Efficient Scheme for the Deterministic Maximal Entanglement of N Trapped Ions

J. Steinbach and C. C. Gerry*

Optics Section, Blackett Laboratory, Imperial College, London SW7 2BZ, United Kingdom

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We propose a method for generating maximally entangled states of N two-level trapped ions. The method is deterministic and independent of the number of ions in the trap. It involves a controlled NOT acting simultaneously on all the ions through a dispersive interaction. We explore the potential application of our scheme for high precision frequency standards. [S0031-9007(98)08028-4]

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The entanglement of quantum states of two or more particles, aside from being of intrinsic interest, is of great practical importance in the fields of quantum cryptography and quantum computation [1]. One other area where entangled quantum states may have a significant impact is that of the improvement of frequency standards [2–4]. Advances in cooling and trapping of ions have given rise to new techniques in high precision spectroscopy which may yield frequency standards with accuracies of the order of one part in 10^{14}–10^{18} [5]. Key to the improvement of frequency standards beyond the shot-noise limit [2] is the establishment of an entangled state of a collection of N two-level atoms. Initial theoretical investigations examined the use of squeezed spin states [2,3]. We concentrate here on a *maximally* entangled N-particle state, having the form [4]

$$|\Psi_M\rangle = \frac{1}{\sqrt{2}}\{|e_1, e_2, \ldots, e_N\rangle + e^{i\phi}|g_1, g_2, \ldots, g_N\rangle\}, \tag{1}$$

where $|e_j\rangle$ and $|g_j\rangle$ denote the excited and ground states of the jth particle, respectively. Using the Dicke angular momentum states [6] this state can be written as

$$|\Psi_M\rangle = \frac{1}{\sqrt{2}}\{|J, J\rangle + e^{i\phi}|J, -J\rangle\}, \tag{2}$$

where $J = N/2$. The above state is an N-particle version of the Greenberger-Horne-Zeilinger state [7] and has been shown to display extreme quantum entanglement [8]. It may also be considered a special case of the atomic Schrödinger cat states [9]. Recently, Turchette *et al.* have reported the generation of a nonmaximally entangled two-particle state using a deterministic method in an ion trap experiment [10].

The maximally entangled state given in Eq. (2) may be used in high precision spectroscopy to measure the transition frequency $\omega_0 = (E_e - E_g)/\hbar$, where E_e and E_g are the respective energies of the electronic excited and ground states [4]. In contrast to measurements with uncorrelated atoms, which yield an uncertainty in the frequency that depends on $N^{-1/2}$, the state $|\Psi_M\rangle$ allows one to measure the transition frequency to an uncertainty of N^{-1}. Huelga *et al.* [11] have described how in principle a standard Ramsey spectroscopy scheme [12] can be modified to achieve this limit. The key to this is a controlled NOT (C-NOT) operation right after the initial and before the final Ramsey pulse, where the electronic state of the ion which is manipulated through the Ramsey pulses acts as a control to flip the electronic state of all the other ions.

As Huelga *et al.* [11] have shown, in the presence of decoherence the standard Ramsey spectroscopy measurements on uncorrelated atoms and measurements on the maximally entangled states yield the same resolution. Of course, if decoherence is not present or if the measurements can be done in a time short compared to the decoherence time, the maximally entangled states will yield higher resolution frequency measurements and thus it is of interest to find efficient mechanisms for their generation.

In view of recent experimental progress [10], perhaps the most promising physical system for the generation of the type of maximally entangled state given in Eq. (2) is a string of laser-cooled ions in a linear rf trap. Previously, Cirac and Zoller [13] have proposed a method which sequentially performs N C-NOT operations between the internal states of pairs of ions in such a string. This approach requires individually addressing all the ions with a well focused laser beam. Bollinger *et al.* [4] have proposed an alternative method that does not require interacting with the ions individually. It *does* require the use of three vibrational modes and the generation of linear couplings between pairs of those modes as electronic transitions are driven for all the ions simultaneously [14]. Both in the scheme by Cirac and Zoller [13] and the method proposed by Bollinger *et al.* [4] the number of steps or laser pulses required to generate the maximally entangled state $|\Psi_M\rangle$ is proportional to the number of ions. Only recently, Wineland *et al.* [3] have proposed for the first time a sequence of operations which accomplishes this with a fixed number of steps.

