# A New Solar System Dark Matter Population of Weakly Interacting Massive Particles

Thibault Damour[1] and Lawrence M. Krauss[2]
[1]*Institut des Hautes Etudes Scientifiques, 91440 Bures-sur-Yvette, France and DARC, Observatoire de Paris-CNRS, F-92195 Meudon, France*
[2]*Departments of Physics and Astronomy, Case Western Reserve University, Cleveland, Ohio 44106-7079*
(Received 11 June 1998)

Perturbations due to the planets combined with the non-Coulomb nature of the gravitational potential in the Sun imply that weakly interacting massive particles (WIMPs) that are gravitationally captured by scattering in surface layers of the Sun can evolve into orbits that no longer intersect the Sun. For orbits with a semimajor axis $<1/2$ of Jupiter's orbit, such WIMPs can persist in the solar system for $>10^9$ years, leading to a previously unanticipated population intersecting Earth's orbit. For WIMPs detectable in the next generation of detectors, this population can provide a complementary signal, in the keV range, to that of galactic halo dark matter. [S0031-9007(98)08075-2]

PACS numbers: 95.35.+d, 96.40.De

The effort to uncover the nature of the dark matter that dominates the gravitational potential well in our Galaxy, and almost all known galaxies, is perhaps one of the most important ongoing experimental programs in cosmology. Among the favored candidates for nonbaryonic dark matter are so called weakly interacting massive particles (WIMPs). Such WIMPs naturally arise in most supersymmetric extensions of the standard model in the form of the lightest supersymmetric partner of normal matter, the neutralino (e.g., [1]). A number of ongoing experiments, involving underground low background detectors, capable of detecting the energy deposited due to the elastic scattering of neutralinos off atoms, are underway. The "tip" of the allowed parameter space of neutralinos is just beginning to be probed.

If a positive detection is made in the next generation of underground detectors, it will be important to search for signatures that definitively distinguish such a signal from the possible background noise due to radioactive contaminants in and around the detectors. Possibilities include searches for a potential annual modulation and/or angular anisotropy of the signal [1–3], or indirect searches involving the annihilation products of the neutralino populations captured by the Sun [1,4–6] or the Earth [7,8].

The fact that WIMPs can be captured by the Sun and planets motivates examining whether various complicated dynamical histories in the solar system might affect the local WIMP density near the Earth (e.g., [9,10]). As we describe here, a careful consideration of the WIMPs that are scattered by a nucleus in the Sun into gravitationally bound orbits indicates that a small population can elude subsequent scattering in the solar interior (which otherwise leads to concentration in the solar core, followed by annihilation). Because of the non-Coulomb nature of the gravitational potential in the Sun, this population involves only WIMPs that scatter near the surface of the Sun. These WIMPs can, due to the perturbing influence of the planets, diffuse out of the Sun and build up over time to produce a density in the region of the Earth that may be comparable to the background WIMP halo density. The range of scattering cross sections and masses for such WIMPs is precisely that which is associated with the range of parameter space just below current experimental limits. Thus, if WIMPs are to be discovered in the next generation of experiments, then the population we describe here should be significant.

We shall focus on the subpopulation of WIMPs that scatter on a nucleus located near the surface of the Sun, and thereby lose just enough energy to stay in Earth-crossing orbits. These will be susceptible to small gravitational perturbations by the planets. We are interested then in the differential capture rate, per energy, and per angular momentum, of WIMPs in the Sun and, in particular, only in the fraction of WIMPs that have angular momenta in a small range $[J_{\min}, J_S]$ where $J_S$ is the angular momentum for a WIMP exactly grazing the Sun. The standard low-energy differential cross section of WIMPs on nuclei of atomic number $A$ is given by $d\sigma_A = \sigma_A^0 F_A^2(Q)\frac{d\Omega_{\rm cm}}{4\pi}$, where $Q = E_{\rm before} - E_{\rm after}$ is the energy transferred during the scattering, $F_A(Q)$ is a form factor, and $d\Omega_{\rm cm}$ is the scattering solid angle element *in the center of mass* frame. Note that $\sigma_A^0$ is by definition independent of the scattering angle and takes a value that depends on the details of the WIMP particle physics parameters. Generalizing the standard [8] capture calculation (see [11]), one can derive the rate with which WIMPs scatter on nuclei with atomic number $A$ to end up into bound solar orbits with semimajor axis between $[a, a + da]$ (corresponding to $[\alpha, \alpha + d\alpha]$ with $\alpha \equiv G_N m_{\rm Sun}/a$), and with specific angular momentum $J_{\min} \leq J \leq J_S$:

