Inadequacies of the Nonrelativistic 3N Hamiltonian in Describing the $n + d$ Total Cross Section

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New high-precision measurements of the neutron total cross section for hydrogen and the deuterium-hydrogen cross section difference were performed for neutron energies between 7 and 600 MeV. The results are compared with state-of-the-art Faddeev calculations of the neutron-deuterium system up to 300 MeV. Above 100 MeV, this comparison reveals significant limitations of the nonrelativistic 3N Hamiltonian using nucleon-nucleon forces only. [S0031-9007(98)06526-0]

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Three-nucleon scattering based on modern NN forces has matured in recent years, and computationally accurate solutions of the three-nucleon (3N) Faddeev equation can be achieved [1]. This allows the comparison of theoretical predictions with a rich set of data in elastic nd scattering and the nd breakup process, including various spin observables. Most of these data lie in the energy range below 100 MeV projectile energy. The agreement of the theoretical predictions with nearly all experimental observables is very good in this energy regime, and little room is left for the action of 3N forces [1]. The next step in testing this approach to the three-nucleon system is to carry out similar calculations at higher energies and compare their predictions with high precision data. It should be emphasized that calculations of the type discussed here use the most modern NN forces which describe the Nijmegen NN data base up to about 350 MeV with a χ^2 per datum very close to 1. It turns out that the theoretical predictions for the three nucleon observables are very stable with respect to the choice of these specific NN potentials [1]. Although these new potentials are of various types (e.g., some are local while others are nonlocal), possible off-shell differences in the NN t matrices are hardly visible in the 3N observables.

In this Letter we present new, fully converged calculations of the total nd cross section in the 10–300 MeV energy range and compare them with experimental results derived from recent high-precision measurements of the nd-np total cross section difference, reported here for the first time. Previous comparisons with nd differential cross section data above 100 MeV [1] showed that the theory underpredicts those differential cross sections at large angles. Therefore we may also anticipate discrepancies in the nd total cross section. In the present work these discrepancies are exhibited by comparing the new high-precision measurements with a fully converged 3N Faddeev calculation, and are found to be significant above approximately 100 MeV.

The new measurement of the difference of the neutron total cross sections for deuterium and hydrogen, $\sigma_{d\text{-}p}$, was undertaken as part of a new survey of total cross sections over a wide mass and energy range carried out at the LANSCE/WNR spallation source as part of the Accelerator Production of Tritium project. The measurements used a slightly modified version of the experimental setup described by Finlay *et al.* [2]. The main variation from Ref. [2] was the addition of a second 5.08-cm-thick plastic scintillator neutron detector mounted approximately 2 m behind the original 1.27-cm-thick detector. The second detector increased the efficiency of the system at the higher neutron energies and provided a useful check on systematic errors, since its count rate was approximately 3 times that of the first detector. The measurements were carried out by measuring the relative neutron transmission of 49.69-cm-long samples of light and heavy water. The samples were contained in nearly identical aluminum cylinders of 3.175 cm inside diameter, which was much larger than the neutron beam diameter of approximately 1.9 cm. Deionized water with naturally occurring abundances of hydrogen isotopes was used for the light-water sample. The heavy water sample was commercially available D_2O with enrichment greater than 99.9%. Just before filling the cans, the water samples were pumped to remove dissolved gases. All results were corrected for density variations with temperature, and the total cross section of oxygen measured in [2] was used to make a small correction for the difference in the areal densities of the oxygen nuclei between the two samples. As in Ref. [2], rapid cycling of the samples gave added confidence to the results. In addition to the difference measurement $\sigma_{d\text{-}p}$, the total cross section σ_p of hydrogen was also measured by comparison of both polyethylene

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(CH_2) and n octane (C_8H_{18}) with carbon samples of appropriate lengths. Systematic errors are estimated as 1% or less. Complete details of these measurements will be published elsewhere.

In Fig. 1 we compare our results for $\sigma_{d\text{-}p}$ in 4%-wide energy bins with those of three earlier measurements [3–5]. The range of the present measurements (7–600 MeV) spans the ranges covered by the first two of these measurements and overlaps the lower end of the range covered by the third. The results of Refs. [3] and [4] were direct measurements of $\sigma_{d\text{-}p}$, while the values shown for Ref. [5] are the differences of separate measurements of σ_d and σ_p. Additional measurements of σ_d, mainly below 100 MeV, are cited in Ref. [1]. Panel (b) of Fig. 1 shows the present and earlier data divided by a 10th order polynomial fitted to the present measurements. This is intended only to exhibit the differences among the various experiments more clearly than in panel (a) and to show the statistical error bars. The present results are in agreement with [4] but disagree in value or slope with the other two experiments. Of particular importance for the comparison with the Faddeev calculations is the disagreement with [3], which approaches 9% near 85 MeV. The present results exhibit significantly improved statistical accuracy compared with the earlier experiments; the statistical errors are less than 1% above 20 MeV.