In this paper, we present a method of generating states of the form of Eq. (2), which is independent of the number of ions, and in contrast to most experiments that generate entangled states, is deterministic [15]. While our scheme is related to the proposal in [3] through the sequence of operations which leads to the maximally entangled state, it is significantly different in that (i) it does not rely on a specific value of the Lamb-Dicke parameter, (ii) it operates only on two electronic levels of

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the trapped ions, and (iii) it points out a C-NOT operation which can operate on multiple ions. Our scheme requires that *one* of the ions be addressed individually for certain manipulations and we assume here that this is done with a well focused laser [16]. We further assume that all the ions can be addressed simultaneously with a laser beam of sufficiently broad waist [10]. The maximally entangled state $|\Psi_M\rangle$ is generated through a sequence of five laser pulses, starting from an initial state $|J, -J\rangle|0\rangle$, where all the ions are in their ground state and the collective motion has been cooled to the ground state [17]. At the heart of our preparation scheme lies a C-NOT operation between the collective vibrational motion and the internal states of *all* the ions. This is generated by a type of dispersive interaction between those degrees of freedom. Previously, this kind of coupling has been discussed in connection with the generation of the vibrational Schrödinger cat states for a single trapped ion [18], and as a degenerate Raman-coupled model in the context of cavity quantum electrodynamics [19].

Before going through the preparation scheme step by step, we consider the four types of pulses involved. We require pulses at the two frequencies ω_0 and $\omega_0 - \nu_x$, where ν_x is the frequency of the ions' collective harmonic motion along the trap axis which we take to be the x axis [20]. We assume that all pulses are performed with laser beams that are derived from the same source, so that at time $t = 0$, all electric fields that excite the system during the preparation scheme are in phase. To derive the transformations caused by the various pulses we generically assume the respective laser beams to be turned on from time $t = t_0$, to $t = t_0 + t_p$, and we give the corresponding unitary transformations $U(t_0, t_p)$, in a rotating frame where $|\Psi_R(t)\rangle = \exp(i\hat{H}_R t/\hbar)|\Psi(t)\rangle$, $\hat{H}_R = \hbar\omega_0 \sum_{i=1}^{N} |e_i\rangle\langle e_i|$, and $|\Psi(t)\rangle$ is the state of the system in the Schrödinger picture. In our analysis of the dynamics generated by the laser excitation we explicitly consider only the collective motion along the trap axis, characterized in terms of the vibrational energy eigenstates $|n\rangle$. The motional state perpendicular to the trap axis remains in the ground state throughout the preparation scheme since we do not excite sidebands of the motion along those directions and in a linear trap $\nu_x \ll \nu_y, \nu_z$. We further assume both the Lamb-Dicke limit [22] and the low excitation regime [23]. The first condition allows us to expand the Hamiltonian describing the interaction of the ions with the laser light in terms of the Lamb-Dicke parameter $\eta_x = k_x \Delta x_0$, where k_x is the projection of the laser wave vector onto the trap axis, and $\Delta x_0 = (\hbar/2M\nu_x)^{1/2}$ is the width of the motional ground state for a single ion along that axis, M being the mass of a single ion. In the low excitation regime we retain only resonant transitions in the analysis of the excitation.

We first consider the action of the pulses involving only one of the ions. The first of these is a resonant $\pi/2$ pulse where a laser beam of frequency ω_0, propagating perpendicular to the trap axis excites, say, the Nth ion. This generates the transformation

$$U(t_0, t_{\pi/2}) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{2}} e^{-in\nu_x t_{\pi/2}} |n\rangle\langle n| \otimes \{|e_N\rangle\langle e_N| + |g_N\rangle\langle g_N| - |g_N\rangle\langle e_N| + |e_N\rangle\langle g_N|\}, \quad (3)$$

where the pulse duration $t_{\pi/2} = \pi/2\Omega$, and Ω denotes the Rabi frequency for that pulse. The phase factor is due to the free evolution of the vibrational degree of freedom during the pulse.

