

## Unambiguous Probabilities in an Eternally Inflating Universe

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“Constants of nature” and cosmological parameters may in fact be variables related to some slowly varying fields. In models of eternal inflation, such fields will take different values in different parts of the universe. Here, I show how one can assign probabilities to values of the “constants” measured by a typical observer. This method does not suffer from ambiguities previously discussed in the literature. [S0031-9007(98)08003-X]

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One of the striking aspects of the inflationary cosmology [1] is that, generically, inflation never ends [2]. The evolution of the inflaton field  $\phi$  is influenced by quantum fluctuation, and as a result thermalization does not occur simultaneously in different parts of the universe. The dynamics of  $\phi$  can be pictured as a random walk superimposed on the classical slow roll and can be described by a Fokker-Planck equation [2–4]. On small scales, the fluctuations in the thermalization time give rise to a spectrum of small density fluctuations, but on large scales they make the universe extremely inhomogeneous. In most of the models, one finds that, at any time, there are parts of the universe that are still inflating and that the total volume of the inflating regions is growing with time. This picture is often referred to as “stochastic inflation” or “eternal inflation.”

Eternally inflating universes may contain thermalized regions characterized by different values of the constants of nature and of the cosmological parameters. For example, the inflaton potential  $V(\phi)$  may have several minima corresponding to different low-energy physics or to different values of the cosmological constant. A more interesting possibility is that the “constants” are related to some slowly varying fields and take values in a continuous range. Examples are the effective gravitational constant in a Brans-Dicke-type theory [5] and the density parameter  $\Omega$  in some models of open inflation [6]. Just like the inflaton field  $\phi$ , the fields  $\chi_j$  ( $j = 1, \dots, n$ ) associated with the constants are subject to quantum fluctuations during inflation, and different regions of the universe thermalize with different values of  $\chi_j$ . An intriguing question is whether or not we can predict which values of the constants we are most likely to observe.

Assume an eternally inflating universe which is inhabited by numerous civilizations that will measure different values of  $\chi_j$ . (For simplicity, I do not distinguish between the fields  $\chi_j$  and the associated constants.) We can define the probability  $\mathcal{P}(\chi)d^n\chi$  for  $\chi_j$  to be in the intervals  $d\chi_j$  as being proportional to the number of civilizations which will measure  $\chi_j$  in that interval [7]. Assuming that we are a typical civilization, we can expect to observe  $\chi_j$  near the maximum of  $\mathcal{P}(\chi)$  [8].

An immediate objection to this approach is that we are ignorant about the origin of life, let alone intelligence, and therefore the number of civilizations cannot be calculated. However, the approach can still be used to find the probability distribution for parameters which do not affect the physical processes involved in the evolution of life. The cosmological constant  $\Lambda$ , the density parameter  $\Omega$ , and the amplitude of density fluctuations  $Q$  are examples of such parameters. We shall assume that our fields  $\chi_j$  belong to this category. The probability for a civilization to evolve on a suitable planet is then independent of  $\chi_j$ , and instead of the number of civilizations we can use the number of habitable planets or, as a rough approximation, the number of galaxies.

If this philosophy is adopted, the problem of calculating  $\mathcal{P}(\chi)$  can be split into two parts. The probability for us to observe certain values of  $\chi_j$  is proportional to the volume of the regions where  $\chi_j$  take specified values and to the density of galaxies in those regions. It is convenient to consider comoving regions and measure their volume at the time of thermalization. Then we can write

$$\mathcal{P}(\chi) \propto \nu(\chi)\mathcal{P}_*(\chi). \quad (1)$$

Here,  $\mathcal{P}_*(\chi)d^n\chi$  is proportional to the volume of thermalized regions where  $\chi_j$  take values in the intervals  $d\chi_j$ , and  $\nu(\chi)$  is the number of galaxies that form per unit thermalized volume. The calculation of  $\nu(\chi)$  is a standard astrophysical problem, which is completely unrelated to the calculation of the volume factor  $\mathcal{P}_*(\chi)$ . Our focus in this paper will be on the volume factor.

The problem one encounters in using Eq. (1) is that in an eternally inflating universe the thermalized volume is infinite, even for a region of a finite comoving size. One can deal with this problem by simply introducing a time cutoff and including only regions that thermalized prior to some moment of time  $t_c$ . One finds, however, that the resulting probability distribution is extremely sensitive to the choice of the time variable  $t$  [4]. This gauge dependence casts doubt on any conclusion reached using this approach.

