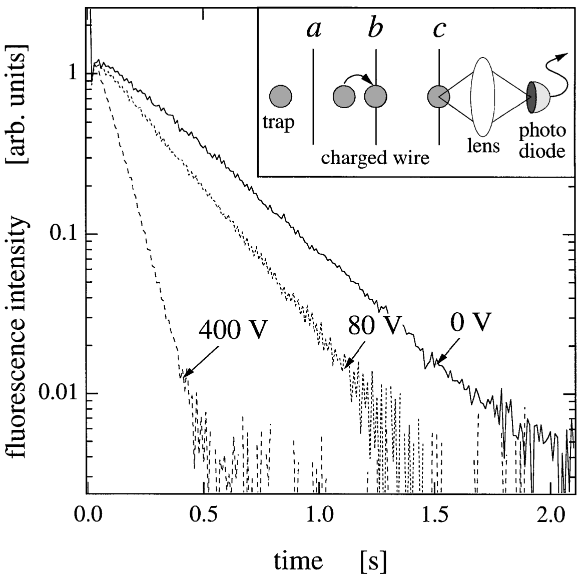

FIG. 2. When moved onto the wire the atom trap decays exponentially, as can be seen by monitoring the atomic fluorescence signal. Charging the wire (100 V $\approx$ 6.4 pC/cm) creates an attractive $1/r^2$ potential and enhances the decay rate. Insert: Steps *a*, *b* and *c* are discussed in the text.

Hamiltonian for the radial motion,

$$H = \frac{p_r^2}{2M} + \frac{L_z^2}{2Mr^2} - \frac{2\alpha q^2}{r^2} = \frac{p_r^2}{2M} + \frac{L_{\rm eff}^2}{2Mr^2}, \quad (2)$$

and together form an effective (attractive or repulsive) potential. Equation (2) has the same form as the free Hamiltonian except that the angular momentum L_z is replaced by an effective angular momentum $L_{\rm eff} = \sqrt{L_z^2 - 4M\alpha q^2}$. There exist no stable orbits for the atom around the line charge. Depending on the sign of $L_{\rm eff}^2$, the atom either falls into the center or escapes away from the wire towards infinity [2] (Fig. 1b). Consider a parallel beam of classical particles of momentum p impinging on an *infinitely thin* charged wire. Particles with an impact parameter of $b = L_z/p < 2q\sqrt{M\alpha}/p$ will fall onto the wire. Assuming that all atoms hitting the wire are adsorbed an absorption cross section ($\sigma_{\rm abs}$) can be defined which is given by twice the impact parameter b (note that in two dimensions the cross section actually has the dimension of *length*). This linear dependence of $\sigma_{\rm abs}$ on the line charge q is one of the characteristics of the $1/r^2$ singularity. For a charged wire with *finite* thickness one obtains the following velocity dependent absorption cross section where the linear dependence on the charge still persists for high q:

$$\sigma_{\rm abs} = 2\sqrt{R_w^2 + \frac{4\alpha}{M}\frac{q^2}{v^2}}. \quad (3)$$

R_w is the radius of the wire and v is the velocity of the incoming particles. This absorption cross section can in principle be measured by placing the wire in an atomic beam and then by counting the number of atoms hitting the wire. In practice this technique is not very feasible. In our experiments the wire is placed in the boxlike MOT mentioned earlier that is filled with a gas of N cold atoms. The charged wire introduces a loss mechanism for atoms in the trap which is related to the absorption cross section by the following rate equation:

$$\frac{dN}{dt} = -N\int \rho(\vec{x}_w, \vec{v})\sigma_{\rm abs}\sqrt{v_x^2 + v_y^2}\, d\vec{v}\, dz + L_+ - L_-, \quad (4)$$

where $\rho(\vec{x}_w, \vec{v})$ is the normalized $[\int \rho(\vec{x}, \vec{v}) d^3x\, d^3v = 1]$ atomic phase space density at the location of the wire $\vec{x}_w$. L_+ is a residual loading rate for the low light MOT and L_- stands for losses due to collisions with the thermal background gas. The density distribution $\rho(\vec{x}_w, \vec{v})$ is time independent if there is an effective mechanism which randomizes the atomic motion in the trap and refills the phase space of the absorbed atoms. In our case randomization is accomplished by light scattering in the MOT. Then as long as all terms of the sum in Eq. (4) are either constant or linearly dependent on N, the number of atoms in the box (N) will decay exponentially:

$$N(t) = N_0 e^{-(D+D_b)t} + \text{const.} \quad (5)$$

D_b stems from the $L_+ - L_-$ term and we call D_b the "background" decay. The decay constant D is due to the atom-wire interaction and equals the integral in Eq. (4). If the atomic density distribution in space and velocity $\rho(\vec{x}, \vec{v})$ is Gaussian (which is typical for a low

