Dynamics of Two-Phase Fluid Interfaces in Random Porous Media

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Explicit account is given of the nonlocal dynamics (in a quasistatic approximation) involved in two-phase fluid dynamics quantifying flow through porous media. The results are used to derive the dynamical equation of motion of a Darcy-scale interfacial fluid front. We consider the cases of invasion and imbibition separately, and point out the features responsible for the different depinning exponents observed in the two cases. A Flory-type scaling analysis is also performed on this model, yielding a roughness exponent $\alpha = 3/4$ in a range of intermediate length scales—in good agreement with experimental observations. Possible reasons are outlined for the different universality classes of exponents observed during imbibition experiments. [S0031-9007(98)06533-8]

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A recent surge of interest has arisen on the effect of disorder on the morphology of moving interfaces, especially in the context of the *fluid flow through porous media*. (For other examples see [1].) The displacement of one fluid by another in a porous medium displays a rich variety of behaviors, depending upon the respective viscosities and wetting properties of the two fluids [2]. In such systems random pinning forces compete with the applied driving force, resulting in a critical threshold force at which a nonequilibrium phase transition occurs from the trapped metastable state (pinned interface) to a depinned moving interface. When the displacing fluid is more viscous and wets the bed particles more effectively than the displaced fluid, pinning by capillary forces leads to a rough interfacial displacement front. The morphology of the resulting self-affine rough interfaces is described by the Family-Viscek dynamical scaling hypothesis [3] relating the width $W(L,t)$ of an initially flat interface to the lateral size L and time t though a scaling relationship, $W(L,t) = L^\alpha g(t/L^z)$, where α and z are called the roughness and dynamical exponents, respectively. The scaling function g exhibits asymptotic behavior such that $W(L,t) \sim t^{\alpha/z}$ for $t \ll L^z$ and $W(L,t) \sim L^\alpha$ for $t \gg L^z$.

Analogies between the behavior exemplified by two-phase fluid and condensed matter systems suggest that the same phenomenological model might be used to characterize the interfacial dynamics in both cases. The model most widely proffered for the depinning transition case is the dynamical version of random field Ising model (RFIM) [4], namely,

$$\frac{\partial h}{\partial t} = F + \gamma \frac{\partial^2 h}{\partial x^2} + \eta(x,h), \tag{1}$$

wherein h denotes the interfacial position of the front in the longitudinal y direction and x the transverse coordinate. γ represents the so-called macroscopic interfacial tension and F the applied force. $\eta(x,h)$ constitutes a quenched random noise, representing the affinity of the disordered medium for one of the two phases. For random field disorder the noise term is assumed to be short-range correlated in both the x and y directions with a zero mean value.

Studies of Eq. (1) have yielded inconclusive results regarding the scaling exponents α and z. At present, two distinct universality classes are believed to exist: (a) The first is forced fluid invasion (FFI), involving the displacement of one fluid by another due to an applied pressure gradient. In such cases renormalization group arguments based on Eq. (1) predict $\alpha = 1$ [5], whereas numerical algorithms based on the above model yield $\alpha = 1.25$ [6,7]. In contrast, experiments performed for such a scenario result in a range of exponents, $\alpha = 0.5$–0.8 [8], depending upon the magnitude of the capillary number. Zhang [9] proposed (corroborated by experiments of Horvath *et al.* [10] and computer simulations by Nolle *et al.* [11]) that a power law distribution of noise amplitudes could rationalize the roughness exponents observed in experiments. Since power law statistics manifest difficulties in interpreting the exponent μ [9] we content ourselves here with representing the porous medium as a quenched random field disorder. (b) The second universality class corresponds to the imbibition invasion (IMI) of one fluid into another by capillary forces. In this case, the experimentally observed depinning exponent of $\alpha = 0.63$ is justified by a phenomenological mapping onto directed percolation depinning (DPD) [12,13]. The continuum model believed to be appropriate for this scenario is obtained by incorporating a nonlinear Kardar-Parisi-Zhang (KPZ)-type term into Eq. (1) [14]:

$$\frac{\partial h}{\partial t} = F + \gamma \frac{\partial^2 h}{\partial x^2} + \lambda\left(\frac{\partial h}{\partial x}\right)^2 + \eta(x,h). \tag{2}$$

The basis for the difference in universality classes of FFI and IMI phenomena, as well as the observed discrepancies between theoretical and experimental values of α in FFI, remain unresolved. Features distinguishing the FFI and IMI cases are (a) the absence of an external driving force in IMI, and (b) the weak strength of the disorder in FFI, contrasted with the strong disorder characterizing IMI.

However, based on these two features it has not yet been possible to justify the continuum Eq. (2).

