Acoustic Phase Conjugation in Superfluid Helium

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We present the first experimental observation of phase conjugate (PC) second sound (SS) waves in superfluid helium. The main feature of a PC wave is that its phase is complex conjugate to the incident wave phase. It is generated and amplified as a result of nonlinear interaction between an incident SS wave and a first sound (FS) pumping wave. At FS amplitudes larger than the threshold value, a parametric instability, i.e., spontaneous decay of a FS wave into two SS waves, takes place. Three main, theoretically predicted, features of phase conjugate waves were verified experimentally. [S0031-9007(98)08054-5]

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Phase conjugation (PC) is the process of nonlinear wave interaction in which the phase of an output wave is complex conjugate to the phase of an input wave. The nonlinear medium which generates this process is called a phase-conjugate mirror. A PC mirror has several unique properties compared to an ordinary mirror. It reflects an incident wave back for any incident angle. The conjugate wave can have a larger amplitude than the incident one. But it is this time-reversed phase property of the reflected wave that makes optical PC so potentially useful for a host of interesting applications, and particularly for correction of wave front distortions [1].

One of the common realizations of PC in optics is the mirror based on four-wave (4W) interactions [1]. But the 4W interaction is not the sole mechanism for obtaining PC. It can also be realized through three-wave (3W) interactions. In this case, PC has been observed in various wave systems manifesting sufficiently strong nonlinear interactions, e.g., microwaves [2] and acoustic waves [3]. In fact, the first observation of PC was made in acoustics, as a result of the nonlinear interaction between sound and electromagnetic waves, long before the observation of optical PC [3]. PC in acoustics results from the interaction between sound waves and the various types of collective oscillations in solids. Interaction of sound waves (either longitudinal or shear) with electromagnetic waves in piezoelectrics or magnets, the phonon-plasmon interaction in piezoelectric semiconductors, and the interaction of electromagnetic waves and spin waves in magnets are a few of many examples. At large enough amplitudes of an external field these systems exhibit a space-homogeneous parametric instability. That is, an externally driven wave is unstable with respect to generation of pairs of waves propagating in almost opposite directions and having almost a half driving frequency.

We recently reported the first experimental observation of the decay of one phonon of first sound (FS) into two phonons of second sound (SS) via a parametric instability in superfluid helium [4]. This effect was theoretically predicted over 20 years ago [5]. In the parametric process, a FS phonon with wave vector $\mathbf{K}$ and frequency Ω decays into two phonons of SS with wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ and frequencies ω_1 and ω_2. The new SS phonons are produced in adherence with energy and momentum conservation (resonance conditions): $\Omega = \omega_1 + \omega_2$ and $\mathbf{K} = \mathbf{k}_1 + \mathbf{k}_2$.

Both types of acoustic excitations, FS and SS sounds in superfluid helium, have linear dispersion laws: $\Omega = c_1 K$ and $\omega_{1,2} = c_2 k_{1,2}$, where $c_{1,2}$ are the first and second sound velocities, respectively [6]. The FS velocity is much larger than that of SS waves, particularly in the vicinity of the superfluid transition: $\eta = c_2/c_1 \ll 1$. Therefore, parametrically generated SS waves have almost half the frequency of the FS wave, and their wave vectors are almost opposite.

The PC phenomenon of SS waves in superfluid helium can be observed below as well as above the parametric instability threshold. An incident SS wave with half the frequency of the FS wave can be amplified by a FS pumping wave, generating a PC wave in the opposite direction. This Letter will describe the first experimental observation of this effect below the instability threshold.

Two main factors distinguish our system from the common manifestation of PC in optics. First, PC in optics is usually examined only below the onset of spontaneous oscillations (instabilities), which is unattainable at the currently available laser intensity [1]. Second, optical systems are of very low dissipation, so that $l \gg L$, where L is the system's size, and $l = c/\gamma$ is the dissipation length with γ being the SS waves attenuation [6]. Our system is in the range of $l/L \geq 1$ depending on temperature [4], so the dissipation is of crucial importance for conjugate wave generation.