$$\frac{d\dot{N}_A}{d\alpha}\bigg|_{J\geq J_{\min}} \simeq \frac{n_X}{v_o}\int_{r\geq r_{\min}} d^3r\, n_A(r)\sigma_A^0 \times \left(1 - \frac{J_{\min}^2}{r^2[v_{\rm esc}^2(r) - \alpha]}\right)^{1/2} K_A(r,\alpha)\,. \tag{1}$$

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Here, the minimum radius $r_{\min}$ (perihelion) is defined in terms of the minimum angular momentum $J_{\min}$ by $r_{\min}\sqrt{v_{\rm esc}(r_{\min})^2 - \alpha} \equiv J_{\min}$, where $v_{\rm esc}(r_{\min})$ is the escape velocity at $r_{\min}$, $n_{A,X}$ is the density of atomic targets $A$, and WIMPs $X$, respectively, $v_o$ is the rms circular velocity of WIMPs in the Galaxy, and the "capture function" $K_A(r,\alpha)$ involves an integral over the WIMP local phase space distribution at the Sun, weighted over the scattering form factor [11]. While the general form (used in deriving our results) is not particularly illuminating, in the limit in which the nuclear form factor is neglected in the scattering cross section, and in which we also neglect the relative motion of the Sun with respect to the galactic halo, $K_A(r,\alpha)$ takes the following simple form:

$$K_A(r,\alpha) = \frac{2}{\pi^{1/2}}\frac{1}{\beta_+^A} \times \left\{1 - \exp\left[-\frac{\beta_-^A}{v_o^2}\left(v_{\rm esc}^2(r) - \frac{\alpha}{\beta_+^A}\right)\right]\right\}, \tag{2}$$

where $\beta_\pm^A \equiv \frac{4m_X m_A}{(m_X \pm m_A)^2}$.

To obtain the total rate, a sum over all the (significant) values of $A$ present in the Sun must be ultimately performed. We shall be interested in values $J_{\min} \simeq J_S = R_S v_{\rm esc}(R_S)$, where $R_S$ is the Sun's radius, and $a \sim 1$ AU, i.e., $\alpha \sim Gm_{\rm Sun}/(1\ {\rm AU}) \sim v_E^2$, where $v_E = 29.8$ km/s is the Earth orbital velocity. Note that for such values of $a$, typically $\alpha \sim v_E^2 \ll v_{\rm esc}^2$, and from the above formula it is clear that the function $K_A(r,\alpha)$ is nearly independent of $\alpha$.

The Sun scattering events create a population of solar-system bound WIMPs, moving (for $a \sim 1$ AU) on very elliptic orbits that traverse the Sun over and over again. For the values of WIMP-nuclei cross sections we shall be mostly interested in here (corresponding to effective WIMP-proton cross sections in the range $4 \times 10^{-42}$–$4 \times 10^{-41}$ cm$^2$), the mean opacity of the Sun for orbits with small perihelions is in the range $10^{-4}$–$10^{-3}$. This means that after only $10^3$–$10^4$ orbits these WIMPs will undergo a second scattering event in the Sun. It is straightforward to show that this second scattering event significantly reduces the semimajor axis of the WIMP, removing them from the population of interest here (they will end up in the core of the Sun where they will ultimately annihilate each other).