This Letter examines an oft-neglected issue (although see He *et al.* [8] and Krug and Meakin [15]), one which distinguishes two-phase fluid flows from random magnets. This relates to the *nonlocal* nature of the flow field characterizing two-phase flows. While models involving local dynamics can be obtained from symmetry considerations, nonlocal models necessitate a more detailed microscale analysis. In our case, such a description of the dynamics is obtained from Darcy's law, which governs the fluid motion. The latter relates the velocity field to the pressure *gradient* rather than to the driving pressure itself. We consider an experimental scenario in which water constitutes the displacing fluid, and air the displaced fluid and wherein the assumption of a quasistatic response of the pressure field to the instantaneous interfacial configuration is valid. Such an approximation is reasonable near the depinning transition, where the mean velocity V of the interface approaches zero. For simplicity, we confine ourselves to the experimentally relevant case of one-dimensional interfaces, for which we derive a new model of interfacial dynamics by adapting the RFIM. Based on the equation we derive from microscopic considerations we outline the expected form of the general equations for FFI and IMI regimes. For the FFI regime a dynamical Flory-type scaling analysis is carried out on the model, carefully delineating the length scales of its validity, thereby enabling us to speculate on the magnitude of the roughness exponent α. This yields $\alpha = 3/4$ which accords well with experimental results. Furthermore, we also heuristically justify the difference in universality classes between FFI and IMI based on the absence of an applied pressure gradient and the strength of the disorder.

Two-phase flows in porous media are governed by Darcy's law, $\mathbf{v} = -\kappa\nabla p$, relating the pressure gradient ∇p to the velocity field $\mathbf{v}$. κ represents the permeability of the porous medium, which in our work will be taken to be spatially uniform. Fluid incompressibility requires that $\nabla \cdot \mathbf{v} = 0$ (cf. Delker *et al.* [16]); thereby we obtain

$$\nabla^2 p = 0. \tag{3}$$

Equation (3) possesses the general solution

$$p = -p_1 y + \frac{1}{2\pi}\int dk\, e^{|k|\tilde{y}} e^{ikx}\phi(k), \tag{4}$$

where p_1 represents the applied average pressure gradient, and $\tilde{y}$ the deviation from a flat interface situated at the mean position. The Fourier components $\phi(k)$ need to be determined from the boundary condition at the interface. The latter is derived starting from the Grinstein-Ma expression for the energy $\mathcal{H}$ of the RFIM [17]:

$$\mathcal{H} = \int dx \left\{ \frac{\gamma}{2}\left(\frac{\partial h}{\partial x}\right)^2 - \int_{-\infty}^{h} dy\,[\,p(x,y) - \eta(x,h)]\right\}. \tag{5}$$

The appropriate boundary condition is obtained by assuming quasistatic dynamics, thereby requiring that

$$\frac{\delta\mathcal{H}}{\delta h} = 0 \Rightarrow p(x,h) = -\gamma\frac{\partial^2 h}{\partial x^2} + \eta(x,h). \tag{6}$$

The field $\phi(k)$ is now obtained by first Fourier transforming Eq. (6) and subsequently invoking Eq. (4). For small deviations from a flat interface ($|k|h \ll 1$), the leading-order term in the expansion adopts the form

$$\phi(k) = p_1 h(k) + \gamma k^2 h(k) + \eta(k); \tag{7}$$

$$h(k) = \int dx\, e^{-ikx} h(x); \quad \eta(k) = \int dx\, e^{-ikx}\eta[x, h(x)]. \tag{8}$$

The latter denote the respective Fourier transforms of $h(x)$ and $\eta[x, h(x)]$. (For notational conciseness, the explicit time dependence of h has been suppressed.)

The dynamical evolution of the interface is described by the normal component of Darcy's law,

$$\frac{\partial h}{\partial t} = -\kappa\left[\frac{\partial p}{\partial y} - \frac{\partial h}{\partial x}\frac{\partial p}{\partial x}\right]_{y=h}. \tag{9}$$

Substitution of (7) into (9), with p given by (4), leads to the Fourier representation of the dynamical equation of motion:

$$\frac{\partial \tilde{h}(k)}{\partial t} = [\kappa p_1 - V]\delta(k) - \kappa p_1|k|\tilde{h}(k) - \kappa\gamma k^2|k|\tilde{h}(k) + \nu(k) - \kappa p_1\int dk'[k - k'][k']\tilde{h}(k - k')\tilde{h}(k') - \kappa\gamma\int dk'[k - k'][k']^3\tilde{h}(k - k')\tilde{h}(k'), \tag{10}$$

with $\tilde{h}(k)$ the Fourier transform of $\tilde{h}(x)$—the deviation from a flat interface moving at a velocity V. The $\nu(k)$ term represents the noise obtained from effecting the substitutions above. Before discussing the individual cases of FFI and IMI in the context of Eq. (10), we consider the noise $\nu(k)$.

