Solar Neutrino Puzzle: An Oscillation Solution with Maximal Neutrino Mixing

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(Received 1 July 1998)

If, as suggested by the Super-Kamiokande results, ν_μ and ν_τ are maximally and "rapidly" ($\Delta m^2 \approx 2.2 \times 10^{-3}$ eV^2) mixed, this alone determines the mapping from current to mass eigenstates up to one rotation angle θ mixing ν_e "more slowly" with a particular, equal-weight combination of ν_μ and ν_τ. For $\sin 2\theta = 1$, the resulting minimal number of free parameters, yet maximal mixing, shows agreement, with minor modifications, between extant observations of solar neutrinos and predictions by the standard solar model. [S0031-9007(98)08023-5]

PACS numbers: 26.65.+t, 14.60.Pq

When Kajita [1] reported at Neutrino '98 evidence for oscillation of atmospheric neutrinos with $\Delta m^2 \approx 2.2 \times 10^{-3}$ eV and large mixing, probably between μ and τ neutrinos, the conceptual landscape for discussion of neutrino mixing changed dramatically. The simplest interpretation consistent with this result is that there is maximal mixing between ν_μ and ν_τ and negligible mixing with ν_e. This remarkable conclusion leads to an important application in that other great arena, where neutrino oscillations have long been suspected but have so far eluded definitive proof, solar neutrinos. We do that here by assuming that the one parameter left free by the new result, the amount of mixing of ν_e, also is maximal, and then comparing deductions from that assumption with current observations, as well as predicting consequences for possible future observations.

At the very beginning of particle-physics efforts to understand the deficit in neutrinos arriving from the Sun, as compared with expectations from the standard solar model (SSM) (see Ref. [2]), it was clear that maximal mixing of ν_e and ν_μ would go a very long way toward solving the puzzle. However, before the new Super-Kamiokande result, there were strong reasons to be cautious about such a hypothesis: (1) Phenomenology: The nearest analog, the Cabibbo-Kobayashi-Maskawa matrix mapping quark electroweak current eigenstates to mass eigenstates shows mixing that is small between adjacent generations and very small between the highest and lowest generations [3]. (2) Theory: The widely accepted seesaw mechanism [4] for neutrino masses also suggests small mixing angles [5]. (3) Superfluity: The MSW effect (so called after Mikheyev, Smirnov, and Wolfenstein) [6] seemed able to give a rigorous explanation for the solar neutrino deficit even with small mixing, provided the relevant values of $\sin 2\theta$ and Δm^2 for ν_e mixing lie in a limited range. (4) Esthetics: Once one knew that there were three generations of neutrinos, why should ν_e be linked mainly with just one other generation? This last objection could be met by completely symmetrical three-generation-maximal mixing as discussed by several authors [7], themselves stimulated by earlier and less definitive indications from Kamiokande of large mixing. (As explained already, this kind of large mixing disagrees with the recent Super-Kamiokande result.) Thus there was neither compelling experimental evidence nor theoretical motivation for large, much less maximal, mixing.

The ideal assumption of maximal mixing between ν_μ and ν_τ for small values of L/E (earth's dimensions and GeV energies) has the immediate consequence that by suitable phase convention choices one mass eigenstate $|\nu_3\rangle$ may be written (as illustrated in Fig. 1)

$$|\nu_3\rangle = (|\nu_\mu\rangle + |\nu_\tau\rangle)/\sqrt{2}\,. \quad (1)$$

The most general form for the two other mass eigenstates then becomes (see also an early review [8])

$$|\nu_1\rangle = \cos\theta|\nu_\mathrm{e}\rangle + \sin\theta|\nu'\rangle \quad (2)$$

and

$$|\nu_2\rangle = -\sin\theta|\nu_\mathrm{e}\rangle + \cos\theta|\nu'\rangle, \quad (3)$$

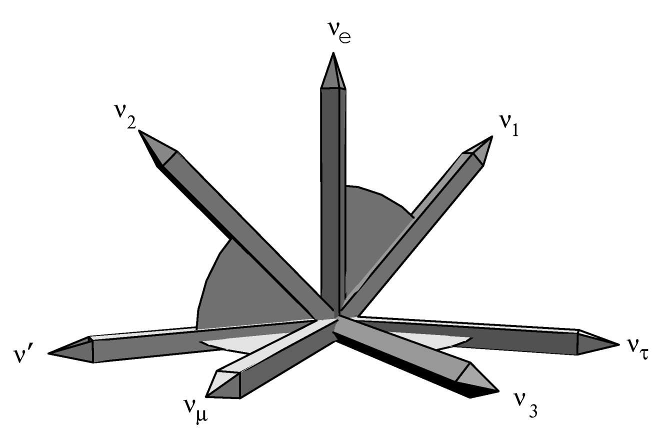

FIG. 1. The figure shows in perspective the three-dimensional principal axis transformation from the current eigenstates to the mass eigenstates. First, the system is rotated 45° about the ν_e direction, thus taking the original ν_τ direction into the final ν_3 direction. Second, the system is rotated 45° about the ν_3 direction, taking the original ν_e direction into the final ν_1 direction.