The only way to save some of these WIMPs from this demise is to consider a fraction of WIMPs that have perihelions $r_{\min}$ in a small range near the radius of the Sun, say $R_S(1-\epsilon) \le r_{\min} \le R_S$. Focusing on such a subpopulation of WIMPs has two advantages: (i) they traverse a small fraction of the mass of the Sun and therefore their lifetime on such grazing orbits is greatly increased, and, *more importantly*, (ii) during this time, gravitational perturbations due to the planets can push them into orbits that no longer cross the Sun.

We first recall some concepts and notation of Hamiltonian dynamics. In standard position-momenta variables the Hamiltonian describing the basic interaction between a WIMP and the Sun reads (for $r \equiv |\mathbf{x}|$), $\mathcal{H}_S(\mathbf{x},\mathbf{p}) = \frac{1}{2}\mathbf{p}^2 - U(r)$, where $U(r) = +G_N \int \rho(\mathbf{x}')d^3\mathbf{x}'/|\mathbf{x}-\mathbf{x}'|$ is the (spherically symmetric) Newtonian potential generated by the mass distribution of the Sun. Note that the mass $m_X$ of the WIMP has been factored out of all quantities. We work with action-angle variables ("Delaunay variables"), traditionally denoted $(L,G,H;\ell,g,h)$ [12]. The action variables $L$, $G$, $H$ are related to $E$, $J$, and $J_z$, respectively. The angle variables (with period $2\pi$) corresponding to $L$, $G$, $H$ are, respectively, denoted $\ell$, $g$, $h$. In these variables the Hamiltonian depends only on $L$ and $G$ and the general evolution equations,

$$\frac{d\ell}{dt} = +\frac{\partial\mathcal{H}}{\partial L}, \qquad \frac{dg}{dt} = +\frac{\partial\mathcal{H}}{\partial G}, \qquad \frac{dh}{dt} = +\frac{\partial\mathcal{H}}{\partial H}, \tag{3}$$

$$\frac{dL}{dt} = -\frac{\partial\mathcal{H}}{\partial \ell}, \qquad \frac{dG}{dt} = -\frac{\partial\mathcal{H}}{\partial g}, \qquad \frac{dH}{dt} = -\frac{\partial\mathcal{H}}{\partial h}, \tag{4}$$

tell us, in the case of the Hamiltonian above, that the action variables $L$, $G$, and $H$ are constant, while, among the angle variables, $h$ is constant, but $\ell$ and $g$ evolve linearly in time: $\ell = nt + \ell_o$, $g = \dot\omega t + g_o$. Here, $n \equiv 2\pi/P$ is the mean angular frequency of the radial motion ($P$ is the perihelion to perihelion period), and $\dot\omega$ is the mean rate of advance of the perihelion.

The crucial point to realize is the following. If we consider a WIMP orbit with a generic perihelion $r_{\min} < R_S$, it will undergo a large perihelion precession $\Delta\omega \sim 2\pi$ per orbit, i.e., $\dot\omega \sim n$, because the potential $U(r)$ *within the Sun* is modified compared to the exterior $1/r$ potential leading to the absence of perihelion motion. In other words, the trajectory of the WIMP will generically be a fast advancing *rosette*. This means that *both* angles $\ell$ and $g$ are *fast variables*. When adding in the small perturbing effect due to the planets, i.e., when considering the total Hamiltonian, $\mathcal{H}_{\rm tot} = \mathcal{H}_S(L,G) + \mathcal{H}_p(L,G,H;\ell,g,h;L_p,\ldots,\ell_p,\ldots)$ where $\mathcal{H}_p$ (which contains a small factor $\mu_p = m_{\rm planet}/m_S$) is the planetary perturbation, we can work out the (first-order) *secular* effects due to the planets by averaging over the fast variables $\ell$ and $g$ (as well as the mean anomalies $\ell_p$ of the planets). Then the evolution equations tell us immediately that the corresponding action variables $L$ and $G$ are secularly constant because planetary perturbations average out to zero (e.g., $\langle dG/dt\rangle = -\langle\partial\mathcal{H}/\partial g\rangle_{\ell,g} \equiv 0$). $L$ is essentially related to the semimajor axis $a$ of the WIMP orbit, while $G/L$ is related to the eccentricity $e$. As a result, when the rosette motion is fast, planetary perturbations do not induce any secular evolution in the semimajor axis and in the eccentricity of the WIMP orbit. Such WIMPs will end up in the core of the Sun.