The second type of pulse required is a π pulse generated by a laser of frequency $\omega_0 - \nu_x$, and where the wave vector has a component k_x along the trap axis. This generates a $|g_N\rangle|n+1\rangle \Leftrightarrow |e_N\rangle|n\rangle$ transition of the Jaynes-Cummings type, where the Rabi frequency $\Omega_{\mathrm{JC}}^{(n)} \propto \eta_x\sqrt{n+1}/\sqrt{N}$. For a given $|g_N\rangle|n+1\rangle \Leftrightarrow |e_N\rangle|n\rangle$ transition a pulse of duration $t_{\mathrm{JC},\pi}^{(n)} = \pi/\Omega_{\mathrm{JC}}^{(n)}$, causes the transformation

$$U_{\mathrm{JC}}^{(n)}(t_0, t_{\mathrm{JC},\pi}^{(n)}) = ie^{-i\nu_x[t_0+(n+1)t_{\mathrm{JC},\pi}^{(n)}]} \times |n+1\rangle\langle n| \otimes |g_N\rangle\langle e_N| + ie^{i\nu_x[t_0-nt_{\mathrm{JC},\pi}^{(n)}]}|n\rangle\langle n+1| \otimes |e_N\rangle\langle g_N|, \quad (4)$$

where the phase factors are due to the free evolution of the vibrational degree of freedom during the pulse and the fact that we have assumed all electric fields to be in phase at time $t = 0$. We note that the state $|g_N\rangle|0\rangle$ is not coupled by the Jaynes-Cummings pulse.

The third type of pulse which we shall refer to as a dispersive π pulse is generated by two laser beams of frequency ω_0 [18]. More specifically, the first beam is propagating perpendicular to the trap axis and the second one has a wave vector component k_x along that axis. While not exciting any vibrational sidebands the pulse exploits the dependence of the generated Rabi oscillations between the states $|g_N\rangle|n\rangle$ and $|e_N\rangle|n\rangle$, on the motional excitation number n, which arises from the spatial variation of the electric field along the x axis [3]. As shown in [18], if the two laser beams have a relative phase difference of π their amplitudes can be chosen such that the (spatially) constant terms of the electric fields associated with the two laser beams cancel each other and the Rabi frequency $\Omega_{\mathrm{dis}}^{(n)} \propto \eta_x^2 n/N$, to leading order in the Lamb-Dicke parameter [24]. In deriving the pulse transformation we have assumed here the second laser beam to be phase shifted by π, with respect to the first. For a given $|g_N\rangle|n\rangle \Leftrightarrow |e_N\rangle|n\rangle$ transition a pulse of duration $t_{\mathrm{dis},\pi}^{(n)} = \pi/\Omega_{\mathrm{dis}}^{(n)}$, then generates the transformation

$$U_{\mathrm{dis}}^{(n)}(t_0, t_{\mathrm{dis},\pi}^{(n)}) = e^{-i\nu_x n t_{\mathrm{dis},\pi}^{(n)}}|n\rangle\langle n| \otimes \{|e_N\rangle\langle g_N| - |g_N\rangle\langle e_N|\}, \quad (5)$$

which is effectively a C-NOT operation between the collective motion being in either of the states $|0\rangle$ and $|n\rangle$,

and the electronic state of the Nth ion. For a motional state $|0\rangle$, the Rabi frequency $\Omega^{(0)}_{\text{dis}} = 0$, and the electronic state *remains unaffected*. For the motional state $|n\rangle$, the above transformation flips the electronic excited and ground state, apart from a phase factor. We note as an important feature of the C-NOT operation proposed here that it remains valid even beyond the Lamb-Dicke limit since it's essential feature of not affecting the motional ground state $|0\rangle$ does not depend on the exact value of the Lamb-Dicke parameter.

The fourth type of pulse required in our preparation scheme is a dispersive π pulse as described above, but acting on all the ions simultaneously. The setup of the two laser beams that generates this pulse is identical to the single ion case described above but where the beam waists are assumed sufficiently broad to excite all the ions with the same strength [10]. We further assume that the spatial variation of the exciting electric fields along the trap axis is small compared to the separation of the ions in the trap, i.e., $k_x \Delta_x \ll 1$, where the separation Δ_x, is typically several μm in current trapped ion experiments [3,16]. This may be realized by having the second laser beam propagating almost perpendicular to the trap axis or through a Raman excitation which allows one to control the effective wave vector [10]. Alternatively, one can consider using two beams whose wave vectors are sensitive to the two (independent) radial directions and employing the ions' collective motion perpendicular to the trap axis [26]. In either situation the phase of the exciting electric fields is equal for all ions [6], and the dynamics generated by the pulse is given by the product of the single ion time evolution operator given in Eq. (5). The dispersive interaction then couples the states $|J,-J\rangle|n\rangle$, and $|J,J\rangle|n\rangle$, and for a given transition the transformation

