Teleportation with Bright Squeezed Light

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We examine the teleportation of continuous quantum variables from a small signal communication point of view. We show that mixed bright squeezed beams can provide the entanglement required for teleportation. Specific experimental criteria for teleportation of bright beams in terms of the information transfer and state reconstruction are proposed. [S0031-9007(98)07946-0]

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The uncertainty principle prevents the simultaneous, precise measurement of the conjugate variables of a quantum state. This would seem to preclude the possibility of sending sufficient information classically to completely reconstruct a measured quantum state. However, in a remarkable discovery by Bennett *et al.* [1] it was found that the unknown state of a spin-$1/2$ particle could be "teleported" to a remote station through the transmission of classical information, provided the sender and receiver share an Einstein-Podolsky-Rosen-type (EPR) entangled quantum pair [2]. Experimental realizations [3] have been restricted by the low efficiency inherent in photon counting experiments.

Recent developments by Vaidman [4] and Braunstein and Kimble [5] have proposed the possibility of teleportation of continuous quantum variables, such as the quadrature amplitudes of the electromagnetic field. This enables high efficiency homodyne detection techniques to be used. As a concrete example they discussed the teleportation of an optical state using parametric down-conversion as an EPR source [6].

In this paper we consider a similar arrangement but show that two bright squeezed sources can be used to produce the required EPR state. The significance of this is threefold: (i) It illustrates that EPR state twin beams can be produced from individual squeezed inputs. This is of practical as well as general interest as compact and reliable, bright squeezed sources (e.g., pump suppressed diode lasers [7]) appear feasible in the short term; (ii) as all beams are "bright," it provides additional degrees of freedom in the experimental setup. This is an improvement to using parametric down-conversion as the EPR source since the necessary quantum correlations for teleportation exist only near threshold [8]; and (iii) it highlights the physics by enabling a direct analogy with electro-optic feedforward [9,10] to be drawn.

We analyze the setup from a small signal, quantum optical communications point of view. Success is measured by the precision with which the spectral variances of the conjugate input variables (intensity and phase) can be reconstructed on the teleported output. Specific experimental criteria for teleportation of bright beams are proposed for the first time.

Let us first examine what we wish to achieve and why this is unallowed if we use only a classical communication channel. In doing so we will propose criteria for deciding if quantum teleportation has been achieved in an experimental situation. We consider a minimum uncertainty state perturbed by small signals as our input. In analogy with the quantum nondemolition (QND) measurement criteria [11] we examine the classical limits to information transfer and state reconstruction and define teleportation as occurring when both exceed the classical limits.

In Fig. 1 we show the idea schematically. An input light beam is detected and the information collected is sent to a remote station. There the information is used to try to reconstruct the state of the original beam. The input field can be written in the form

$$\hat{A}_{\rm in}(t) = A_{\rm in} + \delta\hat{A}_{\rm in}(t), \tag{1}$$

where $\hat{A}_{\rm in}$ is the field annihilation operator; $A_{\rm in}$ is the classical, steady state, coherent amplitude of the field (taken to be real); and $\delta\hat{A}_{\rm in}$ is a zero-mean operator which carries all the classical and quantum fluctuations. For bright beams the amplitude noise spectrum is given by

$$V_{\rm in}^{+}(\omega) = \langle|\delta A_{\rm in}(\omega) + \delta A_{\rm in}^{\dagger}(\omega)|^2\rangle = \langle|\delta X_{\rm in}^{+}(\omega)|^2\rangle, \tag{2}$$

where the absence of hats indicates Fourier transforms have been taken. Similarly, the phase noise spectrum is given by

$$V_{\rm in}^{-}(\omega) = \langle|\delta A_{\rm in}(\omega) - \delta A_{\rm in}^{\dagger}(\omega)|^2\rangle = \langle|\delta X_{\rm in}^{-}(\omega)|^2\rangle. \tag{3}$$

We can write the input light amplitude noise spectrum as $V_{\rm in}^{+} = V_s^{+} + V_n^{+}$ where V_s^{+} is the signal power and V_n^{+}

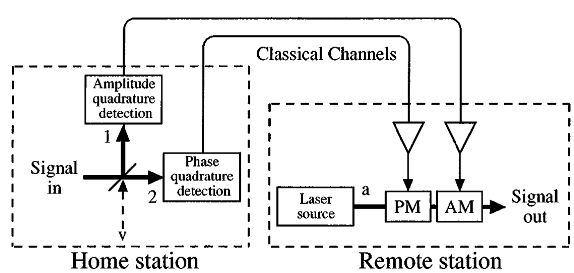

FIG. 1. Schematic of classical teleportation arrangement.