A new situation arises for WIMP orbits that graze the Sun, because these orbits feel essentially a $1/r$ potential due to the Sun, so that their rosette motion will be very slow. Consequently, the variable $g$ will be slow (compared to $\ell$), and we cannot average over $g$. We can split the total Hamiltonian in three parts $\mathcal{H}_{\rm tot} = \mathcal{H}_o + \mathcal{H}_1 + \mathcal{H}_p$, where we take as unperturbed Hamiltonian the one corresponding to a pointlike Sun while the perturbations are $\mathcal{H}_1 = -\delta U(r)$ and $\mathcal{H}_p = \sum_p -G_N m_p (\frac{1}{|\mathbf{x}_X - \mathbf{x}_p|} - \frac{\mathbf{x}_X \cdot \mathbf{x}_p}{|\mathbf{x}_p|^3})$. Here, $\delta U(r) \equiv U(r) - G_N m_S/r$ is the non-$1/r$ part of the potential generated by the Sun, and $\mathcal{H}_p$ denotes the planetary perturbations involving a sum over the planets with masses $m_p$ and heliocentric positions $\mathbf{x}_p$. [The last term comes from the transformation between inertial (barycentric) coordinates and heliocentric ones.]

Using Delaunay variables defined by the Hamiltonian $\mathcal{H}_o$, we can derive the *secular* evolution of $L$, $G$, $H$, under the combined influence of the perturbations $\mathcal{H}_1$ and $\mathcal{H}_p$. This is simply obtained by averaging the canonical equations over the fast angles, i.e., all the mean anomalies of the problem: $\ell$, $\ell_p$. [We denote this average by an overbar.] By averaging the evolution equations one finds that (in first order) $L$ will be secularly constant (i.e., $a =$ const), while $G$, $H$, $g$, $h$ slowly evolve under the averaged perturbed Hamiltonian $\mathcal{H}_{\rm pert}(L, G, H; g) = \overline{\mathcal{H}}_1(L, G) + \overline{\mathcal{H}}_p(G, H; g; L_p)$.

Because $\mathcal{H}_{\rm pert}$ does not depend on the angle $h$, its conjugate momentum $H = J_z$ will be secularly constant. Determining the secular evolution of the remaining canonical pair $(G, g)$, and therefore estimating the minimum values of the WIMP angular momenta $J_{\rm min} \equiv G_{\rm min}$ for which planetary perturbations are strong enough to kick them out of the Sun is then reduced to studying the level curves of $\mathcal{H}_{\rm pert}(G; g)$. Among these level curves, there exists a *separatrix* $S$ such that initial orbits "above" $S$ secularly evolve into orbits with angular momenta $G > G_S$, i.e., large enough to no longer intersect the Sun. We find that it generically takes (for $a \sim 1$ AU) less than $10^3$ WIMP radial periods (i.e., less than $10^3$ yr) for the eccentricity of a WIMP above $S$ to increase sufficiently to exit the Sun. Then, once $G > G_S$ the time scale for the subsequent evolution of $G$ is given by the planetary perturbations alone and is roughly $1/[\sum_p \mu_p (a/a_p)^3]$ longer than one orbital period, i.e., roughly $10^5$ yr for $a \sim a_1 \equiv 1$ AU. After this time, the WIMP would, if it evolved only under the simplified planetary Hamiltonian $\overline{\mathcal{H}}_p$, come back again to low values of the eccentricity, corresponding to Sun-penetrating orbits. Under the influence of $\overline{\mathcal{H}}_1$, it would then again bounce back away from the Sun in $\sim 10^3$ orbits. For the scattering cross sections we shall be discussing below, the opacity of the small outer skin of the Sun is typically smaller than $\sim 10^{-5}$. Therefore the above process could persist for hundreds of cycles before the WIMP gets scattered again in the Sun. However, it is clear that the real gravitational interaction of the WIMP with planets is much more complicated than what is described by $\overline{\mathcal{H}}_p$. In particular, the nonzero eccentricities of the other planets, and the occurrence, once in a while, of a near collision with an inner planet will cause the elliptic elements of the WIMP to diffuse chaotically away from the simplified periodic history described above. As the very high eccentricities (for AU-size orbits) needed to traverse the Sun represent only a very small fraction of the phase space into which the WIMP can diffuse, on time scales of a few million years most of the population of WIMPs we are talking about will have irreversibly evolved onto trajectories that do not intersect the Sun for the entire age of the solar system. We further estimate that the long-term survival of such WIMPs on orbits that stay within the inner solar system is greater than 4.5 Gyr if $a < a_{\rm Jup}/2 = 2.6 a_E$. (This crucial assertion is based on an approximate analytical estimate of the lifetime of WIMPs [11] which should be checked by dedicated numerical simulations.)