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with

$$|\nu'\rangle \equiv (|\nu_\mu\rangle - |\nu_\tau\rangle)/\sqrt{2} \tag{4}$$

and

$$|m_3^2 - m_2^2| \approx 2.2 \times 10^{-3}\ \text{eV}^2 \gg |m_2^2 - m_1^2|. \tag{5}$$

Thus, the issue of ν_e mixing becomes a two-state problem, with the only change from what might have been done years ago being that ν' takes the place of ν_μ as the mixing partner. (Note that ν' is neither a flavor nor a mass eigenstate.) The combination of the atmospheric Super-Kamiokande result and the maximal mixing hypothesis for ν_e uniquely specifies the mapping from the current eigenstates to the mass eigenstates. Note that because we are allowed to choose the mapping as completely real, no *CP* violation arises in the mixing. For that, a necessary requirement would be that each of the three mass eigenstates involves all of the current eigenstates.

It follows from the hypothesis that oscillations of $\nu_e \leftrightarrow \nu_\mu$ as well as $\nu_e \leftrightarrow \nu_\tau$ should be negligible for atmospheric neutrinos. This is compatible with present observations by Super-Kamiokande (see [1]), but the conclusion depends on the absolute number of atmospheric ν_e's predicted. It will be interesting to see whether the results of calculations which take account of the different paths of pions and muons in the Earth's magnetic field will affect this conclusion (see Gaisser [9]).

Compared to the expectations from the published standard solar model [10], the various detectors for solar neutrinos (Homestake [11], GALLEX [12], SAGE [13], Kamiokande, and Super-Kamiokande [14]) have shown deficiencies, often interpreted as due to matter-induced resonant oscillations in the sun (the MSW effect), where the electron neutrinos change flavor to a state for which the detectors are insensitive or less sensitive. These oscillations are characterized by a mixing angle θ and the difference of squared masses $\Delta m^2 = m_2^2 - m_1^2$, where m_1 and m_2 refer to mass eigenstates. A mixed state propagates through the vacuum with oscillation length L_v [2]

$$L_v = 2.48 \times 10^{-3} \frac{E_\nu\ (\text{MeV})}{\Delta m^2 (\text{eV}^2)}\ \text{km}. \tag{6}$$

Various solutions for the parameters θ and Δm^2 are compatible with the data. The MSW effect yields possible central solutions $\Delta m^2 = 5.1 \times 10^{-6}\ \text{eV}^2$, $\sin^2 2\theta = 8.2 \times 10^{-3}$, and $\Delta m^2 = 1.6 \times 10^{-5}\ \text{eV}^2$, $\sin^2 2\theta = 0.63$ (see Hata and Langacker [15]). Since matter enhanced effects in the Sun become unimportant as $\sin 2\theta \rightarrow 1$, the MSW mechanism is not needed for maximal mixing. The special case of a "just-so" vacuum solution has been recently revisited by Krastev and Petcov [16]. For the "just-so" vacuum solutions, there is a large change in the ^{7}Be electron neutrino flux over the year due to the change in phase of order $\pi/2$ in a year brought about by the $\pm 1.67\%$ yearly orbital variation from the mean distance of the Sun to the Earth. GALLEX, where individual experiments represent averages in neutrino absorption over several weeks, did not observe a seasonal effect [17]. For a value of $\Delta m^2 \gtrsim 10^{-9}\ \text{eV}^2$ the oscillation would go through many complete phases in a year and one would attain the region where our phase averaged vacuum mixing model would hold for the ^{71}Ga detectors. For a recent review of the entire current solar neutrino situation see, e.g., Berezinsky [18].