$$U^{(n)}_{\text{N-dis}}(t_0, t^{(n)}_{\text{N-dis},\pi}) = e^{-i\nu_x n t^{(n)}_{\text{N-dis},\pi}} |n\rangle\langle n| \otimes \prod_{i-1}^{N} \{|e_i\rangle\langle g_i| - |g_i\rangle\langle e_i|\}, \quad (6)$$

is generated through a pulse duration $t^{(n)}_{\text{N-dis},\pi} = \pi/\Omega^{(n)}_{\text{N-dis}}$, where $\Omega^{(n)}_{\text{N-dis}} \propto \eta_x^2 n/N$ denotes the Rabi frequency for that transition [24].

We now go through our preparation scheme step by step, starting from the initial state $|\Psi_R(0)\rangle = |J,-J\rangle|0\rangle$. Since the scheme involves acting on one of the ions separately, we shall, when necessary, write $|J,-J\rangle = |J',-J'\rangle|g_N\rangle$, where $J' = (N-1)/2$. First, a resonant $\pi/2$ pulse is applied to the Nth ion to produce the state

$$|\Psi_R(t_1)\rangle = \frac{1}{\sqrt{2}}|J',-J'\rangle\{|g_N\rangle + |e_N\rangle\}|0\rangle, \quad (7)$$

at the time $t_1 = t_{\pi/2}$. Next, a Jaynes-Cummings π pulse transfers the superposition of the electronic state of the Nth ion into the collective motion along the trap axis. From Eq. (4) with $n = 0$, the resulting state at time $t_2 = t_1 + t^{(0)}_{\text{JC},\pi}$ is

$$|\Psi_R(t_2)\rangle = \frac{1}{\sqrt{2}}|J,-J\rangle\{|0\rangle + ie^{-i\nu_x t_2}|1\rangle\}. \quad (8)$$

Then, the superposition of vibrational states is transferred into the electronic degrees of freedom of *all* the ions simultaneously by applying a dispersive π pulse to all the ions. As seen from the pulse transformation $U^{(1)}_{\text{N-dis}}(t_2, t^{(1)}_{\text{N-dis},\pi})$, given in Eq. (6) this effectuates a C-NOT operation between the collective vibrational state and the internal states of all the ions simultaneously, *independent* of the number of ions. The state after the pulse is

$$|\Psi_R(t_3)\rangle = \frac{1}{\sqrt{2}}\{|J,-J\rangle|0\rangle + ie^{-i\nu_x t_3}|J,J\rangle|1\rangle\}, \quad (9)$$

where $t_3 = t_2 + t^{(1)}_{\text{N-dis},\pi}$. We have now generated the required superposition between the Dicke states $|J,-J\rangle$ and $|J,J\rangle$. The remaining two pulses serve to disentangle the vibrational and electronic degrees of freedom. The first of those is a dispersive π pulse acting on the Nth ion. From Eq. (5) with $n = 1$, the state resulting from this pulse at time $t_4 = t_3 + t^{(1)}_{\text{dis},\pi}$ is given by

$$|\Psi_R(t_4)\rangle = \frac{1}{\sqrt{2}}\{|J',-J'\rangle|g_N\rangle|0\rangle - ie^{-i\nu_x t_4}|J',J'\rangle|g_N\rangle|1\rangle\}. \quad (10)$$

Finally a second Jaynes-Cummings π pulse, identical to the one that led us from Eq. (7) to Eq. (8), realizes the maximally entangled state

$$|\Psi_R(t_5)\rangle = \frac{1}{\sqrt{2}}\{|J,-J\rangle + |J,J\rangle\}|0\rangle, \quad (11)$$

at time $t_5 = t_4 + t^{(0)}_{\text{JC},\pi}$, leaving the collective motion along the trap axis in its ground state. In the Schrödinger picture $|\Psi(t_5)\rangle = |\Psi_M\rangle$, where $\phi = N\omega_0 t_5$, in Eq. (2). The phase ϕ in the maximally entangled state can be controlled by changing the phase of the initial $\pi/2$ pulse with respect to the other electric fields. Note that the maximally entangled state is produced deterministically by the procedure described here. Moreover, all pulses considered here drive the same $|e\rangle \Leftrightarrow |g\rangle$ transition. This is important for the experimental realization of our proposal since single transitions can be made independent of magnetic field fluctuations to first order [3].

