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# Fringe Visibility and Which-Way Information in an Atom Interferometer

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We experimentally investigate the reduction of the fringe visibility in an atom interferometer due to the storage of which-way information. We focus on the case of incomplete which-way information and use the distinguishability $D$ to quantify how much information is stored. For a given value of $D$, the fringe visibility $V$ is limited by the duality relation $D^2 + V^2 \leq 1$. We have measured $D$ and $V$ independently. Combining the results, we find good agreement with the duality relation. [S0031-9007(98)08074-0]

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Complementarity expresses the fact that every quantum system has at least two properties, which cannot be observed simultaneously. This is often illustrated by means of interferometers, where "the observation of an interference pattern and the acquisition of which-way information are mutually exclusive" [1]. These two properties are associated with a wave and a particle picture, respectively. Two extreme cases of this wave-particle duality are well known from textbook examples: In the absence of which-way (WW) information, a fringe pattern with perfect visibility can be observed, while in the presence of full WW information, there are no fringes. In this paper we consider intermediate situations in which one obtains incomplete WW information and retains interference fringes with a reduced visibility. One of the most interesting questions in this intermediate regime is the following: How much WW information can be obtained for a given value of the visibility? To answer this question, a quantitative measure for WW information is required. We follow the definitions of Englert [1], who distinguishes between two different methods to obtain WW information in two-way interferometers.

The first method is to set up the interferometer such that the particle flux along the two ways differs. This leads to an *a priori* WW knowledge due to the difference between the probabilities, $w_+$ and $w_-$, that the particle takes one way or the other. The magnitude of this difference, $P = |w_+ - w_-|$, is the predictability. For a given $P$, the visibility $V$ of the interference fringes is limited by $P^2 + V^2 \leq 1$ (see Ref. [1]). Here $V = (I_{\max} - I_{\min})/(I_{\max} + I_{\min})$ is determined from the maximum and minimum intensities of the interference fringes, $I_{\max}$ and $I_{\min}$, respectively. Equivalent expressions for this limit were derived in Refs. [2–4]. Experimental results obtained with neutron interferometers [5] are in good agreement with this limit. We therefore restrict the following considerations to symmetric interferometers, where $w_+ = w_- = 1/2$.

The second method to obtain WW information is to add a second quantum system, the WW detector, to the setup. When the particle passes through the interferometer, an appropriate interaction changes the state of the WW detector depending on the particle's way. Hence WW information is stored and can be read out by measuring a suitable observable $W$ of the WW detector. Let $p(W_\mathrm{i}, +)$ and $p(W_\mathrm{i}, -)$ denote the joint probabilities that the eigenvalue $W_\mathrm{i}$ of $W$ is found and that the interfering object took way "+" or way "−." If $W_\mathrm{i}$ is found, the best guess about the way one can make is to opt for way + if $p(W_\mathrm{i}, +) > p(W_\mathrm{i}, -)$, and for way − otherwise. This yields the "likelihood for guessing the right way" [1]

$$L_\mathrm{W} = \sum_i \max\{p(W_\mathrm{i}, +), p(W_\mathrm{i}, -)\}. \qquad (1)$$

$L_\mathrm{W}$ depends on the choice of the observable $W$, as this determines the fraction of the stored WW information which is read out. Consider, e.g., the case where full WW information is stored: choosing $W$ carefully, one can reach $L_W = 1$, while an unfortunate choice of $W$ could result in $L_W = 1/2$, so that one could just as well toss a coin. To measure quantitatively how much WW information is *stored*, the arbitrariness of the *readout* process must be eliminated. This motivates the definition of the "distinguishability of the ways" [1,6]

$$D = -1 + 2 \max_W \{L_W\}, \tag{2}$$

which uses the maximum of $L_W$ that is reached for the best choice of $W$. The ways cannot be distinguished at all if $D = 0$, and they can be held apart completely if $D = 1$. For a given value of $D$ the visibility $V$ is limited by the duality relation

$$D^2 + V^2 \leq 1 . \tag{3}$$

The equal sign in Eq. (3) holds if the WW detector is initially prepared in a pure state. The duality relation makes a quantitative test of wave-particle duality possible. It has been derived only recently by Jaeger *et al.* [6] and independently by Englert [1], but it has not been tested experimentally. So far, only the reduction of the visibility due to storage of incomplete WW information has been observed in experiments using an optical interferometer [7] or a Ramsey interferometer [8]. However, the distinguishability has not been measured.

