Support for Faraday's View of Circulation in a Fine-Powder Chladni Heap

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Faraday, like Chladni, saw fine powder collect in a circular heap at an antinode of a vibrating plate. In each vibration cycle, the heap experiences a free-flight interval during which pressure gradients in the heap's interior drive powder centerward, as Faraday proposed. When heap-floor collision terminates flight, pressure gradients reverse direction; but passage of a compaction front has locked particles against further movement. Before a next flight interval, an increase in porosity will reverse the compaction that accompanied heap-floor collision. [S0031-9007(98)06672-1]

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Following Chladni's lead, Faraday [1] placed coarse or powdery matter upon a horizontal vibrating plate. He saw the matter migrate to form isolated "Chladni heaps," coarse matter accumulating at nodes, and fine matter at antinodes. Faraday attributed the difference in these behaviors to the absence or presence, respectively, of the influence of air currents created by development of a partial vacuum between plate and heap. In a fine-powder heap, seeing powder move down its conical surface and disappearing inward at its edge, he reasoned that air currents pull particles toward the heap's center and force them upward thereat. Using small bits of paper to deflect air flows, Faraday altered locations of his fine-powder heaps, in support of his argument. Observing the plate in a partially evacuated chamber, he saw them at their usual positions at 13 kPa; at one-half this pressure, powder migrated to nodes, like coarse matter.

Renewing their interest in vibration of granular matter, physicists have discovered nonlinear dynamical phenomena possibly relevant to areas of physics as remote as semiconductors, earthquakes, and clustering of galaxies [2–5]. As recent reviews [2,3] comment, however, the activity has cast doubt upon Faraday's view of the role of air currents in driving fine-powder Chladni heap circulation, several research groups [6–12] offering alternatives thereto.

When these came to our attention, we reviewed the large engineering literature on vibrated granular beds [13] expecting to identify evidence confirming Faraday's thought, but no one seems to have set out to prove Faraday right. From review of our own data [14–18] and their revisualization [19], we can now offer "proofs," by-products of effort directed toward understanding vibrated-bed heat transfer [20] and fashioning a microreactor disclosing the effect of axial gas dispersion upon heterogeneous reaction outcomes [21].

In a glass-walled vessel (25.4 mm × 162 mm in plan, 60 mm in height), we subjected "two-dimensional" granular beds to vertical sinusoidal vibration at 25 Hz [14]: $z = a_0 \sin \omega t$, where $z =$ displacement, $a_0 =$ maximum amplitude, $\omega = 2\pi f$, $t =$ time, and $f =$ frequency. Unless specified otherwise, all data herein are for alumina beads at 30-mm average bed depth, $a_0 = 1.59$ mm, $f = 25$ Hz, and $\Gamma = 4$ ($\Gamma = a_0\omega^2/g$, maximum acceleration over gravity) [22]. We can report phase angles to $\pm 1°$ [14]. Under strobe lighting, our setup permits ready viewing of heap-floor separation (lift-off) and what we call "Faraday circulation": slow powder movement centerward in the bed's interior, accompanied by rapid return flow from peak to sidewall, occurring in a relatively shallow, rarefied surface zone [14,23].

Although others [6,24–27] appreciated the importance of gas floor-pressure data for understanding vibrated-bed dynamics, we are the first, to our knowledge, to measure floor pressures at multiple locations for many types and sizes of particles [17,18]; we record data at 1° phase-angle increments. Qualitative capacitance data [26,28] suggested that beds expand at lift-off. Our pressure data permit us to confirm and quantify the effect. We have discovered that a bed does not lift when a net upward force first acts upon it: e.g., a bed of 177-μm alumina beads lifts at ~93°, while the first action of the net upward force upon the bed occurred ~78° earlier. During this lag, it imbibes a quantity of air that we can estimate from our pressure data, causing it to undergo an absolute porosity increase of ~0.8%.