Two linear (in respect to the incident and conjugate waves amplitudes) problems can be formulated in regard to the parametric instability of FS: (1) Determination of the threshold of spontaneous SS wave generation via a 3W interaction process; (2) the generation of the conjugate SS wave below the instability onset due to nonlinear interaction of the incident SS wave with the FS pumping field.

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The first problem, studied by us in [4], deals with the spatially uniform, rotationally invariant state in a FS resonance cavity. The instability occurs at the following value of the FS amplitude [4,7–9]:

$$a_{\mathrm{th}} = \frac{\gamma}{U}\,\zeta\sqrt{\left[1 + \left(\xi\,\frac{l}{L}\right)^2\right]}. \qquad (1)$$

Here, $\zeta = 1/2$ or $1/\sqrt{2}$ at $\Omega\eta^2/\gamma \gg 1$ or $\ll 1$, respectively [7–9], U is the interaction amplitude of the FS and SS waves, and the value of ξ depends on the ratio l/L and lateral reflection coefficient [4].

The second problem is related to a system with its rotational invariance externally broken by the incident SS wave having spatially dependent amplitude. Here, the relevant question is the following: What is the amplitude of the conjugate SS wave which is generated as a result of a 3W interaction? Depending on the boundary conditions and the value l/L for the same cell geometry, this problem can be described in two ways. One method is to consider the problem similar to PC in optics, i.e., an inhomogeneous problem for the space-dependent amplitude distribution of the probe and conjugate waves in a resonance cavity with dissipation. The second method is to consider a homogeneous problem for the probe and conjugate waves with linear decay. As our calculations and comparison with the experimental data show [8], the second approach is relevant to our experiment. This is due to the relatively large dissipation and, particularly, due to the nonreflecting lateral boundaries. Thus, the nonlinear process of PC wave generation via 3W interaction is described by the following set of linear equations:

$$\begin{aligned}&[\partial/\partial t + \gamma + i\omega_1]b_1 + iUab_2^* = 0,\\ &[\partial/\partial t + \gamma - i\omega_2]b_2^* - iU^*a^*b_1 = 0.\end{aligned} \qquad (2)$$

Here, incident and conjugate SS waves have amplitudes b_1 and b_2 and frequencies ω_1 and ω_2, respectively. $\omega_{1,2} = \Omega/2 \pm \delta$, where δ is the frequency shift of the parametrically generated SS waves. The nonlinear power reflection coefficient, $r = |b_2(0)|^2/|b_1(0)|^2$, is obtained from Eqs. (2) as

$$r = \frac{|Ua|^2}{\gamma^2 + \delta^2}. \qquad (3)$$

Here, $b_{1,2}(0)$ are the amplitudes of the incident and reflected SS waves, respectively, at the entrance to the cell.

In order to describe experimental data by Eq. (3), obtained in a finite lateral geometry FS resonance cavity, one can rewrite Eq. (3) by incorporating Eq. (1) in the following form:

$$r = \left(\frac{a}{a_{\mathrm{th}}}\right)^2 \frac{\zeta^2[1 + (\xi l/L)^2]}{1 + (\delta/\gamma)^2}. \qquad (4)$$

This approach is justified in the linear regime below the parametric instability threshold. The effective attenuation $\Gamma = \gamma - \sqrt{|Ua|^2 - \delta^2}$ tends to zero at the threshold and becomes negative above it, causing the SS wave amplitude to diverge exponentially. Saturation occurs due to higher-order nonlinear effects of SS wave interaction [7–9]. This regime will not be considered here.

The main theoretical predictions for PC waves generated parametrically *below* the onset are as follows.

(1) The amplitude of the conjugated SS wave is proportional to (a) the FS amplitude at fixed value of the frequency shift and (b) the amplitude of the incident SS wave.

(2) The sum of the phases of one FS and two SS waves involved in the resonance interaction is $\phi + \phi_1 + \phi_2 = \pi/2$. For a pair of SS waves this leads to $\phi_1 + \phi_2 = \mathrm{const}$.