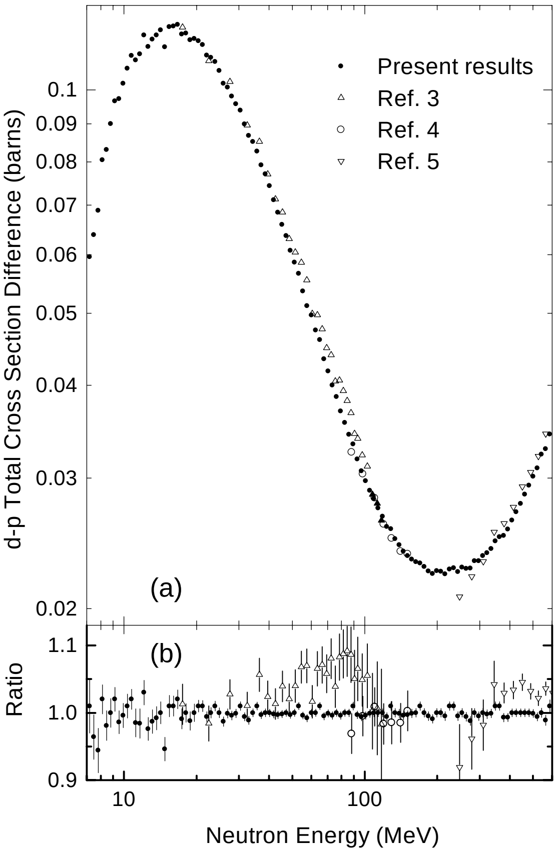

FIG. 1. Results of the present measurements of the deuterium-hydrogen total cross section difference compared with those of Refs. [3–5]. For clarity of presentation, panel (a) shows the data points without error bars. In panel (b) we present the data with error bars as ratios to a 10th-order polynomial fitted to the present measurements.

In carrying out the calculations, we use the Faddeev equations in the form

$$T|\Phi\rangle = tP|\Phi\rangle + tPG_0T|\Phi\rangle, \tag{1}$$

where T is part of the $3N$ breakup operator; see Ref. [1] for details. The iteration of this equation generates the well known multiple scattering series. In Eq. (1) t is the NN off-shell transition operator, P represents permutation operators which take into account the identity of the three nucleons, and G_0 is the free three-nucleon propagator. Finally, Φ stands for the nd channel state composed of a deuteron state and the projectile momentum state. Knowing T, one can obtain the nd forward elastic scattering amplitude by quadrature:

$$\langle\Phi|U|\Phi\rangle = \langle\Phi|PG_0^{-1}|\Phi\rangle + \langle\Phi|PT|\Phi\rangle. \tag{2}$$

Using the optical theorem for the forward scattering amplitude,

$$\mathrm{Im}\langle\Phi|U|\Phi\rangle = \frac{-1}{(2\pi)^3}\,\frac{3}{4}\,q_0\,\frac{1}{m}\,\sigma_{\mathrm{tot}}\,, \tag{3}$$

we determine the nd total cross section. Here m is the nucleon mass and q_0 the asymptotic relative projectile momentum with respect to the deuteron. Details of the numerical calculations will be presented in a forthcoming article.

In this study we use the CD-Bonn NN potential [6]. Our experience below 100 MeV [1] is that the predictions of all modern phase-equivalent NN potentials lead to nearly identical results for the $n + d$ total cross section. Moreover, the predictions of nearly all 3-nucleon scattering observables using these various potentials are in excellent agreement. Thus the entire picture of 3-nucleon reactions using a nonrelativistic Hamiltonian with two nucleon forces only is extremely stable when different phase-equivalent potentials are considered. We expect this to be true at higher energies also. Indeed, at two energies (140 and 200 MeV) we have calculated the $n + d$ total cross section using other modern NN potentials (Nijm I and Nijm II [7], and AV18 [8]) and found the same results as for CD-Bonn within less than 1%. A complete survey for all energies will be included in a more extensive publication.