An alternative procedure, suggested in [10,11], is to introduce a  $\chi$ -dependent cutoff at the time  $t_c(\chi)$ , when all

but a small fraction  $\epsilon$  of the comoving volume destined to thermalize with  $\chi_j$  in the intervals  $d\chi_j$  has thermalized. The limit  $\epsilon \rightarrow 0$  is taken after calculating the probabilities. It was shown in [10,11] that the resulting probability distribution is essentially insensitive to the choice of time parametrization. However, the same problem appears in a different guise. Linde and Mezhlumian [12] have found a family of gauge-invariant cutoff procedures which includes the  $\epsilon$  procedure described above. This indicates that the invariance requirement alone is not sufficient to define the probabilities uniquely.

Another source of uncertainty is associated with large values of the inflaton potential  $V(\phi)$ . Usually,  $\phi$  is assumed to vary in an infinite range, with  $V(\phi) \rightarrow \infty$  at  $\phi \rightarrow \pm\infty$ . Higher values of  $V(\phi)$  correspond to higher rates of expansion, and thus the regions with Planckian and super-Planckian values of  $V(\phi)$  expand at the highest rate. The effect of these regions on the dynamics of stochastic inflation is therefore very important [13]. Since the physics at Planckian energy density is completely unknown, this results in a significant uncertainty [4,11].

In the face of these difficulties, serious doubts have been expressed that a meaningful definition of probabilities in an eternally inflating universe is even, in principle, possible [4,9,12]. However, these pessimistic conclusions may have been premature. In this paper, I am going to argue that unambiguous probabilities can in fact be defined in a wide class of models of eternal inflation.

The factor  $\mathcal{P}_*(\chi)$  in Eq. (1) is the probability distribution of the fields  $\chi_j$  on the thermalization hypersurface  $\Sigma_*$  which separates inflating and thermalized spacetime regions. It is a three-dimensional spacelike surface which plays the role of the big bang for the thermalized regions. In the case of several discrete vacua,  $\Sigma_*$  consists of a number of disconnected pieces, each connected component corresponding to thermalization into a single vacuum [14]. Each connected piece of  $\Sigma_*$  has an infinite volume. (The situation here is somewhat similar to open inflation, where thermalized regions inside the bubbles have the form of infinite, open Robertson-Walker universes.) In order to determine the relative probability of different vacua, one has to compare the infinite volumes of the corresponding components of  $\Sigma_*$ , which is an inherently ambiguous task. This ambiguity is the source of the problems encountered in Refs. [4,10–12].

Now, the key observation is that the situation may be greatly improved in the case of continuous fields  $\chi_j$  varying in a finite range,  $0 \leq \chi_j \leq \Delta_j$ . In this case, each connected part of  $\Sigma_*$  is still infinite, but now different parts are not characterized by different values of  $\chi_j$ . On the contrary,  $\chi_j$  run through all of the range of their values on each connected part. Since the inflationary dynamics of the fields  $\chi_j$  has a stochastic nature, the distributions of  $\chi_j$  on different components of  $\Sigma_*$  should be statistically equivalent. It is therefore sufficient to consider a single connected component. Moreover, since  $\chi_j$  have a finite range, they will run through all of their values many times

on any sufficiently large part of  $\Sigma_*$ . Hence, there is no need to deal with infinite hypersurfaces. The probability distribution  $\mathcal{P}_*(\chi)$  can be determined by examining a large finite piece of  $\Sigma_*$ .

The above argument is an adaptation, to the case of stochastic inflation, of the argument given earlier by Garriga, Tanaka, and Vilenkin [15] for models of open inflation with a variable  $\Omega$ . In such models, each bubble contains regions with all possible values of  $\Omega$ , and we argued that the probability distribution for  $\Omega$  can be determined by considering a single bubble.

The argument can also be extended to fields with an infinite range of variation, provided that the probability distributions of  $\chi_j$  are concentrated within a finite range, with a negligible probability of finding  $\chi_j$  very far away from that range.

Models with a finite range of  $\chi_j$  are not difficult to construct. For example,  $\chi_j$  could play the role of angular variables, with the inflaton field  $\phi$  being the radial variable in the field space. Another attractive possibility is a  $\sigma$ -model-type theory in which both  $\chi_j$  and  $\phi$  take values on a compact manifold. In such models, the potential is necessarily bounded and problems with super-Planckian densities are easily avoided.

