Controlling Turbulence in the Complex Ginzburg-Landau Equation

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(Received 5 May 1998)

Controlling turbulence in the complex Ginzburg-Landau equation (CGLE) is investigated. The CGLE is generalized to include gradient force. Local injections (pinnings) are applied for turbulence control. It is found that local injections are effective in eliminating turbulence. In particular, for large gradient force, it is possible to suppress fully developed turbulence by adding a few injections much less in number than the number of positive Lyapunov exponents of the system. The high efficiency of controlling is heuristically explained, based on the spatial correlation length and space-time-variable transformation. [S0031-9007(98)07967-8]

PACS numbers: 47.27.Rc, 05.45.+b, 47.27.Eq

The complex Ginzburg-Landau equation (CGLE)

$$\partial_t A = A + (1 + ic_1)\partial_x^2 A - (1 + ic_2)|A|^2 A \tag{1}$$

has been extensively investigated for the problems of pattern formations and transitions to chaos and turbulence [1–3]. On the other hand, chaos control and synchronization have attracted much attention in recent years [4–14]. In this Letter, we will study turbulence control by injecting external signals in the CGLE system. For simplicity, we consider only one-dimensional (1D) CGLE with the periodic boundary condition $A(x + L, t) = A(x, t)$. An extension to more general cases will be briefly discussed at the end of the Letter.

In Eq. (1) the spatial coupling appears to be diffusive and dispersive. This system can be generalized to include a gradient coupling (bias)

$$\partial_t A = A + r\partial_x A + (1 + ic_1)\partial_x^2 A - (1 + ic_2)|A|^2 A\,. \tag{2}$$

It is emphasized that the gradient force is of practical importance. On one hand, this force is common in nature; it appears in hydrodynamic flows in sloping channels, in plasma systems with electromagnetic fields, and so on. And it has attracted central attention in the study of open-flow systems. On the other hand, this force introduces nontrivial new features in the system dynamics (e.g., convective instabilities), which have been extensively investigated for coupled-map-lattice systems and coupled oscillators [15,16]. In this Letter, we will study in detail the influence of gradient coupling on the effect of turbulence control of the CGLE.

Without injecting signals, Eq. (2) is identical to Eq. (1) for the space-time transformation

$$y = x + r\tau\,, \qquad \tau = t \tag{3}$$

[i.e., the dynamics of Eq. (2) in the moving frame (y, τ) is exactly the same as that of Eq. (1) for the static frame (x, t)]. It is an easy matter to show that Eq. (2) possesses uniform traveling wave solutions

$$A(x, t) = A_0 \exp[i(kx - \omega t)]\,,$$
$$A_0 = \sqrt{1 - k^2}\,, \qquad \omega = c_2 + (c_1 - c_2)k^2 - rk\,, \tag{4}$$

which are stable in the wave number region

$$k^2 < k_c^2 = \frac{1 + c_1 c_2}{3 + c_1 c_2 + 2c_2^2} \quad \text{if } 1 + c_1 c_2 > 0\,.$$

For $1 + c_1 c_2 < 0$, all plane waves (4) are unstable, and turbulence appears. According to the different natures of turbulence, one can classify phase turbulence and defect turbulence in different parameter regions [for the detail, see Refs. [1–3]; the existence of the constant gradient force in (2) does not change the distributions of various regions for regular and turbulent patterns of (1)].

Assume that the CGLE (2) is in a turbulence region, and now our central task is to perform turbulence control by making local feedback injections (pinnings) at a few points in x space in Eq. (2), and driving the whole system from the turbulence state to certain regular target state. Therefore, we modify Eq. (2) to

$$\partial_t A = A + r\partial_x A + (1 + ic_1)\partial_x^2 A - (1 + ic_2)|A|^2 A + \varepsilon \sum_{i=1}^{N} \delta(x - x_i)[\overline{A}(x, t) - A]\,, \tag{5}$$

where $\overline{A}(x, t)$ is our target state which has been chosen as one of the traveling wave solutions (4). It can be easily accepted that for sufficiently large ε we can effectively make $A(x_i, t)$ approach $\overline{A}(x_i, t)$. Therefore, for facilitating our numerical algorithm we simply represent the control (5) by identifying $A(x_i, t) = \overline{A}(x_i, t)$, $i = 1, 2, \ldots, N$. Actually, this kind of pinning control is equivalent to

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boundary driving control that can be conveniently applied in practical situations. Note that the symmetry of Eq. (2) with respect to the transformation (3) no longer exists in Eq. (5), where the injection term definitely breaks this symmetry. An then, changing r may essentially change the topological structure of the solution of Eq. (5). We start from arbitrary initial conditions, and the global behavior of the system evolution can be solved only numerically. In the following simulations we use the hopscotch and classical central finite difference approach [17,18]. The validity of the results is confirmed by reducing the time and space grid lengths.