In this Letter, we report on measurements of $V$ and $D$ in an atom interferometer, where WW information is stored in internal atomic states. By varying the parameters of the experiment, $D$ can be adjusted continuously. We determined $D$ by an independent measurement of the atom's way and observed the reduction of the visibility. Combining the results for $D$ and $V$, we find good agreement with the duality relation.

We start with a brief description of the atom interferometer, which has been presented in more detail in Refs. [9,10]. Figure 1 shows a scheme of the setup, which uses Bragg reflection of atoms from standing light waves. A first standing light wave splits the incoming atomic beam, $A$, into two beams, $B$ and $C$, of equal atomic flux. After free propagation, a second standing light wave splits each atomic beam into two components. Now two beams, $D$ and $E$, are traveling to the left, while beams $F$ and $G$ are traveling to the right. In the far field, each pair of overlapping beams produces a spatial interference pattern. The envelope of the interference pattern is given by the collimation properties of the incoming atomic beam. For detection, the atoms are illuminated with laser light and the fluorescence photons are observed.

The experiment employs a beam of $^{85}$Rb atoms, whose ground state is split into two hyperfine components with total angular momentum $F = 2$ and $F = 3$, labeled $|2\rangle$ and $|3\rangle$, respectively. In these long-lived internal states, WW information can be stored. For that purpose, the frequency of the standing light wave, $\omega_{\text{light}}$, is tuned halfway between the $|2\rangle \leftrightarrow |e\rangle$ and $|3\rangle \leftrightarrow |e\rangle$ transition (see Fig. 2). Hence atoms in state $|2\rangle$ ($|3\rangle$) see a red (blue) detuned light field creating a negative (positive) ac-Stark shift potential, corresponding to an optically thicker (thinner) medium. In analogy to light optics one therefore expects [10] that the wave experiences a $\pi$ phase shift if reflected from an optically thicker medium, i.e., if an atom is Bragg reflected in state $|2\rangle$. However, a detailed calculation [11] shows that here this $\pi$ phase shift occurs if an atom is transmitted in state $|2\rangle$.

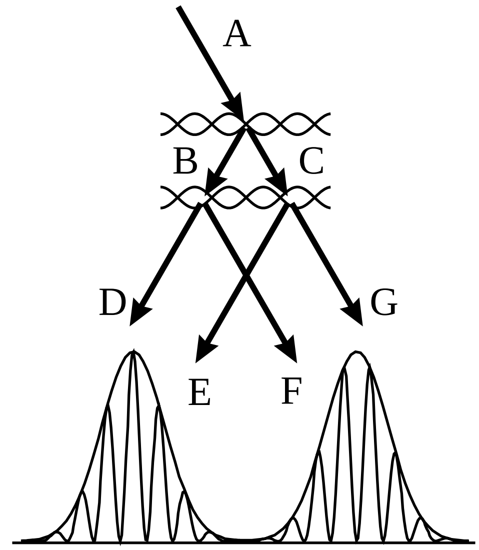


FIG. 1. Scheme of the atom interferometer. Bragg reflection from a standing light wave splits the incoming atomic beam, $A$, into two beams, $B$ and $C$. A second standing light wave splits the beams again. In the far field, a spatial interference pattern is observed.

This phase shift is converted into a population difference between states $|2\rangle$ and $|3\rangle$ by applying a microwave field, at frequency $\omega_{\text{mw}}$, resonant with the $|2\rangle \leftrightarrow |3\rangle$ transition. Two $\pi/2$ pulses of the microwave are required. They form a Ramsey scheme: one is applied before and one after one of the two standing light waves. We first consider the case, where the first standing light wave is sandwiched between the microwave pulses. The internal atomic state is initially prepared in state $|2\rangle$ and then converted to the superposition state $(|2\rangle + |3\rangle)/\sqrt{2}$ by the first microwave pulse. Next, the standing light wave splits the beam, so

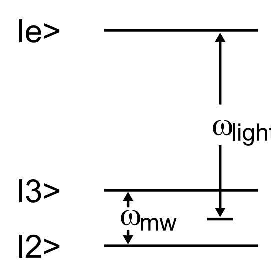


FIG. 2. Level scheme. The ground state is split into two components $|2\rangle$ and $|3\rangle$. The light frequency is chosen midway between the corresponding resonances to the excited state $|e\rangle$.