The expression for p obtained by substituting (7) into (4) contains the term

$$\zeta(x,y) = \frac{1}{2\pi}\int dk\, e^{ikx} e^{|k|y}\int dx'\, e^{-ikx'}\eta[x', h(x')], \tag{11}$$

representing the manifestation of the noise term η. From (11), the resulting two-point correlation of ζ is

$$\langle\zeta(x,y)\zeta(x',y')\rangle \sim \frac{y + y'}{(x - x')^2 + (y + y')^2}. \tag{12}$$

As is intuitive, translational symmetry in the y direction is lost. We observe that the noise term $\zeta(x, y)$ exhibits long-range correlations in both x and y space. This results from the quasistatic assumption used to project out the fast relaxing variables, together with the special form of the driving field p, which necessarily needs to satisfy the Laplace equation (3). Noise terms with long-range correlations have been studied in the past, albeit in a different context [18]. Chow [19] has proposed that $\alpha > 1/2$ would require the presence of long-range correlations in the noise. Our analysis is the first to postulate noise accompanying fluid flow through porous media. Long-range correlated noise of the type (12) modifies the scaling of the noise, whence ζ scales as $l^{-1/2}$. The inverse Fourier transform of $\nu(k)$, viz. $\nu(x)$, involves a dominant term of the form $\partial\zeta(x, h)/\partial h$, which can be expected to scale as $h^{-1}l^{-1/2}$. This behavior contrasts with random field noise, which scales as $h^{-1/2}l^{-1/2}$.

In contrasting Eqs. (10) and (1), note that the last two terms of (10) are nonlinear in h. One might expect, and indeed confirm, that similar nonlinear terms arise from the next-order term in the expansion of $\phi(k)$. For example, a nonlinear term of the form $h\nabla^2 h$ (in physical space) is generated by such a procedure. However, for the eventual scaling analysis we have performed, the results are not modified by the presence of these additional terms (i.e., the leading nonlinear term scales as $h^2 l^{-2}$). Accordingly, we do not dwell upon them here. More importantly, one should note the structure of the terms appearing in Eq. (10). Explicitly, the first two terms exemplify the nonlocal nature of the dynamics in the normal driving force. The $|k|$ term has been predicted in the context of capillary line depinning, which also involves nonlocal dynamics [20]. Thus, from our microscale analysis of the equations of motion, we discern at least two main features distinguishing two-phase flows from RFIM dynamics: (a) the structure of the noise term, and (b) the nonlocal nature of the terms present in the dynamical equations.

FFI depinning.—In this case, p_1 is finite even though the mean velocity V may approach zero. Hence, the third term in (10), representing a KPZ type of nonlinearity, is nonzero despite the proximity of the system to the depinning transition. Several mechanisms have been proposed to justify the presence of such a nonlinear term in the dynamics [13]. One possible source arises from the anisotropy of the driving force, coupled with the fact that the driving force at every interfacial point acts normal to the interface at that point. However, as we subsequently find, such a nonlinear term is irrelevant in the dynamical description of FFI depinning.

What is the nature of the roughness exponent that would be expected to arise from Eq. (10)? Two important features arise in this context.

(i) In addition to those terms explicitly indicated in the equations, the capillary term $\kappa\gamma k^2|k|\tilde{h}$ might, in the presence of a nonlinear term ν, generate a Darcy-scale "surface tension"-like term. In such circumstances, we expect a general equation of the form,

$$\frac{\partial h(k)}{\partial t} = P_1|k|h(k) + P_2k^2h(k) + \nu(k) + P_1\int dk'[k - k'][k']h(k - k')h(k'), \quad (13)$$

wherein the other terms in Eq. (10) have been discarded on the basis of power counting arguments. Algebraic sign issues for the above coefficients necessarily play a crucial role in interfacial stability, requiring a more sophisticated treatment than that attempted here.

(ii) The $|k|h(k)$ term dominates in the hydrodynamic (large wavelength) limit. However, flow experiments in porous media have persistently shown that the observed roughness exponents constitute an intermediate length scale phenomenon, rather than being a manifestation of asymptotic behavior [8]. Thus, we employ a dynamical Flory-type scaling [21] to analyze the intermediate length scales representative of experimental conditions. For instance, the small capillary number regime suggests a scenario wherein P_1 is small compared with the capillary terms (represented by P_2).