We can finally estimate the density of WIMPs that diffuse out to solar-bound orbits and which can survive to the present time by simply integrating our differential capture rate over all trajectories $J > J_{\rm min} \equiv G_{\rm min}$ that end up out of the Sun, suitably averaged over the initial WIMP distribution. The rate (per $\alpha = G_N m_S/a$) of solar capture of WIMPs that subsequently survive out of the Sun to stay within the inner solar system then depends on the $A$-dependent combination: $g_A \equiv \frac{f_A}{m_A} \sigma_A^0 K_A^s$, where $f_A$ is the fraction (by mass) of element $A$ in the Sun, and $K_A^s$ is the Sun-surface value of the capture function mentioned above. Note that the $A$ dependence is entirely contained in $g_A$ with dimensions [cross section]/[mass]. The total capture rate is then dependent on $\sum_A g_A = g_{\rm tot}$.

By integrating this capture rate and making simplifying assumptions about the orbital evolution of the considered WIMPs, we can estimate both the present space and velocity distributions of this new WIMP population. We find a local enhancement in density near the Earth, compared to the halo WIMP density:

$$\delta_E \equiv \frac{n(a_1)}{n_X} = \frac{0.21}{(v_o/220\ {\rm km\,s^{-1}})}\, g_{\rm tot}^{(-10)}, \qquad (5)$$

where $g_{\rm tot}^{(-10)} \equiv 10^{10} g_{\rm tot} ({\rm GeV})^3$ (i.e., for masses given in GeV and cross sections in units of $10^{-10}$ GeV$^{-2}$, with $\hbar = c = 1$).

The *a priori* reasonableness of the final density enhancement (5) can be seen as follows. From Eqs. (1) and (2) the total rate (for $J_{\rm min} = 0$) of WIMPs scattered within 1 AU is of order $v_E^2 dN/d\alpha \sim n_X m_S g_{\rm tot} v_E^2/v_o$. Integrating over the lifetime $t_S$ of the solar system and dividing by the number of galactic WIMPs present within 1 AU gives a maximum possible density enhancement $\delta_{\rm max} \sim (3/4\pi)(m_S/a_E^3)(v_E^2/v_o) t_S g_{\rm tot} \sim 180 g_{\rm tot}^{(-10)}$. Out of this, detailed calculations of planetary perturbations of the WIMP's captured in the Sun's outer regions show [11] that a fraction *parametrically* of order $(\epsilon_m)^{0.4}(a_1/R_S)^{0.3} \times [\sum_p \mu_p (a_1/a_p)^3]^{0.6} \sim 10^{-3}$ is extracted. (Here $\epsilon_m = 0.022$ accounts for the density decrease of the Sun's outer regions of interest here.)