First, let us study the Lyapunov spectrum of Eq. (1) in different parameter regions by applying the standard approach extensively used for discrete systems [19], which is of great importance for the system dynamics. In Fig. 1(a) we fix $c_1 = 2.1$ and plot the largest Lyapunov exponent λ_m of Eq. (1) vs c_2 for $L = 64$ and 256. Arbitrary initial conditions are used for the plots. In the stable traveling wave region ($c_2 > -0.48$), we have $\lambda_m = 0$. In the turbulence regions ($c_2 < -0.48$) λ_m increases to positive value. In Fig. 1(b), we plot the number of positive Lyapunov exponents M vs c_2 for the same parameter conditions as 1(a). We have $M = 0$ in the laminar phase region, and M becomes nonzero in the phase turbulence region and then increases rapidly when c_2 decreases. It is emphasized that for Eq. (2) we can get Fig. 1 for arbitrary r since both Eqs. (1) and (2) are equivalent with the transformation (3). Moreover, in our numerical simulations, Fig. 1 is not affected by changing the time and space grid lengths when these grids are sufficiently small.

With Fig. 1 in mind we now start to make pinnings, and to drive the system to a target state chosen from the traveling wave solutions (4). In Figs. 2(a), 2(b), and 2(c), we consider three parameter combinations in the laminar phase, phase turbulence, and defect turbulence regions, respectively, and plot d vs k for different r, where d is the largest distance of two adjacent pinnings for successful control and k is the wave number of the target traveling wave state. Successful control is regarded to be achieved if the following condition can be satisfied and maintained for time period T, $T \geq 400$.

$$|\mathrm{Re}[A(x,t) - \overline{A}(x,t)]| + |\mathrm{Im}[A(x,t) - \overline{A}(x,t)]| < 0.2\,.$$

In Fig. 3, various time-space evolutions of the system states are shown. The following interesting features are observed in Figs. 2 and 3.

(i) Pinning control is effective for eliminating turbulence. The control efficiency depends sensitively on the gradient force. It is really striking that with relatively large bias r, the very wild turbulence with a huge number of positive Lyapunov exponents M can be successfully controlled by applying local pinnings of which the number N (proportional to L/d) can be very much smaller than M [in Fig. 3(d), we have $M/N \approx 80$].

(ii) The effect of control depends not only on the bias, but also on the reference state $\overline{A}(x,t)$. Therefore, choosing a proper target state is also important for enhancing control efficiency.

(iii) In Fig. 3(d), we apply only a time-dependent driving on a single space point which does not contain any spatial information. It is interesting that the system responds to this purely time-varying injection with a spatially well ordered wave. Then, spatial order can arise from a time coherent injection, and one has the option of realizing a variety of spatial patterns by varying the injecting timing only.

(iv) Feedback injections can be applied not only for eliminating turbulence, but also for migrating spatiotemporal patterns [Fig. 2(a)]. In the stable region there are many attracting traveling wave solutions [Eq. (4) for $k^2 < k_c^2$], and each has its own basin of attraction. From an arbitrary initial condition one does not know which solution can be asymptotically approached. If we have an ensemble of identical CGLE's it is difficult to realize

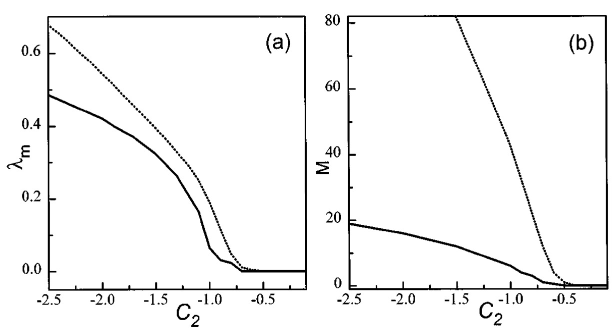

FIG. 1. (a) The largest Lyapunov exponents λ_m vs c_2 for Eq. (1). $c_1 = 2.1$; $L = 64$ (solid line) and $L = 256$ (dotted line), respectively. (b) The number of positive Lyapunov exponents M vs c_2. The same parameters as (a).