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is the quantum noise power. Similarly, the phase noise spectrum can be written $V_{\rm in}^{-} = V_s^{-} + V_n^{-}$. Suppose the input light is split into two equal halves with a beam splitter (see Fig. 1). The amplitude spectrum is detected in one arm and the phase spectrum is detected in the other, leading to the following spectra: $V_1^{+} = V_{\rm in}^{+}/2 + V_v^{+}/2$ and $V_2^{-} = V_{\rm in}^{-}/2 + V_v^{-}/2$. As the amplitude and phase quadratures are conjugate observables, it is not possible to obtain perfect knowledge of both simultaneously. This is ensured by the noise penalties, V_v^{+} and V_v^{-}, introduced by the beam splitter. For the case of only vacuum entering at the empty port of the beam splitter $V_v^{+} = V_v^{-} = 1$. The measurement penalty may be reduced for one quadrature by introducing squeezed vacuum into the empty port such that either $V_v^{+} < 1 < V_v^{-}$ or $V_v^{+} > 1 > V_v^{-}$, but any improvement in the measurement of one quadrature necessarily leads to a degradation of the measurement of the other. To quantify this we consider the transfer coefficients of the two quadratures defined by $T^{+} = {\rm SNR}_1^{+}/{\rm SNR}_{\rm in}^{+}$ for the amplitude quadrature and $T^{-} = {\rm SNR}_2^{-}/{\rm SNR}_{\rm in}^{-}$ for the phase quadrature. Here SNR stands for the signal-to-noise ratios of the input quadratures, in, and the detected fields, 1, 2. We find quite generally

$$T^{+} + T^{-} = \frac{V_n^{+}}{V_n^{+} + V_v^{+}} + \frac{V_n^{-}}{V_n^{-} + V_v^{-}} = 1 + \frac{V_n^{+}V_n^{-} - V_v^{+}V_v^{-}}{V_n^{+}V_n^{-} + V_n^{+}V_v^{-} + V_v^{+}V_n^{-} + V_v^{+}V_v^{-}} \,. \tag{4}$$

We wish to derive a quantum limit so we assume our input beam is in a minimum uncertainty state ($V_n^{+}V_n^{-} = 1$). Also using the uncertainty relation ($V_v^{+}V_v^{-} \geq 1$) we find

$$T^{+} + T^{-} \leq 1 \tag{5}$$

for any simultaneous measurement of both quadratures. This places an absolute upper limit on the quantum information that can possibly be transmitted through the classical channel.

The information arriving at the receiver is imposed on an independent beam of light using amplitude and phase modulators. We now wish to consider how well this can be achieved. The problem is that the light beam at the receiver must carry its own quantum noise. For small signals the action of the modulators can be considered additive, and we will assume that they are ideal in the sense that loss is negligible and the phase modulator produces pure phase modulation and similarly for the amplitude modulator. The output field is given by

$$\hat{A}_{\rm out} = \hat{A}_a + \delta\hat{R}_{+} + i\delta\hat{R}_{-} \,. \tag{6}$$

The fluctuations imposed by the modulators can be written as the following convolutions over time [12]:

$$\delta\hat{R}_{\pm} = \int_0^t k_{\pm}(\tau)\,\frac{1}{2}\,A_{\rm in}[\delta\hat{X}_{\rm in}^{\pm}(t-\tau) + \delta\hat{X}_v^{\pm}(t-\tau)]d\tau \,, \tag{7}$$

where k_{+} and k_{-} describe the action of the electronics in the amplitude and phase channels, respectively. The amplitude and phase quadrature fluctuations of the receiver beam are represented by $\delta\hat{X}_a^{+}$ and $\delta\hat{X}_a^{-}$, respectively. The quadrature noise spectra of the output field are

$$V_{\rm out}^{\pm} = V_a^{\pm} + |\lambda_{\pm}(\omega)|^2(V_{\rm in}^{\pm} + V_v^{\pm}) \,, \tag{8}$$