This increase seems to be required to release particles from a compacted condition in which forces acting upon the particles are insufficient to initiate particle motion in shear: We report sighting a compaction front (inclined at ~28° to horizontal) traveling across the bed from the sidewall centerward during bed-floor collision. In cinematographs taken at 2500 frames per second (fps) and viewed at 25 fps, we can see the front in particles 177 μm and larger [29]. Cinematographs at 1250 fps reveal that powder circulates in start-stop fashion: Except in the surface layer, a particle moves only during bed flight.

Figure 1 gives the status of a center-high 177-μm alumina bed (a Chladni heap truncated by side walls) at two phase angles during flight. Shortly after lift-off, at 105°, downward drag per unit bed weight is greater near the wall than at the center. The discrepancy has two effects, each contributing to maintenance of the heap. First, downward drag works against the "throw" the bed

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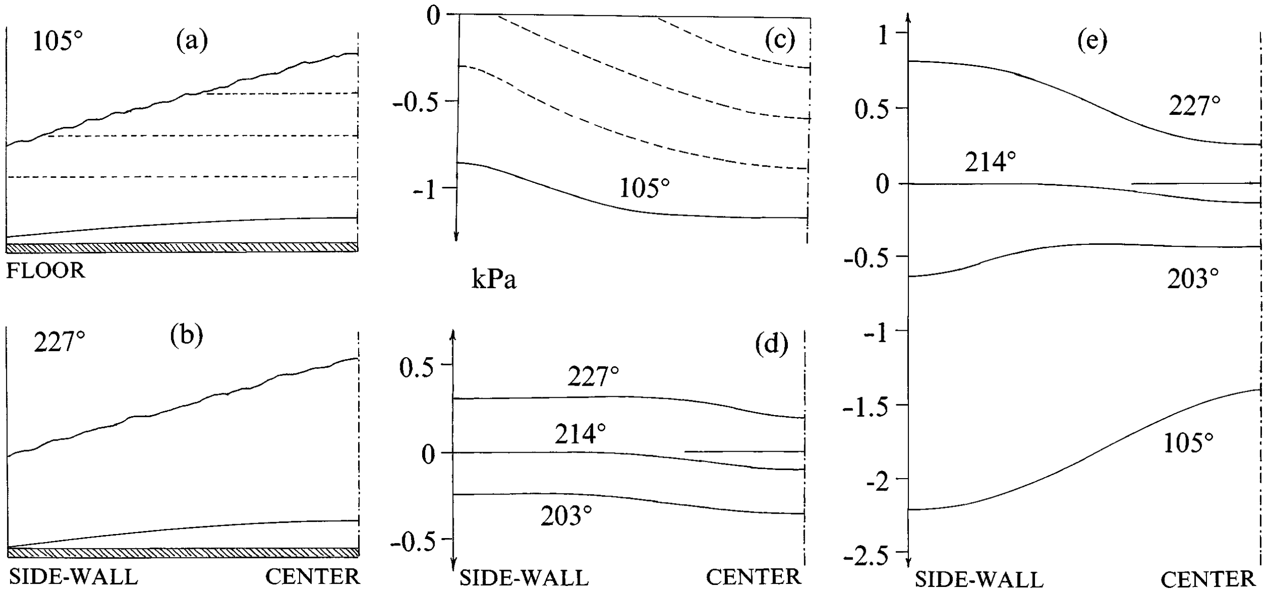

FIG. 1. Status of "two-dimensional" center-high vibrated bed (a slice of a truncated Chladni heap) at selected sinusoidal phase angles during free flight. Bed = 177-μm "Master Beads" (crude alumina, nearly spherical), 25.4 mm $\times$ 162 mm in plan, 30 mm in average depth; vibration at 25 Hz; vibrational intensity $\Gamma = 4$ (maximum acceleration divided by gravity). Diagrams represent left one-half of the bed. Curves of gas pressure (departure from atmospheric in kilopascals) are drawn to pass through three measured values (at wall, center, and halfway in between) and to display horizontal tangents at wall and center (no gas can flow across these boundaries). For reference in (c), dashed horizontal lines in (a) are drawn at $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ distance from peak to bottom of bed at its center. (a) and (b) give bed profiles at designated phase angles: (a) early in gap formation; (b) bed-floor collision just commencing (gap just closed at side wall). In (c) and (d), solid curves: floor pressure; in (c), broken curves: pressure profiles along dashed horizontal lines in (a). (e) plots a dimensionless ratio, vertical drag divided by bed weight (drag is downward if negative).