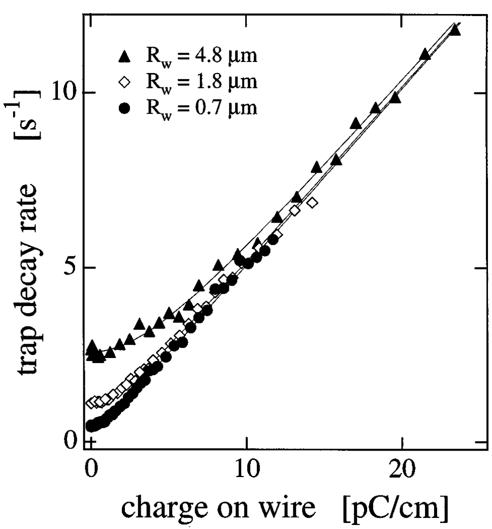

FIG. 3. Dependence of the trap decay rates on the wire charge for different wire thickness. The decay rate for uncharged wires is proportional to their actual diameters. For increasing charges the absorption rate becomes a linear function of the charge, a characteristic of the $1/r^2$ singularity. The slope is independent of the wire diameter. The data are well described by Eq. (6). Because the trap parameters $\sigma_v, \sigma_x, \sigma_y$ were slightly different for the three measurements, the original curves have been normalized to equal temperatures and trap sizes.

density MOT like that used in our experiment) and one assumes an isotropic velocity distribution, $\sigma_v = \sigma_{v_x} = \sigma_{v_y}$ (velocity widths), the integral can be solved analytically. For the wire being centered on the atomic cloud this integral can be very well approximated (better than 3%) by the following simple and very intuitive relation which is exact in the limit of $q = 0$ and $q \to \infty$:

$$D = \frac{1}{\sqrt{2\pi}} \frac{\sigma_v}{\sigma_x \sigma_y} \sqrt{R_w^2 + \frac{8}{\pi} \frac{\alpha}{M} \frac{q^2}{\sigma_v^2}}. \quad (6)$$

Here σ_x, σ_y, and σ_v are the spatial and velocity widths (standard deviations) of the atom trap.

In our experiments we use cold lithium atoms trapped in a MOT. The atoms are loaded into the trap for 20 sec out of an effusive thermal beam at a red laser detuning of 25 MHz and a total beam power of about 150 mW. An electro-optic modulator produces sidebands of 812 MHz (30%), one of which is used as a repumper. We use an additional 20 mW slower beam with a red detuning of about 100 MHz directed through the MOT into the oven which increases the total number of atoms in the trap by a factor of 5. At a vacuum pressure of 6×10^{-10} Torr we obtain typically 2×10^7 atoms with a lifetime of about 10 sec.

The absorption experiments are carried out in three steps (see insert of Fig. 2). In a first step (*a*) we load the lithium atoms into the MOT which is displaced (typically 1 mm) from the wire. After loading the atoms (*b*) the slower beam is shut off and the center of the MOT is shifted within 5 ms onto the charged wire [6] by applying an additional magnetic bias field. The charge on the wire is controlled by the voltage $U = 2q \ln(R_g/R_w)$, where R_w is the radius of the wire and R_g is the radius of the grounded cylinder, in our case the vacuum chamber. The frequency and the power of the trap laser are set to predefined values. In this way we control the size (density) of the trap as well as the mean velocity (temperature) of the atoms. During the absorption experiment the MOT is operated at a low light level which has the effect of practically turning off the loading rate. This is accomplished by either dimming or chopping the trap light. In the chopped mode the light beams are turned on and off at a rate of typically 1 to 20 kHz with chopping time ratios ($t_{\rm off}/t_{\rm on}$) of up to 50. In this manner we guarantee that the atoms move freely most of the time. In the third step (*c*) we monitor the exponential decay of the number of trapped atoms by measuring the fluorescence of the atomic cloud as imaged onto a photodiode (Fig. 2). For each charge that is applied to the wire steps *a, b, c* are repeated (Fig. 3).