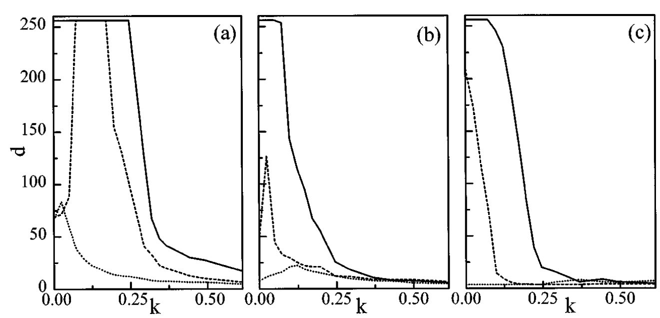

FIG. 2. (a)–(c): d plotted against k for different r for Eq. (5), with d being the largest distance between two adjacent pinnings for successful control, and k the wave number of the target traveling wave solution of (4). $L = 256$. (a) $c_1 = 2.1$, $c_2 = -0.1$, stable region. (b) $c_1 = 2.1$, $c_2 = -0.7$, phase turbulence region. Dotted lines: $r = 0$; dashed lines: $r = 1$; solid lines: $r = 2$; for both (a) and (b). (c) $c_1 = 2.1$, $c_2 = -1.5$, defect turbulence region. The number of positive Lyapunov exponents without control is $M = 82$. Dotted line: $r = 0$; dashed line: $r = 2.0$; solid line: $r = 2.5$.

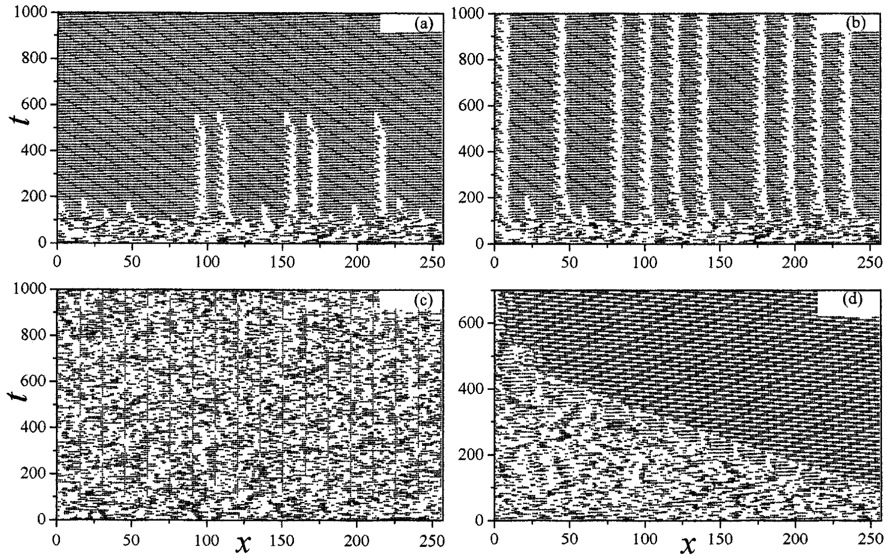

FIG. 3. Spatiotemporal evolutions of CGLE under control. Black regions correspond to $\mathrm{Re}(A) > 0.6$, white regions otherwise. $L = 256$, $c_1 = 2.1$, $c_2 = -1.5$, $k = 0.031\,25\pi$. (a), (b): $r = 2$, $d = 15$ and 17, respectively. $d = 15$ is the maximal d for successful control for the given parameters. (c) $r = 0$; $d = 15$; fully developed turbulence is practically not affected. (d) $r = 2.6$; $d = 256$ (only one point is injected). A time-dependent injection develops a regular spatiotemporal pattern, which eats the original wild fully developed turbulence.

synchronization between different CGLE sets. With proper pinnings one can eliminate multibasin structure and select a specific basin of attraction, and then realize synchronization of identical CGLE systems by driving the systems to the wanted state; that may again be very useful in practice.