that the state vector is changed to

$$|\psi\rangle \propto |\psi_B\rangle \otimes (|3\rangle + |2\rangle) + |\psi_C\rangle \otimes (|3\rangle - |2\rangle), \quad (4)$$

where the minus sign is due to the $\pi$ phase shift, and $|\psi_B\rangle$ and $|\psi_C\rangle$ denote the state vectors of the center-of-mass motion for the reflected and transmitted beams ($B$ and $C$ in Fig. 1), respectively. The second microwave pulse, acting on both beams ($B$ and $C$), converts the state vector to

$$|\psi\rangle \propto |\psi_B\rangle \otimes |3\rangle - |\psi_C\rangle \otimes |2\rangle. \quad (5)$$

Obviously, the atom's internal and external degrees of freedom are entangled. This entanglement is the key point for the storage of WW information. If later a measurement of the internal state is performed, the result of this measurement reveals in which of the beams the atom is: if the internal state is found to be $|3\rangle$, the atom was Bragg reflected, otherwise transmitted. Equation (5) indicates that full WW information is stored.

Generalizing this scheme to arbitrary microwave pulse areas allows us to store incomplete WW information. It is sufficient to consider the case where the areas $\varphi$ of both microwave pulses are identical. In this case, the state vector after the second microwave pulse is

$$|\psi\rangle \propto |\psi_B\rangle \otimes (\cos\varphi|2\rangle + \sin\varphi|3\rangle) - |\psi_C\rangle \otimes |2\rangle. \quad (6)$$

For $\varphi = 0$ no WW information is stored, while for $\varphi = \pi/2$ full WW information is obtained. For intermediate values of $\varphi$, incomplete WW information is stored.

Alternatively, the *second* (instead of the *first*) standing light wave can be sandwiched between the two microwave pulses. Then the internal state contains the information whether the atom was reflected or transmitted in the *second* standing light wave. There is no essential difference between these two schemes: both produce two pairs of beams which overlap in the far field thereby creating a spatial interference pattern with visibility

$$V(\varphi) = |\cos\varphi|. \quad (7)$$

In order to measure the distinguishability $D$, a suitable observable $W_{\mathrm{opt}}$ has to be found which maximizes $L_{\mathrm{W}}$. It can be shown that this observable must have the eigenvectors

$$\cos\left(\frac{\pi}{4} - \frac{\varphi}{2}\right)|2\rangle - \sin\left(\frac{\pi}{4} - \frac{\varphi}{2}\right)|3\rangle, \quad (8)$$

$$\sin\left(\frac{\pi}{4} - \frac{\varphi}{2}\right)|2\rangle + \cos\left(\frac{\pi}{4} - \frac{\varphi}{2}\right)|3\rangle, \quad (9)$$

and that a measurement of this observable yields $L_{\mathrm{W}} = (1 + |\sin\varphi|)/2$, so that

$$D(\varphi) = |\sin\varphi|. \quad (10)$$

To measure the likelihood $L_{\mathrm{W}}$, we perform the experiment with only one standing light wave sandwiched between the two microwave pulses, while the other standing light wave is removed. This allows us to measure the atom's external and internal states simultaneously in the following way: first, the far-field position of an atom now reveals whether the atom is in beam $B$ or $C$. Second, it is possible to detect atoms in either state $|2\rangle$ or state $|3\rangle$ by an appropriate choice of the frequency of the detection laser. This frequency is tuned into resonance with either a transition $|2\rangle \leftrightarrow |e\rangle$ or a transition $|3\rangle \leftrightarrow |e\rangle$, where $|e\rangle$ denotes an excited level. This measurement yields the joint probabilities $p(2,B)$, $p(3,B)$, $p(2,C)$, and $p(3,C)$ which determine $L_{\mathrm{W}}$. Note that a position measurement without internal state detection is also possible. For that purpose the atoms are illuminated with light at both frequencies simultaneously, so that all atoms are detected.

The internal state measurement discussed so far is a measurement of an observable $W_0$ with eigenvectors $|2\rangle$ and $|3\rangle$. In order to measure an arbitrary observable $W$ of the internal state, a third microwave pulse with suitable parameters is applied before measuring $W_0$ [12]. For example, to measure $W_{\mathrm{opt}}$ according to Eqs. (8) and (9), the area of the third microwave pulse must be $\pi/2 - \varphi$ (or $3\pi/2 - \varphi$, etc.). This allows us to measure the probabilities required to determine $L_{\mathrm{W}}$, from which $D$ can be inferred.