Figure 2 shows three typical fluorescence decay curves. Exponential decay is easily observed over more than 2–3 orders of magnitude. The decay rate increases strongly with increasing line charge. The decay constants obtained from the fits still include the small contribution D_b corresponding to the $L_+ - L_-$ terms in Eq. (4). In our measurements great care was taken to measure this "background decay" D_b and subtract it from the measured decay in order to obtain D. D_b was measured by moving the trap away from the wire as opposed to toward the wire, keeping all other parameters constant. Typical decay constants ranged from below 1 to 100 sec^{-1}, with the zero decay D_b usually being around 0.1–0.3 sec^{-1} [7].

Figure 3 shows sets of experimentally determined decay constants D obtained by varying the charge on the wire which follow the anticipated "square-root" behavior of Eq. (6). The three different curves correspond to wires with radii of approximately 4.8, 1.8, and 0.7 μm. The decay constant for uncharged wires is directly proportional to their respective radii. As discussed earlier the linear slope for higher charges is a characteristic sign of the $1/r^2$ potential.

In order to obtain a more quantitative comparison of our data with Eq. (6) we performed a sequence of experiments (charge scans similar to those shown in Fig. 3) with very different trap parameters. The size, $\sigma_{x(y)}$, of the atom cloud was measured with a calibrated and triggered charge-coupled device (CCD) camera. The temperature ($\propto \sigma_v^2$) of the atoms was determined with the same camera by measuring how fast the atomic cloud expands within a few milliseconds after the laser and the anti-Helmholtz coils were shut off.

Square root fits of the form $D(q) = \sqrt{O^2 + S^2 q^2}$ to these charge scans provided us with two parameters, offset O and slope S. As expected from Eq. (6) the offset O shows a linear dependence on R_w and scales like $\frac{\sigma_v}{\sigma_x \sigma_y}$ (Fig. 4a). The slope S (Fig. 4b) is independent of σ_v and of the radius R_w but linearly dependent on $\frac{1}{\sigma_x \sigma_y}$. The independence of S on σ_v corresponds to an absorption cross section $\sigma_{\rm abs}$ which is proportional to $1/\sigma_v$. This

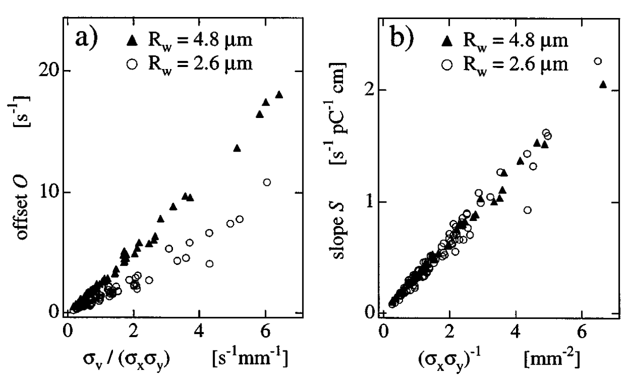

FIG. 4. Offset O and slope S are linear functions of $\sigma_v/\sigma_x\sigma_y$ and $1/\sigma_x\sigma_y$, respectively. Typical trap sizes $\sigma_{x(y)}$ range between 0.2 and 2 mm. The velocity σ_v of the atoms varies almost randomly between 0.5 and 1.5 m/sec. Two sets of data are given, corresponding to an early measurement with a $R_w = 2.6$ μm wire and a more recent measurement with a $R_w = 4.8$ μm wire. Comparison of these two data sets also shows the linear dependence of the offset O on R_w, whereas S is independent of R_w.

$1/v$ dependence of σ_{abs} is expected for the $1/r^2$ potential where atoms fall into the singularity as shown in our previous discussion of the impact parameter b.