(3) The power reflection coefficient has a Lorentzian shape, as a function of the frequency shift, with a width equal to the SS wave linear attenuation.

The experimental setup consisted of an open FS resonance cavity formed by two round FS capacitive transducers of 52 mm in diameter held 2.8 mm apart (Fig. 1). The resonance cavity has a quality factor of $Q \approx 150$ for the main resonance that allows it to reach large FS amplitudes and to neglect the influence of all other acoustic cavity modes. The cell was laterally open, and several layers of crumpled paper were placed around the cell

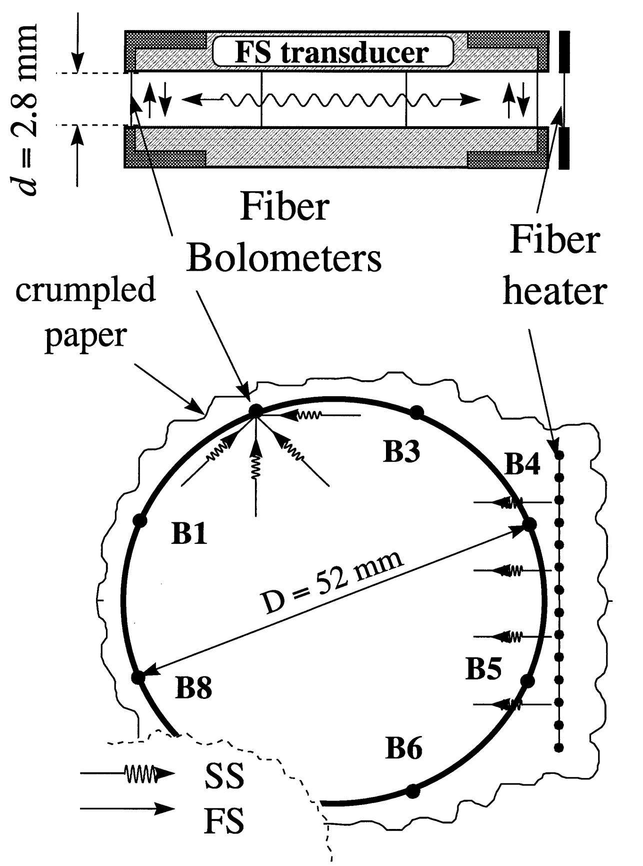

FIG. 1. Experimental cell and geometry of FS and SS waves propagation.

to absorb acoustic and thermal waves. The first resonance FS frequency, F_p, of the cavity was in the region of 39.4 kHz with a slight temperature variation in the vicinity of the superfluid transition temperature, T_λ. The cell was placed into a container, filled with approximately 300 ml of He^4, and vacuum sealed. A three-stage temperature regulated cryostat, having its acoustic cell at the third stage, was used [10,11]. In the experimentally investigated temperature range $10^{-4} < t < 10^{-3}$, where $t = (T_\lambda - T)/T_\lambda$, the cell had a temperature stability of approximately 0.1 μK, achieved by using a phase of SS as a thermometer in the SS phase-locked technique [12].

The greatest technological challenge behind construction of the new cell was the development of the fiber bolometers and heater [13]. Superconducting gold-lead (2:1 composition) films of approximately 200 nm thickness were evaporated on 8 μm diameter glass fibers. The fiber diameter was much smaller than the SS wavelength which was in the range of $0.1 < \lambda < 0.5$ mm. In contrast to flat bolometers, having very narrow angular sensitivity and used in a previous set of experiments [4,7,8,14], these fiber bolometers had uniform angular sensitivity and could detect all waves propagating in the cell plane. At the same time, the new fiber bolometers had figures of merit ($R^{-1}dR/dT \approx 50$ K^{-1}) comparable to flat bolometers and also possessed sufficient resistance. In order to achieve long-term room-temperature stability, the superconducting layer was covered by 40 nm of MgF_2 to decrease the oxidation rate. As a result of this treatment, the bolometers could survive many hours of mounting in the cell and approximately a month in a vacuum dessicator without a noticeable change in their properties.