In Fig. 2 we compare theory and data for the nd total cross section σ_d as well as the difference of the cross sections for nd and np scattering $\sigma_{d\text{-}p}$. The data for σ_d were obtained by adding the measurements of $\sigma_{d\text{-}p}$ and σ_p described above. The figure also shows the data for σ_p compared with the values calculated from CD-Bonn. We see that for both σ_d and $\sigma_{d\text{-}p}$ the theoretical calculations fall below the data at higher energies. For

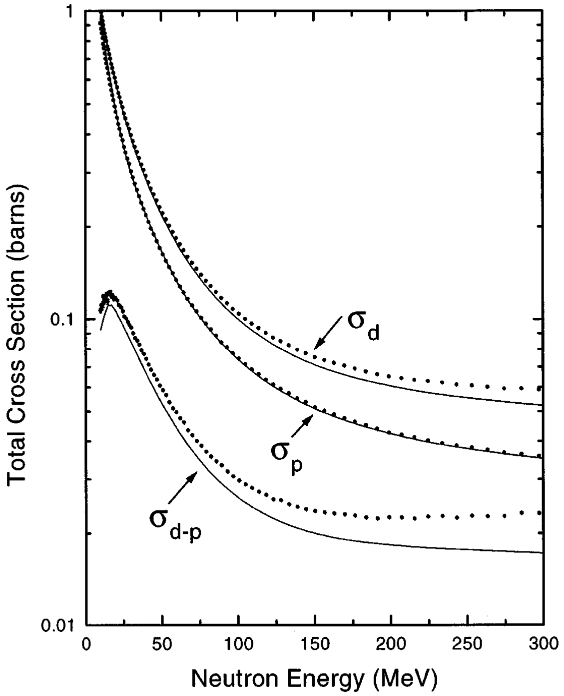

FIG. 2. Comparison of the results of the present measurements (dots) with the calculations (solid lines) described in the text. Total cross sections for deuterium σ_d were obtained by adding the separately measured values of the hydrogen cross section σ_p and the deuterium-hydrogen cross section difference $\sigma_{d\text{-}p}$.

σ_d the calculation begins to underestimate the data around 100 MeV by about 4%. This discrepancy increases to about 11% at $E_{\mathrm{lab}} = 300$ MeV. In the case of the cross section difference $\sigma_{d\text{-}p}$, the discrepancy is magnified by the simple effect that the calculated $\sigma_{d\text{-}p}$ is small compared to the magnitudes of the values being subtracted.

The newly measured *np* total cross section data shown in Fig. 2 are in excellent agreement (at the 1% level) with the CD-Bonn potential, which has been tuned to the overall set of *NN* data. While the new data for σ_p were chosen to construct the experimental value of σ_d from $\sigma_{d\text{-}p}$, other representations of σ_p, such as the experimental data of Lisowski *et al.* [9] or the values derived from CD-Bonn and the other modern phase equivalent *NN* potentials quoted above, would serve as well for the present purpose. The σ_p predictions for the four potentials agree to better than 1%. As a consequence, the variations in σ_d resulting from these different determinations of σ_p are small compared to the discrepancies between the experimental and calculated σ_d.

The question remains as to the origin of the discrepancy, which increases with energy, in the predicted and measured *nd* total cross sections. The calculations were not taken beyond 300 MeV because there is no feature in the *NN* potential model to account for pion production, which is clearly evident in the data at higher energies (see, e.g., the rise at high energies in Fig. 1). In the energy regime between 100 and 300 MeV there are two obvious possibilities for the discrepancy between the calculations and experimental *nd* data. The first is the neglect of relativity in the theoretical calculations and the second the neglect of 3*N* forces. Preliminary calculations using only a small number of partial waves and employing a two-pion exchange 3*N* force model [10] did not lead to effects larger than 1% in the $n + d$ total cross section. This conclusion might still change if a sufficient number of partial waves is included or if different types of three-nucleon forces are used. However, we believe that the more likely picture emerging here is that the onset of relativistic effects is seen. Indeed, we find that a simple inclusion of relativistic kinematics in the optical theorem [Eq. (3)] leads to a sizable change in the calculated cross sections. This result is based on calculating the current density relativistically, which results in a change of the kinematical factor in Eq. (3), while leaving the forward scattering amplitude unchanged. This substitution shifts the calculation toward the data by approximately 3% at 100 MeV and 8% at 300 MeV, which is a large fraction of the observed discrepancy. We do not suggest that this is the complete solution to the problem, since the forward scattering amplitude is still calculated entirely nonrelativistically; we rather take this quite large effect as an indication that relativistic effects are essential in a theoretical description. This calls for a strong effort to develop a relativistic framework for 3*N* scattering, which is also badly needed in the context of electron scattering from ^{3}He at high energy and momentum transfers.

In summary, we have performed for the first time fully converged 3*N* Faddeev calculations above 100 MeV projectile energy. New measurements of the *nd*-*np* total cross section difference and the *np* total cross section have been performed with sufficient accuracy to test the calculations stringently. The comparison of the theoretical calculations and experimental observables exhibits a discrepancy with respect to the *nd* data that starts around 100 MeV with a few percent and reaches about 10% at 300 MeV. Since the theoretical predictions are not strongly dependent on the choice among the most recent phase-equivalent forces, the discrepancies uncovered by the present work call for new ingredients in the theory. We feel that the onset of relativistic effects is a strong candidate for the resolution of this problem.

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