We now discuss effects of the trap light on the scattering measurements in detail: The wire will cast a shadow in the 6 MOT laser beams. There the light forces are not balanced and atoms are pushed onto the wire, which effectively increases the absorption cross section. This light induced increase in the absorption cross section should play a large role for thick wires which cast long shadows and should be negligible for thin wires with a radius smaller than the wavelength of light. Figure 5 demonstrates this influence of light on our scattering measurements. According to Eq. (6) the quantity $O/(S\sigma_v)$ [8] should be a constant and depend only on α and R_w. For the thick wire ($R_w = 4.8\ \mu$m) this is not the case (Fig. 5a). For higher light intensity the cross section increases by up to 30%, while for the thin 0.7 μm radius wire no such increase is detectable. For low light levels $O/(S\sigma_v)$ approaches a constant even for the thick wire. By decreasing the laser light intensity the mean free path of the atoms increases and atoms pass through the shadow of the wire without being pushed towards the wire. This picture is supported by experiments where the light beams were chopped with various chopping ratios (Fig. 5b). By increasing the ratio $t_{\mathrm{off}}/t_{\mathrm{on}}$ the atoms are guaranteed to move freely in the $1/r^2$ potential for longer times. The increase of σ_{abs} due to the shadow can be suppressed for the $R_w = 4.8\ \mu$m wire by a chopping ratio greater than 10. For higher $t_{\mathrm{off}}/t_{\mathrm{on}}$ ratios $O/(S\sigma_v)$ stays constant and has the same value as that of the continuous trap. Again no shadow effect is detectable for the thin $R_w = 0.7\ \mu$m wire.

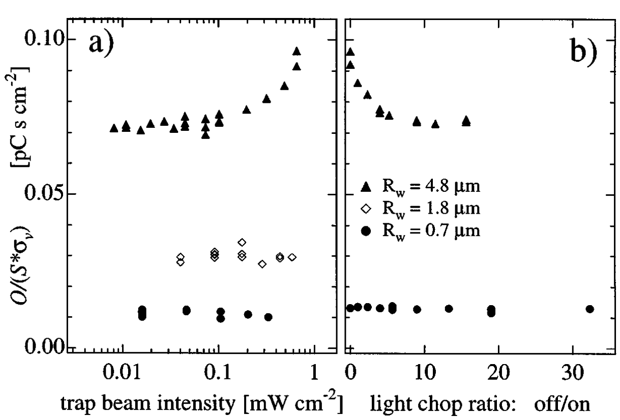

FIG. 5. The influence of the MOT light on the scattering experiment. In contradiction to Eq. (6) the quantity $O/(S\sigma_v)$ is not a constant but grows with increasing light intensity for the thick wire ($R_w \approx 4.8\ \mu$). This effect is shown as a function of the laser beam power (a) or the chopping ratio $t_{\mathrm{off}}/t_{\mathrm{on}}$ (b) and can be explained as due to the shadow cast by the wire. The laser detuning for the measurements was 12 MHz. In (b) the laser power per beam was set to 0.6 mW cm^{-2}.

Given $\alpha_{\mathrm{Li}} = 24.3$ [Å^3] [9] our measurements for the thick wire agree quantitatively to within about 5% of our theoretical predictions [8]. The main uncertainties in our experiments stem from the lack of precision in the trap parameters σ_v, σ_x, σ_y, and the wire diameter itself, caused by nonuniformity of the wire thickness and dirt particles on the wire.

In conclusion, we have realized an attractive cylindrical $1/r^2$ potential and presented a novel method of probing singular potentials with a gas of cold atoms. The strength of the $1/r^2$ potential can be varied by applying a corresponding voltage on the wire. We confirmed the theoretical predictions of the absorption cross section such as its linear increase with line charge and $1/\sigma_v$ which are both signs for atoms *falling* into the singularity. We are convinced that working with potentials created by small material objects as presented in this Letter will lead to new microscopic traps and guides for cold atoms [10–12], the future building blocks of mesoscopic atom optics. Because these potentials can be constructed only near a material surface, further experiments will also develop new possibilities for studying cold-atom/surface interactions.

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[6] To ensure two-dimensional geometry our wire was more than 10 cm long (ratio length to radius larger 10 000 : 1) and the experiments were performed in the central part of the wire.

[7] In our experiments we always checked against non-exponential decay, which occurs in regimes where the

atomic density in the trap is high. In those cases it was possible to get back to an exponential decay regime by starting out with fewer atoms in the trap.

[8] In order to attain the highest precision in our evaluation we analyse our data in terms of O/S where errors due to the trap size measurement drop out. Also imperfections in the trap itself like a non-Gaussian density distribution of atoms or misalignments in centering the trap on the wire cancel. The uncertainties of our analysis is then mainly due to the error in determining the temperature of the atomic cloud.

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