Eight bolometers were mounted at equally spaced intervals on the circumference of the cell, and the transition temperature of each bolometer was adjusted by varying a bias current. In the experiment reported in this Letter, only two bolometers were used: bolometer B4 located close to a heater, and bolometer B8 located on the opposite side of the cell (Fig. 1).

The nonreflecting SS emitter-heater was constructed from 31 glass fibers of 8 μm diameter spaced 1 mm apart. This grid produced a plane wave at a distance of the order of the fiber spacing, and SS waves emitted by the heater could pass through the cell without reflection from the bolometers. Moreover, since the bolometers were sensitive to incident waves from the heater and those coming from the cell, this design is well suited to study PC in SS waves. As in previous experiments, we performed SS spectra measurements in a narrow bandwidth around $F_p/2$ using a lock-in amplifier fixed to this reference frequency [4]. The experiment was conducted at a fixed temperature by pumping the acoustic cavity with FS waves at frequency F_p and amplitude below the threshold value for the parametric instability [4,7,8]. While this pumping was occurring, a small amplitude SS wave was emitted by the heater. In order to separate SS signals coming directly from the heater and those coming from the cell, the incident wave frequency was shifted from $F_p/2$ to $f_1 = F_p/2 + \delta f$. Bolometer B8, located far from the heater, then detected only SS waves with frequency f_1. Bolometer B4, on the other hand, located between the heater and the cell, detected two signals: one with the frequency f_1 coming from the heater, and another with frequency $f_2 = F_p/2 - \delta f$ coming from the cell. This dual detection is clearly seen from the power spectrum of the signal read by bolometer B4 (see inset of Fig. 2). The spectra were measured at the reduced temperature $t = 7.07 \times 10^{-4}$ and the FS amplitude $A = 75$ Pa. This amplitude was below the threshold value $A_{\mathrm{th}} = 162$ Pa measured at the same temperature [4,7,8]. The dependence of amplitudes of both peaks, at the same temperature, as a function of the FS amplitude is also presented in Fig. 2, the

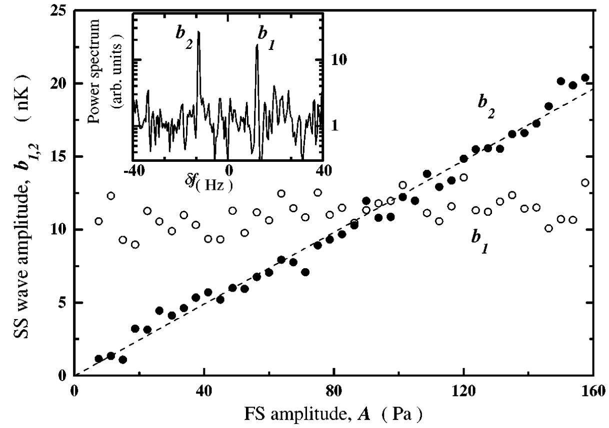

FIG. 2. Amplitudes of incident, b_1, (open circles) and conjugate, b_2, (full circles) SS waves as a function of FS amplitude A at $t = 7.07 \times 10^{-4}$ and frequency shift $\delta f = -12$ Hz. Inset: The spectrum of the SS signal measured on the bolometer B4 at FS amplitude $A = 75$ Pa.

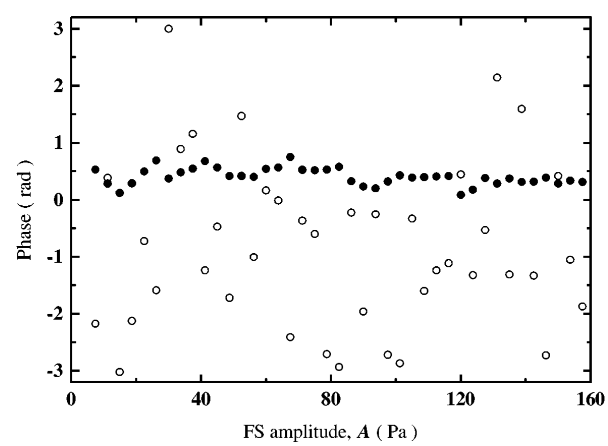

FIG. 3. Dependence of the phase of the conjugate SS waves, ϕ_2, on FS amplitude (open circles), and dependence of the sum of phases, $(\phi_1 + \phi_2)$, of the incident and conjugate SS waves on the FS amplitude (solid circles).