Respective scalings of various terms on the right-hand side of Eq. (13) are as follows: $|k|h(k) \sim h/l$; $k^2h(k) \sim h/l^2$; $\int dk'(k - k')k'h(k - k')h(k') \sim h^2/l^2$; $\nu(k) \sim \Delta_0^{1/2}/hl^{1/2}$ (wherein Δ_0 represents the strength of the disorder). These scaling relations permit us to calculate the roughness exponents by matching the scalings of the individual terms with those of the noise. At long length scales the noise can be expected to occur as white noise (cf. Narayan and Fisher [5] and Horvath *et al.* [8]). However, here we consider only the regime wherein the noise manifests as a quenched noise. The results of such an analysis are as follows: (a) Matching the second term with the noise yields $h \sim l^{3/4}$ for $l \ll \mathrm{Min}(P_2^2/\Delta_0^{1/3}P_1^{4/3}, P_2/P_1)$. The scaling exponent obtained in this regime agrees well with the experimentally observed value of $\alpha = 0.8$. The length scales also appear to correspond to the experimental conditions as a consequence of the small capillary numbers characterizing such experiments. (b) A similar exercise in matching the nonlinearity with the noise yields $h \sim l^{1/2}$ for $P_2^2/\Delta_0^{1/3}P_1^{4/3} \ll l \ll \Delta_0^{1/3}/P_1^{2/3}$. The self-consistency condition for the existence of such a regime requires that $\Delta_0^{2/3}P_1^{2/3}/P_2^2 \gg 1$, whence the regime probably does not exist in the FFI case at low capillary numbers due to the weak strength of the disorder. However, manifestations of this regime are likely to appear at higher capillary numbers. Thus, we propose that *the long-range correlated nature of the noise, unique to two-phase fluid flows, is responsible for the anomalous exponents observed in experiments. The observed range of exponents is a manifestation of crossover behavior at higher capillary numbers. Furthermore, the nonlinear terms which are generated prove to be irrelevant at low capillary numbers due to the weak nature of the disorder and the presence of long length scale viscous smoothing effects.*

IMI Depinning.—Here, we propose an explanation for the different behaviors exemplified by the FFI and IMI regimes, hitherto unresolved. In the case of IMI, due to the strong disorder forces, the motion of the interface at any point depends only on the local disorder forces. In two dimensions the depinning transition was argued [12] to be a DPD transition with the interface being pinned when strong impurities that stop its motion span the system. The main feature of IMI is the lack of an external driving force, in lieu of which capillary forces drive the flow. Consequently, $P_1 = 0$ in the above equations. In such circumstances we expect a general equation of the form

$$\frac{\partial h(k)}{\partial t} = P_2 k^2 h(k) + P_3 |k| k^2 h(k) + \nu(k) + P_5 \int dk' [k - k'][k']^3 h(k - k') h(k') . \quad (14)$$

The latter equation reveals that rotational symmetry about the surface normal is broken due to the presence of the nonlinear term $\nabla h \cdot \nabla(\nabla^2 h)$ [22]. Thus, even in a geometrically isotropic porous media, anisotropy is generated by the dynamics. Reasons for the presence of such a term can be attributed to a combination of two factors: (a) The driving force for the dynamics is the gradient of the pressure field, rather than the pressure itself. (b) The fact that the pressure field p satisfies the Laplace equation leads to an exponential decay of the pressure field in the direction normal to the interface. As postulated by Tang *et al.* [13], the presence of anisotropy in the dynamics can lead to the generation of a KPZ nonlinearity. Such terms, which were shown to be irrelevant in the FFI case, can be shown to be relevant in this case, the reasons for which can be attributed to the strength of the disorder and the absence of the $|k|$ term. The presence of anisotropy presumably puts this model in the same universality class as DPD, which also constitutes an example of anisotropic depinning, thereby explaining the observed roughness exponent of $\alpha = 0.63$. Thus, we propose that *the anisotropy generated in the description of dynamics is responsible for the universality class of imbibition, corresponding to* $\alpha = 0.63$. We believe that our arguments provide the first theoretical rationale for the distinction between FFI and IMI.

In summary, we have derived a new model for two-phase frontal displacement flows in random porous media. The underlying analysis utilizes a quasistatic approximation for the pressure field, along with Darcy's law governing the dynamical evolution of the interface. The resulting evolution equation contains nonlocal terms as well as a noise term, the latter exhibiting long-range correlations. For FFI, a Flory-type scaling analysis was performed and the possible scaling regimes carefully delineated. Such an analysis yielded results in good agreement with experimental observations. Despite the success of our scaling analysis it is to be cautioned here that dynamical Flory-type analysis does not enjoy the same success as equilibrium Flory scaling [18]. It does, however, provide a lower bound for RG calculations. We also studied the IMI regime, pointing out the distinctions between FFI and IMI, and justifying anisotropic depinning in the IMI case. Our analysis emphasizes the marked contrast between two-phase fluid dynamics in random media and random magnets, thereby rationalizing existing discrepancies between experimental measurements of two-phase flows and analytical calculations thereof based on RFIM.

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