Let us assume that the neutrino deficiencies found are partially due to oscillations of electron neutrinos to different flavors, and partially due to an overestimate of the last, and probably weakest, link in the main neutrino chain of the SSM, viz. the emission intensity of ^{8}B neutrinos. The minimum required deficiency in emission is obtained for maximal neutrino mixing. If a detector integrates over a sufficient range of energies and/or a sufficient range of distances, phase averaging leads, after many oscillations, to a reduction of the expected signal by a factor of 2. Since the number of ^{8}B neutrinos is found by Super-Kamiokande [14] to be less than half of the SSM value the assumed vacuum solution would imply that there is a deficit in emission of ^{8}B neutrinos, compared with expectations from the SSM.

For the chemical detectors (^{37}Cl and ^{71}Ga) the maximal mixing vacuum solution would lead after phase averaging to a halving of the expected neutrinos detected, as the experiments are not sensitive to mu or tau neutrinos. In the water Ĉerenkov detectors mu or tau neutrinos are both detected at a rate reduced to about 14.7% of the detection rate for electron neutrinos, when averaged over the part of the spectrum detected by Super-Kamiokande. Assuming the rate of ^{8}B neutrinos emitted by the Sun to be $(1 - x)$ times the value predicted by the SSM, the ratio $R(^8\text{B})$ of electron recoils observed by Super-Kamiokande, relative to the expectation from the SSM without oscillations, can be written as

$$R(^8\text{B}) = \tfrac{1}{2}(1 + 0.147)(1 - x) = 0.368 \quad \text{or} \quad (0.474), \tag{7}$$

giving a reduction $x \sim 0.36$ or (0.17) for the ^{8}B neutrinos, when the 1995 [10] or (1998 [19]) version of the Bahcall-Pinsonneault SSM is considered.

The effect of reduction in ^{8}B from the SSM predictions is shown in Fig. 2. This allows us, as explained in the legend, to test the consistency of our model with the results obtained by the ^{37}Cl and ^{71}Ga experiments. For BP95 SSM we find a 36% reduction of the ^{8}B neutrinos emitted by the Sun. This leads to a prediction in agreement with the calibrated ^{71}Ga results but misses the pioneering ^{37}Cl result of 2.56 ± 0.23 SNU (solar neutrino unit), overestimating it by three and one half σ. It should be noted that there is no consensus whether the large fluctuations over time in the ^{37}Cl data are purely statistical. The recently revised SSM (BP98) makes use of a ^{7}Be (p,γ) ^{8}B cross section reduced by 15% from BP95 and of revised solar dynamics that reduce the

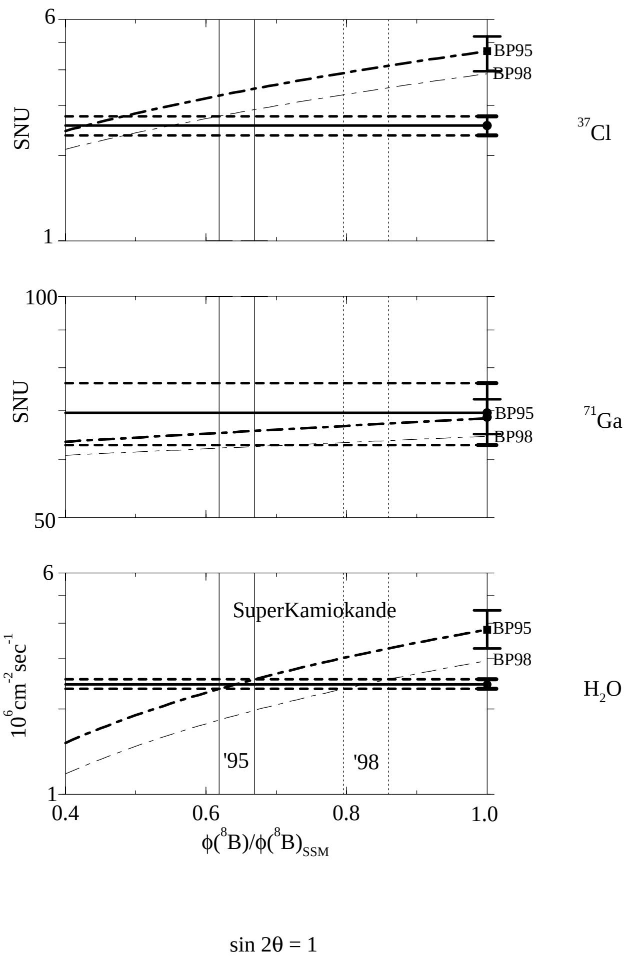

FIG. 2. Rates observed by the solar neutrino detectors compared with rates predicted for maximal neutrino mixing as a function of the reduction of the ^{8}B neutrino flux in the Sun from the predictions of the SSM BP95 (heavy dot-dashed line) and BP98 (faint dot-dashed line) are shown in all three boxes. Note that the vertical scale is logarithmic. Heavy horizontal lines represent the experimental values, with dashed lines the errors. Errors shown on the right side for BP95 are similar to those for BP98 (not shown). The ^{71}Ga data are an average of the GALLEX and SAGE data.