where various parameters have been rolled into the electronic gains, $\lambda_{\pm}$, which are proportional to the Fourier transforms of $k_{\pm}$. By making both $|\lambda_{\pm}|^2 \gg 1$ the signal transfer coefficient for the output, $T_s^{\pm} = {\rm SNR}_{\rm out}^{\pm}/{\rm SNR}_{\rm in}^{\pm}$, can satisfy the equality in Eq. (5), thus realizing the maximum allowable information transfer. However, then the output beam would be much noisier than the input beam and hence a very dissimilar state. A measure of the similarity of the input and output beams is given by the amplitude and phase conditional variances [13],

$$V_{\rm cv}^{\pm} = V_{\rm out}^{\pm} - \frac{|\langle\delta X_{\rm in}^{\pm}\delta X_{\rm out}^{\pm}\rangle|^2}{V_{\rm in}^{\pm}} \,. \tag{9}$$

If $V_{\rm cv}^{+} + V_{\rm cv}^{-} = 0$ then the input and output are maximally correlated. For our system we find

$$V_{\rm cv}^{+} + V_{\rm cv}^{-} = V_a^{+} + V_a^{-} + |\lambda_{+}|^2V_v^{+} + |\lambda_{-}|^2V_v^{-} \,. \tag{10}$$

Once again any attempt to suppress the noise penalty in one quadrature, say, by squeezing the receiver beam, results in a greater penalty in the other quadrature. The best result is obtained for $\lambda_{+} = \lambda_{-} = 0$ and a coherent receiver beam giving

$$V_{\rm cv}^{+} + V_{\rm cv}^{-} \geq 2 \,. \tag{11}$$

That is, the best correlation between the states is achieved by not transferring any information. This rather strange result occurs because we have already optimized the correlation between input and output by choosing a coherent receiver beam. Any attempt to transfer signal information inevitably adds additional uncorrelated noise to the output which degrades the correlation. In principle, one could measure $V_{\rm cv}^{+}$ directly by performing a perfect QND measurement of the amplitude quadrature of the input field and electronically subtracting it from an amplitude quadrature measurement of the output field. In a similar way, $V_{\rm cv}^{-}$ could in principle be measured using a perfect QND measurement of the phase quadrature of the input field. Clearly this is impractical. However, the correlations can be inferred quite easily from individual measurements of the transfer coefficients and the absolute noise levels of the output field via

$$V_{\rm cv}^{\pm} = (1 - T_s^{\pm})V_{\rm out}^{\pm} \,. \tag{12}$$

These results are summarized for a coherent state input in Fig. 2 where $T_s^{+} + T_s^{-}$ versus $V_{\rm cv}^{+} + V_{\rm cv}^{-}$ are plotted as a function of increasing $|\lambda_{\pm}|$. The dotted lines represent the limits set by purely classical transmission. For clarity we have considered only a symmetric scheme, i.e., one which

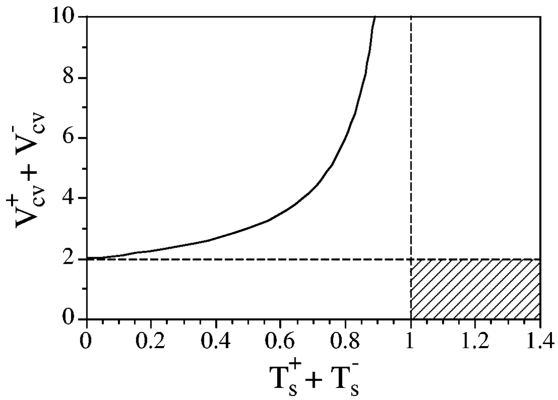

FIG. 2. Performance of classical teleportation arrangement with a coherent input. Information transfer ($T_s^+ + T_s^-$) is plotted versus state reconstruction ($V_{\text{cv}}^+ + V_{\text{cv}}^-$) for $\lambda_+ = -\lambda_-$ running from 0 to 2.0. Dashed lines indicate the classical limits.

detects and transmits information about both quadratures equally. Asymmetric detection and transmission allows access to the region between the curve and the dotted lines in Fig. 2. For example, with detection and transmission of only one quadrature of a coherent state the gain curve will be the line segment $V_{\text{cv}}^+ + V_{\text{cv}}^- = 2$, $0 \leq T_s^+ + T_s^- \geq 1$. Also, the region between the curve and the dotted lines can be accessed in a symmetric scheme with a squeezed input state. However, for no detection-transmission scheme or input state can one go below $V_{\text{cv}}^+ + V_{\text{cv}}^- = 2$ or (for a minimum uncertainty state) above $T_s^+ + T_s^- = 1$.