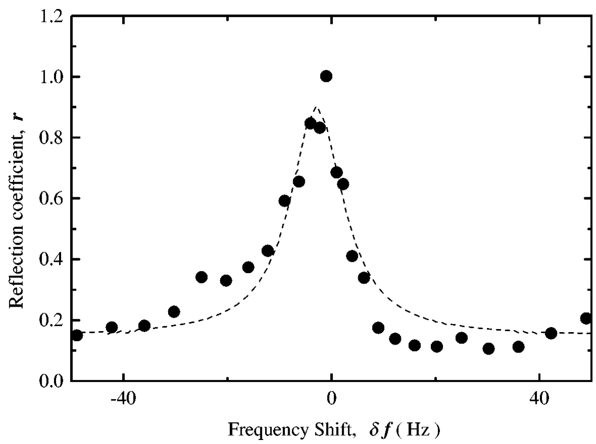

FIG. 4. Frequency dependence of the nonlinear reflection coefficient at constant FS amplitude $A = 75$ Pa and $t = 7.07 \times 10^{-4}$. The dashed line is a fit of the experimental data by a Lorentzian curve with a bandwidth of $\Delta = 14$ Hz.

frequency shift is $\delta f = -12$ Hz. The amplitude of the incident SS waves b_1, arriving directly from the heater on the bolometer B4, should clearly not depend on the FS amplitude, since these waves do not interact in the region between the heater and the resonance cavity. On the other hand, the amplitude of the SS waves arriving from the cell, b_2, is a linear function of the FS amplitude below the parametric instability threshold, as follows from the theory [see Eqs. (3) and (4)], and is also a linear function of the SS wave amplitude emitted by the heater b_1. The estimate of the slope of b_2/b_1 as a function of A/A_{th} from Fig. 2 gives 1.7 ± 0.15. This value is in fair agreement with the theoretical value of 2.05 obtained from Eq. (4). Thus the first property of the conjugated SS signal, following from Eq. (4), is quantitatively verified.

Particular efforts were made to verify the phase relation between the incident and conjugate SS waves. The phases of each of the SS signals at frequencies f_1 and f_2 were arbitrary with respect to the pumping field, and changed randomly from one measurement set to another (open circles in Fig. 3). However, the sum of the two phases of both signals was constant for all FS and SS incident wave amplitudes (Fig. 3). The standard deviation from the average value of the phase sum was $\delta(\phi_1 + \phi_2) = 0.15$ rad, as compared to the uniform distribution of the phase, in a 2π bandwidth, for each separate signal. This result is the main evidence for PC in SS waves.

The dependence of the nonlinear reflection coefficient, r, on the frequency shift, δf, at the reduced temperature $t = 7.07 \times 10^{-4}$ and the FS amplitude $A = 75$ Pa is presented in Fig. 4. Fitting of the experimental data by a Lorentzian function gives a bandwidth of $\Delta = 14$ Hz compared with the theoretical width $\Delta_{\mathrm{th}} = 19$ Hz. The theoretical width is defined solely by the SS attenuation in an infinite cell at the experimental values of t and the SS frequency $F_p/2 \approx 20$ kHz. The main reason for the discrepancy is a near-field configuration, used in the experiment. The latter can lead to distortions of the linear decay.

In conclusion, we have presented the first direct evidence of the PC which occurs for parametrically driven SS waves, as a result of 3W interaction between a FS pumping field and the SS waves, below the threshold of the parametric instability. Since PC is a fairly general phenomenon in parametrically generated waves, we are convinced that it can be observed in other parametrically driven systems. One of these easily accessible systems is that of surface waves parametrically excited by vertical vibration [15]. An obvious advantage of the latter system is the easy visualization of surface waves which can be used to verify the PC effect experimentally.

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