In order to have a complete idea on how the degree of turbulence and the strength of the bias influence the controllability, we investigate the effect of pinnings by varying various system parameters. In Fig. 4(a) [4(b)] we plot d_m vs $c_2(r)$ for different $r(c_2)$ by fixing $c_1 = 2.1$, and $L = 256$, where d_m is the maximum d with respect to all k (i.e., the peak height of Fig. 2) for the given parameters. In all cases one finds that increasing r can very sensitively enhance the control efficiency (i.e., increase d_m).

The most significant observation of this Letter is the strikingly high efficiency of pinning control to suppress turbulence in the presence of bias. An intuitive understanding on the mechanism underlying this efficiency is very useful. The essences of local feedback control are, first, to drive the pinned points (and their vicinities, of course) to the local values of the target state, and then, to bring the entire system to this state through spatial coupling. Therefore, it is acceptable that for successful control the distance between two adjacent pinnings should not exceed the correlation distance of the system; that restricts the minimal pinning density of successful control. Without bias ($r = 0$), the correlation distance of the system reduces quickly as the system goes deeply into the defect (fully developed) turbulence region with a large number of positive Lyapunov exponents. Applying bias does not affect the intensity of turbulence (i.e., does not affect all the values of positive Lyapunov exponents); however, it builds up convective correlation, and considerably enlarges the spatial correlation length. In other words, for Eq. (2) the correlation length in y space ($y = x + r\tau$) is equal to that of Eq. (1) in x space, and then is very small, but the correlation between two x space points of Eq. (2), a large distance apart, may be large with shifted times due to the convective interaction; that is the key point for the high efficiency of turbulence control of the biased CGLE.

The above explanation can be described in another way. With the transformation (3) we can transform (5) to

$$\partial_\tau A = A + (1 + ic_1)\partial_y^2 A - (1 + ic_2)|A|^2 A + \varepsilon \sum_{i=1}^{N} \delta[y - y_i(\tau)][\overline{A}(y - r\tau, t) - A]. \quad (6)$$

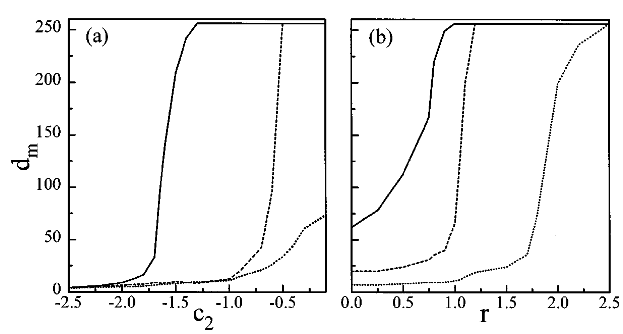

FIG. 4. $c_1 = 2.1$; $L = 256$. (a) [(b)] d_m plotted vs $c_2(r)$ for different $r(c_2)$. d_m is the maximum d with respect to all k for given parameters (the peak height in Fig. 2). (a) Dotted line: $r = 0$; dashed line: $r = 1$; solid line: $r = 2$. (b) Dotted line: $c_2 = -1.5$; dashed line: $c_2 = -0.7$; solid line: $c_2 = -0.1$.

Now in y space we no longer have bias, but the pinning points are no longer fixed; they move in the manner $y_i(\tau) = x_i + r\tau$. These moving pinnings go through the entire y space while staying in each given ΔL space segment for small time portion $N\Delta L/L$. A good match between the moving velocity, which is determined by the bias r, and the moving fashion of the target state, determined by k, produces an optimal control efficiency.

In conclusion, we emphasize that gradient force is very common in nature, and it is thus significant to reveal its influence on turbulence control. On the other hand, turbulence may appear in systems without bias. For effectively controlling this turbulence it might be possible (not always possible, of course) to intentionally apply certain bias for making control easier. For instance, it is not difficult to apply uniform electrical or magnetic fields to do it if the system consists of a charged medium. At any rate, we expect that the study in this Letter may initiate the extremely useful investigation to control fully developed turbulence with a huge number of positive Lyapunov exponents in extended systems continuous in both time and space.

In this Letter, we focus on turbulence control in the 1D CGLE system. The approach can be directly applicable to 2D systems by using line pinnings. For systems with higher dimensions it becomes more difficult as well as more interesting to design proper distribution of pinnings that is worthwhile for further investigation.

This work is supported by the National Natural Science Foundation of China, and Project of Nonlinear Science.

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