In Fig. 3, the measured values of $D$ (dots) are displayed as a function of the area $\varphi$ of the first two microwave pulses. Simultaneously, the area of the third microwave pulse was varied, in order to keep it equal to the optimum value $\pi/2 - \varphi$ (or $3\pi/2 - \varphi$, etc.). Hence $W_{\mathrm{opt}}$ is measured and the measured data maximize $L_{\mathrm{W}}$. We checked experimentally that no larger value of $L_{\mathrm{W}}$ could be obtained by varying the parameters of the third microwave pulse.

According to Eq. (10), $D$ should be unity at $\varphi = \pi/2$. In the experiment, we find $D_{\max} = 0.81 \pm 0.02$. This reduction is mainly due to two effects: first, the presence of background counts in the fluorescence detection reduces $D$ by a factor of 0.90. Second, the standing light wave is not a perfect plane wave but has a transverse profile

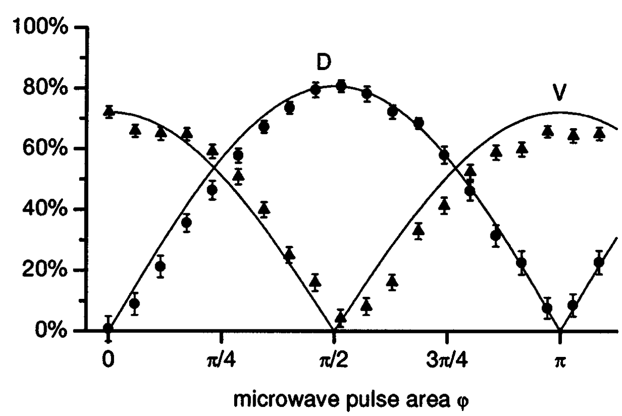


FIG. 3. Visibility $V$ (triangles) and distinguishability $D$ (dots) as a function of the microwave pulse area $\varphi$. The solid lines are the theoretical expectations Eqs. (11) and (12).

which exhibits intensity wiggles caused by imperfections in the optical elements. We measured the relative intensity variations and obtained $\sigma_{\mathrm{I}} \approx 12\%$ (rms). Hence the light intensity $I$ seen by an individual atom depends on its transverse position. A detailed calculation shows that the amount of WW information stored is independent of $I$, but that the parameters of the third microwave pulse must depend on $I$ in order to maximize $L_{\mathrm{W}}$. As $I$ is not known for each atom, the best one can do is to choose the parameters of the third microwave pulse such that the ensemble average of $L_{\mathrm{W}}$ is maximized. This reduces $D$ by a constant factor $1 - (\pi\sigma_{\mathrm{I}})^2/2 = 0.93$. Multiplication of both factors gives $D = 0.84$ at $\varphi = \pi/2$, which is in reasonable agreement with the experimental result $D_{\max} = 0.81 \pm 0.02$.

Because the background as well as the intensity variations reduce the measured value of $D$ by a constant factor, independent of $\varphi$, the measured quantity is

$$D(\varphi) = D_{\max}|\sin\varphi| \tag{11}$$

instead of Eq. (10). Equation (11) with $D_{\max} = 0.81$ is plotted as a solid line in Fig. 3.

In order to observe an interference pattern, both standing light waves are turned on. The second standing light wave is sandwiched between the two microwave pulses. The triangles in Fig. 3 show the measured values of the fringe visibility $V$, when all atoms are detected. $V$ was determined by measuring the atomic flux at a maximum and a minimum of the interference pattern. We observed a maximum visibility of $V_{\max} = 0.72 \pm 0.02$ at $\varphi = 0$. The reduction from unity can be explained by two effects: first, background counts reduce the visibility by a constant factor 0.92. Second, the detection laser beam and the atom source have a finite size, so that the fringes wash out. This reduces the visibility by a constant factor 0.82. Both factors together give $V = 0.75$ at $\varphi = 0$, which is in reasonable agreement with the experimental result $V_{\max} = 0.72 \pm 0.02$. Again, the two factors are independent of $\varphi$. Hence Eq. (7) should be modified to

$$V(\varphi) = V_{\max}|\cos\varphi|. \tag{12}$$

The result is shown as a solid line in Fig. 3 with $V_{\max} = 0.72$.