^{8}B neutrino flux to 78% of that predicted by BP95. Our maximal mixing model then calls for only a 17% reduction of the ^{8}B neutrino flux from BP98. Again our prediction is in agreement with the ^{71}Ga results, but misses the ^{37}Cl result by similarly overestimating it.

The solution of maximal mixing, along with a reduction in the emission of ^{8}B neutrinos, is consistent with a large range of possible values of Δm^2. The value of Δm^2 must be large enough to achieve phase averaging of the oscillations for the various neutrino sources in the Sun. At a value of $(5{-}9) \times 10^{-11}$ eV^2 there is a just-so vacuum oscillation solution relying on the oscillation phase [15], corresponding to several ($\sim$2–4) full wavelength oscillations on the way from the Sun to the Earth (mean distance $= 1.49 \times 10^8$ km). The vacuum oscillation formula for survival of an electron neutrino with maximal mixing is [2]

$$P(\nu_e) = 1 - \sin^2 2\theta \, \sin^2 \frac{\pi L}{L_v}\,, \tag{8}$$

where L is the distance from the Sun and L_v is given by Eq. (6). For the scattering by electrons of the monoenergetic ^{7}Be neutrinos, which BOREXINO intends to observe, the detection rate (normalized to unity for no oscillations) becomes

$$R(^7\mathrm{Be}) = 1 - 0.79 \sin^2 2\theta \, \sin^2 \frac{\pi \Delta m^2(\mathrm{eV}^2) L(\mathrm{km})}{(0.862)2.48 \times 10^{-3}}\,, \tag{9}$$

where the mu or tau neutrino scattering relative to electron neutrino scattering at 0.862 MeV is 0.21 [2]. Thus we take our solution to span approximately the mass region $10^{-9} < \Delta m^2 \ll 0.9 \times 10^{-3}$, using the CHOOZ upper limit [20].

The variation in orbital distance (5×10^6 km) may be compared to the average source size of the shell in the Sun whence the ^{7}Be neutrinos originate ($\sim 10^5$ km) [10]. As the phase change in a year due to the variation in the Sun-Earth distance is $\sim$50 times the phase averaging due to the source size, the Sun-Earth variation dominates on a yearly average. However, if one had sufficient statistics to measure the ^{7}Be intensity on, say, a daily basis, then the change in phase from day to day due to the Earth's orbit would be of the same order of magnitude as the phase variation (averaging) at the source, thus allowing an island of Δm^2 at $\sim 10^{-8}$ eV^2 to be explored.

We summarize here some experimental consequences of our solution which can be tested by existing or soon to be completed neutrino detectors:

(1) There is no distortion of the ^{8}B neutrino spectrum of the kind demanded by an MSW effect in the Sun. Suzuki [14] reports a hint of higher than expected counts near the high end of the ^{8}B spectrum. It has recently been suggested that this may be due to hep neutrinos [21].

(2) A deficit of $\sim$36% (or $\sim$17%) for ^{8}B neutrinos from the SSM predictions can be tested when neutral current interactions are studied at SNO.

(3) Our value for $R(^7\mathrm{Be})$ can be tested at BOREXINO.

(4) There should be no day-night effect (see [22]) following from matter oscillations in the Sun. (Since Super-Kamiokande has seen no statistically significant day-night effect, the region $3 \times 10^{-7} \lesssim \Delta m^2 \lesssim 10^{-5}$ eV^2 is already excluded for maximal mixing [23].)

We thank M. L. Horner for the design of Fig. 1. We thank M. Diwan, W. J. Marciano, P. G. Langacker, S. T. Petcov, S. P. Rosen, R. E. Schrock, and A. Yu. Smirnov for valuable discussions. This manuscript has been authored under Contract No. DE-AC02-98CH10886 with the U.S. Department of Energy and under National Science Foundation Grant No. PHY-9802453.

Note added.—While this paper was being completed, our attention was drawn to a manuscript posted by V. Barger, S. Pakvasa, T. J. Weiler, and K. Whisnant [24], which has some similar considerations.

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