The gauge dependence of the probability distributions obtained using a constant-time cutoff can be understood as follows. Suppose that the time coordinate  $t$  has been chosen to be the proper time,  $t = \tau$ , and that regions with  $0 < \chi_1 < \Delta_1/2$  take a shorter time  $\tau$  to thermalize than regions with  $\Delta_1/2 < \chi_1 < \Delta_1$ . Then, in the vicinity of the cutoff ( $t = t_c$ ), the surface  $\Sigma_*(t_c)$  will tend to include regions with  $\chi_1 < \Delta_1/2$  and exclude regions with  $\chi_1 > \Delta_1/2$ . Suppose now that, despite the shorter roll-down time, regions with  $\chi_1 < \Delta_1/2$  expand by a larger factor than regions with  $\chi_1 > \Delta_1/2$ . [This can be arranged with a suitable choice of the potential  $V(\phi, \chi)$ .] Then, if we choose the scale factor to be the time coordinate,  $t = a$ , the surface  $\Sigma_*(t_c)$  will tend to include regions with  $\chi_1 > \Delta_1/2$  and exclude those with  $\chi_1 < \Delta_1/2$ . Thus, the cutoff at a fixed time  $t_c$  is biased with respect to different types of regions, depending on the choice of the time coordinate. One might think that the probability distribution of  $\chi$  on a hypersurface  $\Sigma_*$  should be insensitive to variation of the boundary of  $\Sigma_*$ , provided that the volume of  $\Sigma_*$  is sufficiently large. However, in the case of a fixed-time cutoff,  $\Sigma_*$  is a multicomponent hypersurface whose volume is dominated by small, newly formed thermalized regions. It is therefore no surprise that the resulting distribution  $\mathcal{P}_*(\chi)$  is sensitive to the choice of the time variable.

Thus, what we need is an unbiasedly selected portion of  $\Sigma_*$ . Such an unbiased cutoff is hard to implement in the Fokker-Planck formalism [2–4], which deals with probability distributions on equal-time surfaces. A more promising approach to the calculation of  $\mathcal{P}_*(\chi)$  is to use a numerical simulation of stochastic inflation. Simulations of this sort have been introduced in Ref. [16] and have

been further developed in Ref. [4]. In the present paper, I will not attempt to pursue this numerical approach. Instead, I am going to argue that in a wide class of models the probabilities can be calculated analytically.

Suppose the maximum of the potential  $V(\phi, \chi)$  is at  $\phi = 0$ . We shall first consider models in which  $V(\phi, \chi)$  is very symmetric near the top, so that it is essentially independent of  $\chi$  in the range  $0 \leq \phi < \phi_1$ , but may have significant  $\chi$  dependence elsewhere. We shall assume that  $\phi_1$  is in the slow-roll range,  $\phi_q \ll \phi_1 < \phi_*(\chi)$ , where  $\phi_q$  is the boundary between the quantum diffusion and deterministic slow-roll regimes, and  $\phi_*(\chi)$  is the value corresponding to the end of inflation. Let us consider a constant- $\phi$  hypersurface  $\Sigma_0$ :  $\phi = \phi_0$  with  $\phi_q \ll \phi_0 < \phi_1$ . Since  $\phi_0$  is in the slow-roll range, this hypersurface is (almost everywhere) spacelike [10]. Moreover, all values of  $\phi$  in the past of  $\Sigma_0$  belong to the range  $0 \leq \phi < \phi_0$  where the potential is independent of  $\chi$ . Therefore, we expect the distribution of  $\chi_j$  on  $\Sigma_0$  to be flat,  $\mathcal{P}_0(\chi_j) = \text{const}$ , with all values of  $\chi_j$  equally probable [17].