Perhaps a more relevant quantity, from the point of view of experiment, is the differential rate, $dR/dQ$, per keV per kg per day, of scattering events in a laboratory sample made of element $A$. Comparing this differential rate due to our new WIMP population with that due to the parent galactic halo population we find a ratio which reaches the flat maximum,

$$\rho(Q) \equiv \frac{(dR/dQ)^{\rm new}}{(dR/dQ)^{\rm standard}} = 1.1 g_{\rm tot}^{(-10)}, \qquad (6)$$

for energy transfers $Q$ smaller than $Q_E = 2[m_X/(m_X + m_A)]^2 m_A v_E^2$. For a target of germanium (the present material of choice in several ongoing cryogenic WIMP detection experiments), $Q_E$ is in the keV range. If $g_{\rm tot}^{(-10)} \sim 1$ this yields a 100% increase of the differential event rate below the value $Q = Q_E \sim$ keV which is typical for the energy deposit due to the new WIMP population, whose characteristic velocity $v_E$ is $\sim$7 times smaller than that of galactic halo WIMPs.

This is our central result. Its relevance depends completely on the actual value of $g_{\rm tot}^{(-10)}$. As an example of the possibilities, in Fig. 1 we display the range of $g_{\rm tot}$ derived by sampling over the allowed minimal supersymmetric standard model phase space [values for supersymmetric (SUSY) parameter $\mu > 0$ are chosen here], and for WIMP density in the range $0.05h^{-2} < \rho_X/\rho_{\rm closure} < 1$ (where $h$ is the Hubble parameter in units of 100 km s$^{-1}$ Mpc$^{-1}$). Values of $g_{\rm tot}$ in excess of 1 are thus possible.

Existing detectors tend to lose sensitivity in the range of a few keV. However, our result provides motivation to consider pushing hard in this direction. The new population will have a strongly anisotropic velocity distribution. Not only might this greatly help distinguish it from backgrounds, but a comparison of the angular anisotropy and annual modulation of this distribution with the corresponding features of the higher energy signal from a halo WIMP distribution would be striking. We emphasize once again that if neutralinos exist in the range detectable at the next generation of detectors, this new WIMP population should exist at detectable levels as well. Finally, we note that the indirect neutrino signature of such WIMPs that might subsequently be captured by the Earth, and annihilate, could be dramatic. The new population has a characteristic velocity that more closely matches the escape velocity from the Earth than does the background halo population. As a result, resonant capture off elements such as iron in the Earth could be greatly enhanced.

This work was supported in part by the U.S. DOE. L. M. K. acknowledges the hospitality of the IHES, where much of this work was carried out, and CERN, where this work was initiated. We thank D. Devaty and P. Kernan for assistance with various numerical routines.

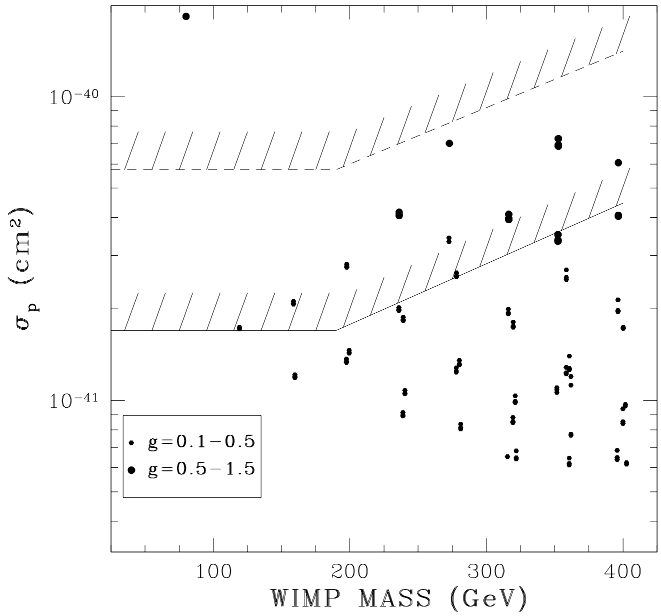


FIG. 1. Values of $g_{\rm tot}^{(-10)}$ as a function of WIMP nucleon cross section and WIMP mass for a sample range of allowed SUSY models ($\mu$ parameter $> 0$). Also shown are approximate experimental upper limits on WIMP nucleon cross section from direct detection experiments [13], assuming two different values for the local galactic halo dark matter density. $\rho = 0.3$ GeV cm$^{-3}$ (lower) and $\rho = 0.1$ GeV cm$^{-3}$ (upper).

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