We now discuss whether the variations of the light intensity influence the visibility. As was mentioned above, there is no influence onto the amount of WW information stored. Hence one might assume that the intensity variations do not influence the visibility either. However, it is easy to see that the intensity variations actually reduce the visibility in the right part of the interference pattern which is formed by beams $F$ and $G$ (see Fig. 1). This reduction is due to the fact that the probability for Bragg reflection of an atom from the standing light wave depends on the light intensity. Beam $F$ is reflected twice, while beam $G$ is transmitted twice. This leads to different atomic fluxes in beams $F$ and $G$ and reduces the visibility in the right part of the interference pattern.

This argument does not apply in the left part of the interference pattern, because beams $D$ and $E$ are transmitted and reflected once, so that any deviations from the exact 50:50 beam-splitter ratio compensate each other. Hence the atomic fluxes in beams $D$ and $E$ are equal and the visibility is unaffected. For this reason, the results for $V$ displayed in Fig. 3 were determined in the left part of the interference pattern, namely, from the atomic flux at the central maximum and at one of the neighboring minima.

We conclude that the reduction of the measured distinguishability $D$ and visibility $V$ is well understood. In order to test the duality relation, it is therefore justified to divide the measured data from Fig. 3 by $D_{\max}$ and $V_{\max}$, respectively. The result is shown in Fig. 4, where

$$\left(\frac{D(\varphi)}{D_{\max}}\right)^2 + \left(\frac{V(\varphi)}{V_{\max}}\right)^2 \tag{13}$$

is plotted as a function of $\varphi$. The data at $\varphi = 0$ and $\varphi = \pi/2$ (open circles) are close to unity by definition. All other data (full circles) are below unity which means that we find no violation of the duality relation.

The initial state of the WW detector is a pure state, so that all data in Fig. 4 should reach unity. The data for $\varphi < \pi/2$ agree well with this prediction. The deviations for $\varphi > \pi/2$ are due to the fact that the measured visibility is smaller here than for $\varphi < \pi/2$ (see Fig. 3). This reduction of $V$ can be explained by noting that the contrast of the interference pattern is reversed for $\varphi > \pi/2$; i.e., the positions of interference maxima and minima are exchanged. As a consequence, the height of the interference maximum for $\varphi > \pi/2$ is smaller than for $\varphi < \pi/2$ because of the finite width of the beam envelope. Hence background counts and the finite position resolution have a slightly larger effect on $V$ for $\varphi > \pi/2$.

To summarize, we performed the first quantitative test of the recently published duality relation with an atom interferometer. The duality relation sustained the test.

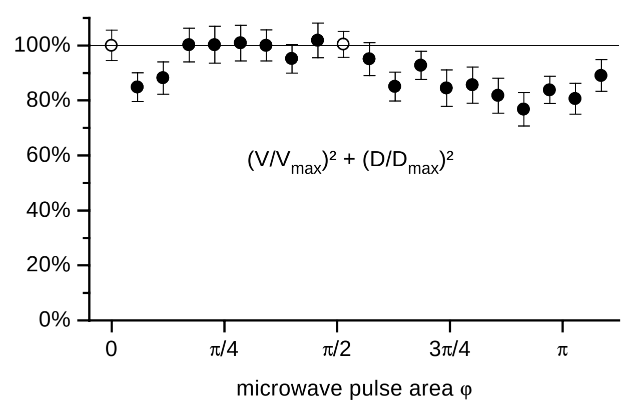


FIG. 4. Experimental test of the duality relation based on the data from Fig. 3. $[D(\varphi)/D_{\max}]^2 + [V(\varphi)/V_{\max}]^2$ is plotted as a function of $\varphi$. According to the duality relation the data points may not exceed unity.

We finally mention that a similar experiment could be performed with a light interferometer, where a half-wave plate is used to rotate the light polarization in one interferometer arm. However, such an experiment can be described in terms of classical electrodynamics, without any quantum theory. Hence, it could not be considered as a test of quantum-mechanical complementarity. In contrast to this, our experiment employs *massive* particles, whose interference phenomena can be explained only in quantum theory.

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[12] For technical reasons, it is difficult to detect only atoms in state $|2\rangle$. The use of the third microwave pulse solves this problem, because for a pulse area of $\pi$, the subsequent detection of atoms in state $|3\rangle$ corresponds to the direct detection of atoms in state $|2\rangle$.