We now consider the electro-optical arrangement that is shown in Fig. 3. It is similar to that proposed by Braunstein and Kimble in Ref. [5]. However, in contrast to Ref. [5], we have replaced the parametric down converter with two coherently related amplitude squeezed sources which are mixed on a 50:50 beam splitter (BS1). One source is of much lower intensity than the other, and they are combined with a $\pi/2$ phase shift. One of the beams is sent to where we wish to measure the input signal. There it is mixed with the input signal beam (which is of similar intensity) on another 50:50 beam splitter (BS2). We combine them in phase such that there are bright and "dark" outputs. The bright beam is directly detected to obtain its amplitude quadrature. The dark beam is mixed with a local oscillator (LO) and homodyne detection is used to measure its phase quadrature (represented schematically in Fig. 3). The photocurrents thus obtained are sent to amplitude and phase modulators situated in the other beam coming from the mixed squeezed sources.

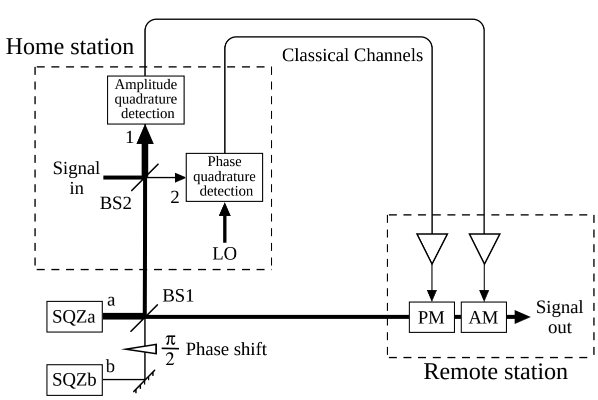

FIG. 3. Schematic of quantum teleportation arrangement. SQZa and SQZb are coherently related squeezed sources with the intensity of a much greater than that of b. The signal input and local oscillator (LO) must also be coherently related to the squeezed sources. BS1 and BS2 are 50:50 beam splitters.

Following the approach of Ref. [10], the amplitude and phase noise spectra of the output field are found to be

$$V_{\text{out}}^{\pm} = \left| \frac{1}{\sqrt{2}} + \frac{1}{2}\lambda_{\pm} \right|^2 V_a^{\pm} + \left| \frac{1}{\sqrt{2}} - \frac{1}{2}\lambda_{\pm} \right|^2 V_b^{\mp} + \left| \frac{1}{\sqrt{2}}\lambda_{\pm} \right|^2 V_{\text{in}}^{\pm}. \tag{13}$$

Here the amplitude (phase) spectra of beams a and b are given by V_a^+ (V_a^-) and V_b^+ (V_b^-), respectively. The cross coupling of the phase spectrum of the weak beam, b, into the amplitude spectrum of the output is due to the $\pi/2$ phase shift. Consider first the situation if beams b and the signal are blocked so that just vacuum enters the empty ports of the beam splitters. The setup is then just a feedforward loop. Lam *et al.* [10] have shown that the measurement penalty at the feedforward beam splitter (BS1) can be completely canceled by correct choice of the electronic gain, allowing noiseless amplification of V_a^+ to be achieved. This cancellation can be seen from Eq. (13) with the electronic gain set to $\lambda_+ = \sqrt{2}$. The remaining penalty is due to the in-loop beam splitter (BS2) which, here, is allowing us to detect both quadratures. But now suppose we inject our signal into the empty port of the in-loop beam splitter. With $\lambda_+ = \sqrt{2}$ we find Eq. (13) reduces to

$$V_{\text{out}}^+ = 2V_a^+ + V_{\text{in}}^+, \tag{14}$$

and if beam a is strongly amplitude squeezed such that $V_a^+ \ll 1$ then

$$V_{\text{out}}^+ \simeq V_{\text{in}}^+. \tag{15}$$

Now consider the phase noise spectrum, Eq. (13). If we impose the same electronic gain condition on the feedforward phase signal as we have for the amplitude signal we will get an output spectrum

$$V_{\text{out}}^- = 2V_a^- + V_{\text{in}}^-. \tag{16}$$

If beam a is strongly amplitude squeezed then the uncertainty principle requires $V_a^- \gg 1$ so this is not a useful arrangement. However, if we perform negative rather than positive feedforward on our phase signal such that $\lambda_- = -\sqrt{2}$ then we will cancel the phase noise of beam