receives from the floor; eventual gap height is less at wall than at center. Second, some gas entering the developing gap near the wall must flow horizontally centerward within the gap. A horizontal pressure gradient at the floor drives this flow, as Faraday hypothesized. For the bed of Fig. 1, when floor pressures are negative, gas compressibility effects upon a vertical pressure profile are negligibly small [24]. To obtain a pressure within the bed along such a profile, linear interpolation between pressures at floor and bed surface provides a good approximation. Using such interpolation, we have derived the broken curves in Fig. 1(c), which correspond to horizontal dashed lines in Fig. 1(a). Horizontal pressure gradients within the bed easily move the "flying" alumina beads centerward. Floor pressures become positive shortly after 214°. Thereupon, as Fig. 1(e) illustrates, an upward drag cushions the bed-floor collision, which just commences at $\sim$227°.

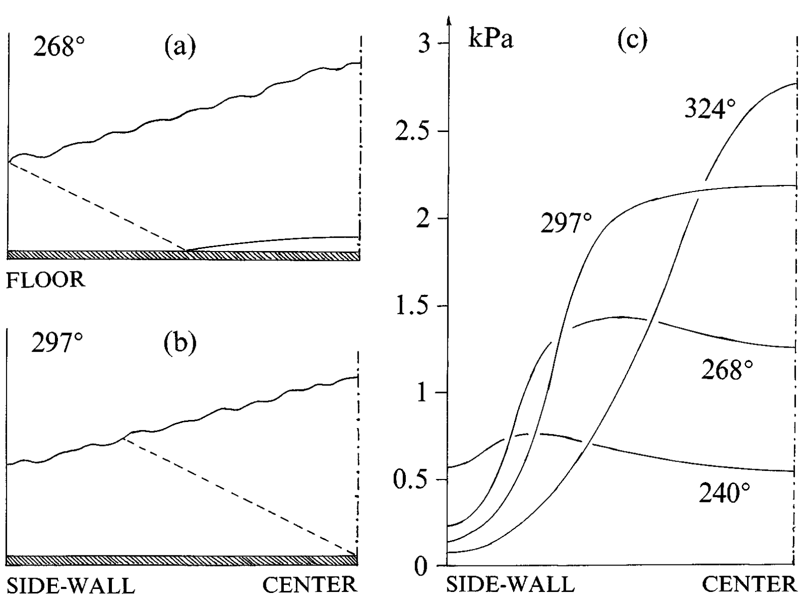

FIG. 2. Status of center-high vibrated bed at phase angles during gap closure. (See Fig. 1 caption for bed description and vibration parameters.) (a) and (b) Show bed profiles at two phase angles: gap closed halfway to center and gap just closed at center, respectively. Dashed lines indicate positions of compaction front. (c) Gives approximate floor pressure profiles at designated phase angles.

Figure 2 illustrates the bed status during bed-floor collision. At a given point along the floor, the compaction front's arrival is simultaneous with gap closure. Ahead of the front, gradient in floor pressure is centerward. Behind, the gradient reverses direction, but a concomitant gas flow cannot cause the compacted particles to move. At a given point, floor pressure peaks later than gap closure: The lag is 13° at the wall, 29° midway between wall and center, and 27° at center. These delays can be understood: Because the compaction front is inclined toward the sidewall, the front arrives earlier at a given point along the floor than at an elevation within the bed above the given floor point. Compaction of particles at the higher elevation drives gas downward as well as toward bed surface; eventually, gas driven downward reaches the surface, horizontal pressure gradients behind the compaction front having first driven this gas laterally, toward the wall.