As we have said earlier, the maximally entangled state generated in Eq. (11) may be used in high precision spectroscopy [4,11]. In Ref. [4] Bollinger *et al.* describe a Ramsey technique where once the maximally entangled state has been established two Ramsey pulses are applied to all ions simultaneously, and the expectation value of the product operator $\prod_{i=1}^{N}\{|e_i\rangle\langle e_i| - |g_i\rangle\langle g_i|\}$ serves to extract the transition frequency ω_0. This is measured by determining the number of ions in the excited or ground states. In order to not degrade the signal-to-noise ratio, the uncertainty in this measurement must be $\ll 1$ atom which requires that the number of ions in the trap be

small. In contrast, the Ramsey technique described by Huelga *et al.* [11] relies on population measurements on a single ion and the state generation is an integral part of the modified Ramsey scheme. In this situation the role of the initial $\pi/2$ pulse at frequency ω_0 is taken over by a $\pi/2$ Ramsey pulse at frequency ω, which is detuned by a small amount $\Delta = \omega_0 - \omega$, from the resonance frequency ω_0 which one aims to determine. We therefore assume that the pulses which prepare the maximally entangled state are generated by laser beams at the frequencies ω and $\omega - \nu_x$. Thus the frequency ω_0 is replaced by ω in the pulses, and we describe the time evolution in a frame rotating with the frequency ω of the Ramsey pulses. We denote the state of the system in this frame by $|\Psi_{R'}(t)\rangle$, and under the assumption $|\Delta| \ll \Omega, \Omega_{\mathrm{JC}}^{(n)}, \Omega_{\mathrm{dis}}^{(n)}, \Omega_{\text{N-dis}}^{(n)}$, the pulse transformations in that frame are the same as given in Eqs. (3)–(6). Starting from the state $|\Psi_{R'}(0)\rangle = |J, -J\rangle|0\rangle$ the pulse sequence described above leads to the state $|\Psi_{R'}(t_5)\rangle = \{|J, -J\rangle + |J, J\rangle\}|0\rangle/\sqrt{2}$. We then assume that the system is let free to evolve for a time T, resulting in the state $|\Psi_{R'}(t_6)\rangle = \{|J, -J\rangle + e^{-iN\Delta T}|J, J\rangle\}|0\rangle/\sqrt{2}$, where $t_6 = t_5 + T$. Then the pulse sequence which generates the maximally entangled state is applied again but in *reverse* order. This results in the final state

$$|\Psi_{R'}(t_7)\rangle = \frac{1}{2}|J', -J'\rangle\{[1 + (-1)^N e^{-iN\Delta T}]|g_N\rangle + [1 - (-1)^N e^{-iN\Delta T}]|e_N\rangle\}|0\rangle , \quad (12)$$

at the time $t_7 = T + 2t_5$. From this state the resonance frequency ω_0 can be determined with uncertainty $\delta\omega_0 \propto 1/N$ [11], by measuring the internal state of the Nth ion, and where $P = \{1 - (-1)^N \cos[N\Delta T]\}/2$ gives the probability of finding the Nth ion in its excited state.

To address the susceptibility of our scheme to imperfections in the state generation procedure we emphasize that the experiment reported in [10] has demonstrated the generation of almost maximally entangled states through unitary manipulations. The observed fidelities of approximately 0.7 indicate that deviations from unitary evolution do not destroy the sought-after state and affirm the validity of the unitary analysis presented here.

In summary, we have proposed an efficient method for generating maximally entangled internal states of a system of N trapped ions. The method has the further advantage of being deterministic. Finally, we have shown how such a state and the generation scheme described here may be used for high precision Ramsey spectroscopy.

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*Permanent address: Lehman College, The City University of New York, 250 Bedford Park Boulevard West, Bronx, NY 10468-1589.

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