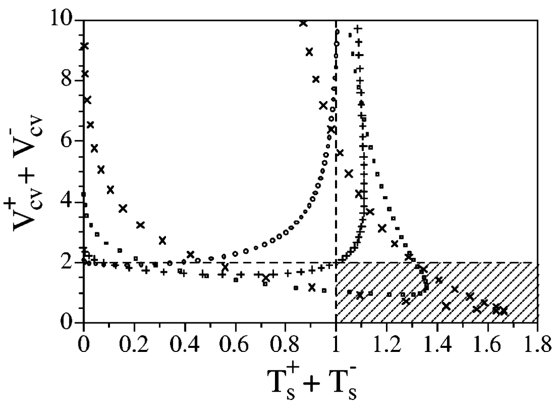

FIG. 4. Performance of quantum teleportation arrangement with a coherent input. Information transfer ($T_s^+ + T_s^-$) is plotted versus state reconstruction ($V_{cv}^+ + V_{cv}^-$) for $\lambda_+ = -\lambda_-$ starting from 0 with increments of 0.05. Circles, pluses, squares, and crosses are for 25%, 50%, 75%, and 90% squeezing from both sources, respectively. Dashed lines indicate the classical limits, and the shaded region is the region of successful quantum teleportation.

a and instead see the vacuum noise entering at the empty port of the feedforward beam splitter. Finally, by injecting our low intensity beam b at this port we find

$$V_{\text{out}}^- = 2V_b^+ + V_{\text{in}}^- . \tag{17}$$

Beam b can be made strongly amplitude squeezed without affecting Eq. (15) thus giving us

$$V_{\text{out}}^- \simeq V_{\text{in}}^- . \tag{18}$$

Hence we have the remarkable result that we can satisfy both Eqs. (15) and (18) simultaneously even though the only direct connection between the input and output fields is classical, i.e., teleportation of our input field. More generally, the spectral variance at some arbitrary quadrature phase angle (θ) is given by

$$\begin{aligned} V_{\text{out}}^\theta &= \langle |\delta A_{\text{out}}^\dagger e^{+i\theta} + \delta A_{\text{out}} e^{-i\theta}|^2 \rangle \\ &= V_{\text{in}}^\theta + 2\cos^2\theta V_a^+ + 2\sin^2\theta V_b^+ . \end{aligned} \tag{19}$$

This form makes it clear that, provided beam a and beam b are both strongly amplitude squeezed, the input and output spectral variances will be approximately equal for any arbitrary quadrature angle (not just amplitude and phase). Note that, as for other teleportation schemes, no quantum limited information about the input field can be obtained from the classical channels. This is because it is "buried" by the large antisqueezed fluctuations that are mixed with the input beam at the measurement site. The strong EPR correlations carried by the quantum channel enable this quantum information to be retrieved on the receiver beam.

Experimental conditions will in general be nonideal. We propose defining teleportation as having been achieved unconditionally when both the correlation and the information transfer have exceeded the classical limits (i.e., $T_s^+ + T_s^- \geq 1$ *and* $V_{\text{cv}}^+ + V_{\text{cv}}^- \leq 2$) at some rf detection frequency. In Fig. 4 we plot $T_s^+ + T_s^-$ versus $V_{\text{cv}}^+ + V_{\text{cv}}^-$ for a coherent input as a function of feedforward gain for various values of squeezing. Notice that although moderate values of squeezing allow either information transfer or state reconstruction to be superior to the classical channel limit, squeezing must be greater than 50% before both conditions can be met simultaneously and hence unconditional teleportation achieved. This limit remains valid for asymmetric detection-transmission or arbitrary minimum uncertainty input states [14].

In summary, we have shown that the EPR type correlation needed to produce teleportation of continuous variables can be established using two bright squeezed sources. We have analyzed the setup from a small signal quantum optical point of view. We have established criteria for deciding if teleportation has been achieved and have shown that the mechanism can be understood in terms of a special sort of feedforward loop.

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