While adding particulars, Figs. 1 and 2 confirm Faraday's basic ideas: that circulation in a fine-powder Chladni heap is aerodynamically driven, primarily while floor pressures are negative (between 93° and ~214° in our example).

Hypothesizing an inclined "condensation front" analogous to our compaction front, Laroche *et al.* [6] suggested particles move centerward by an "internal avalanche flow" [7] sliding downward along the hypothesized front. We see no such avalanche in high-speed cinematographs. *From the onset of free flight*, long before a compaction front appears, we see particles moving with largely horizontal components of velocity centerward. Afterward, behind the front, particles are stationary. Cinematographic evidence also rules out the suggestion [12] that heaping arises from positive pressure effects accompanying gap closure.

The discussion so far has focused upon 177-μm alumina beads. We deal briefly with beads of smaller and larger size.

At 30-mm average depth, although no gap forms beneath 88-μm beads, they form a heap displaying Faraday circulation during a "weightless" interval. Floor pressure data [30] confirm the presence of the driving gradients and reveal cyclic variation in porosity (an absolute ~2% rise and fall). Pressure data even disclose passage of the "compaction front" that we do not see in a cinematograph.

Before lift-off, 707-μm beads undergo an absolute porosity increase of ~0.13%; they exhibit a horizontal compaction front originating at floor and traveling upward. In 272-μm beads, the front is inclined at ~14° to horizontal. Fronts seen cinematographically may be traveling passive failure planes: Between $\Gamma = 3$ and 5, their angles are constant, each a fundamental property, apparently, of particle and orientation of principal stresses. Although Faraday circulation is not evident in coarse matter, nevertheless, it will form a heap slowly: 505-μm alumina beads require 3 min at $\Gamma = 2$. In all granular solids, heap slopes approach a dynamic angle of repose as Γ approaches one from above [6,12,17].

Withdrawing gas (air, helium, or propane) from closed chambers housing vibrated beds, Pak *et al.* [12] saw sharp declines in heap slope. Calling attention to the nonlinear partial differential equation for pressure in one-dimensional vibrated-bed theory, they suggested that the declines occur when a term (with pressure in the denominator) can no longer be safely ignored when the equation is linearized [26,31]. Mean-free-path calculations disclose another possibility. In each experiment, upon progressive reduction in pressure, a decline in slope coincides with departure from viscous flow [32], first, to slip flow with progressive reduction in viscous drag, and, finally, to molecular streaming and zero drag. Slopes must decline when within-bed gas flow loses its power to shape and maintain a heap and to cushion bed-floor impact.

All particulates that we studied (10 to 707 μm) experience zero drag at 8 Pa. All behave just the same. There is no heaping. All display a gap. All exhibit a friction-driven circulation characteristic of coarse powders at all pressure levels [33]. All are noisy: We hear the sharp crack of each uncushioned bed-floor collision.

Some researchers [9–11] reported upon nonsymmetrical, wall-high heaps. Evesque [11] pointed out the practical impossibility of establishing strictly vertical vibration, unbiased by horizontal velocity components. If such components are large, formation of a symmetrical, center-high heap may become an impossibility. In our work we succeeded in reducing the effect of horizontal velocity components to a degree allowing useful study of center-high heaps. An "indicator" glass duct [21] displaying substantially uniform distribution of powder in the "coherent-expanded" state [15] provides an acutely sensitive indication that bias in the vibration is sufficiently small such that symmetrical, center-high heaping can be reliably achieved.

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[23] Photographs [14] of 177-μm glass beads at selected phase angles under stroboscopic top lighting show how this zone's existence depends upon a sharp upward rush of air across bed surface during heap-floor collision. Lateral surficial flow varies along with expanded zone thickness, each reaching a maximum just after the collision. The zone then shrinks twofold to threefold before its renewal in the following cycle.
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