The hypersurface  $\Sigma_*$  is a spacelike hypersurface in the future of  $\Sigma_0$ . The probability distribution for  $\chi_j$  on  $\Sigma_*$  is related to that on  $\Sigma_0$  by

$$\mathcal{P}_*(\chi) \propto \mathcal{P}_0(\chi^{(0)}) \exp[3N(\chi^{(0)})] \det(\partial\chi^{(0)}/\partial\chi). \quad (2)$$

Here,  $\chi_j^{(0)}(\chi)$  are the values on  $\Sigma_0$  that evolve, along the slow-roll trajectory, to the values  $\chi_j$  on  $\Sigma_*$ ,  $N(\chi^{(0)})$  is the number of inflationary  $e$ -foldings along that trajectory,  $\exp(3N)$  is the volume expansion factor, and  $\det(\partial\chi^{(0)}/\partial\chi)$  is the Jacobian transforming from variables  $\chi_j^{(0)}$  to  $\chi_j$ . Now,  $\mathcal{P}_0(\chi^{(0)}) = \text{const}$ , and the functions  $\chi^{(0)}(\chi)$  and  $N(\chi^{(0)})$  can be straightforwardly determined from a given potential  $V(\phi, \chi)$ . Hence, Eq. (2) solves the problem of finding the probability distribution for  $\chi_j$  on  $\Sigma_*$ . This equation is significantly simplified in models where  $\chi_j$  do not change much during the slow roll, so that  $\chi_j^{(0)} \approx \chi_j$ . In such models,

$$\mathcal{P}_*(\chi) \propto \exp[3N(\chi)]. \quad (3)$$

Equation (2) disregards quantum fluctuations of  $\chi_j$  and  $\phi$  in the slow-roll regime at  $\phi \geq \phi_0$ . For  $\chi_j$  this is justified, provided that the average cumulative fluctuation is small compared to  $\Delta_j$ . This condition is not difficult to satisfy. Small quantum fluctuations of  $\phi$  are not correlated with  $\chi_j$  and their effect vanishes to linear order in  $\delta\phi$ . However, quadratic and higher-order terms will give some corrections to Eq. (2). One may also be worried about the effect of very large fluctuations which bring  $\phi$  back to the quantum diffusion regime [4]. Although extremely rare, such fluctuations are likely to be present for a sufficiently large piece of  $\Sigma_0$ . If the probability of large fluctuations were independent of  $\chi_j$ , then small patches of  $\Sigma_0$  which have such fluctuations in their future could be left out, without any effect on the distribution (2). If there is some  $\chi$  dependence, its effect will be to reduce  $\mathcal{P}_*(\chi)$  for those values of  $\chi_j$  which give

a higher probability of large backwards jumps. I expect this effect to be very small, but more work is needed to obtain a quantitative estimate.

Large fluctuations back to the quantum regime may occur even after thermalization [18]. Then each thermalization surface  $\Sigma_*$  may have an infinite number of other thermalization surfaces in its future. However, Eq. (2) can still be used to derive the probability distribution for  $\chi_j$  on any of these surfaces.

As an illustrative example, let us consider a two-field model with a potential

$$V(\phi, \chi) = V_0 \cos^2(\alpha\phi) [1 + \lambda \sin^4(\alpha\phi) \cos(\beta\chi)]. \quad (4)$$

Here,  $|\lambda| < 1$ ,  $\alpha = \pi/\Delta_\phi$ , and the range of  $\phi$  is  $0 \leq \phi \leq \Delta_\phi$ , with the values  $\phi = 0$  and  $\phi = \Delta_\phi$  identified. Similarly,  $\beta = \pi/\Delta_\chi$  with  $\chi = 0$  and  $\chi = \Delta_\chi$  identified. We shall assume that  $\beta \lesssim \alpha \lesssim 1$ ; that is,  $\Delta_\chi \gtrsim \Delta_\phi \gtrsim 1$ . (I use Planck units throughout the paper.) The top of the potential is at  $\phi = 0$ , and the true vacuum is at  $\phi = \Delta_\phi/2$ . The field  $\chi$  is massless in the true vacuum, while the mass of  $\phi$  is  $\chi$  dependent,

$$m_\phi^2(\chi) = 2V_0\alpha^2 [1 + \lambda \cos(\beta\chi)]. \quad (5)$$

The boundary  $\phi_q$  between the quantum diffusion and slow-roll regimes is determined by the condition  $|\partial H/\partial\phi| \sim H^2$ , where  $H(\phi, \chi) = [8\pi V(\phi, \chi)/3]^{1/2}$  is the inflationary expansion rate. We shall assume that  $V_0 \ll \alpha^2$ ; then  $\phi_q \sim V_0^{1/2}/\alpha^2 \ll \Delta_\phi$ . Since  $\alpha \lesssim 1$ , the problem of super-Planckian energies does not arise in this case.

The potential (4) is essentially independent of  $\chi$  for  $\phi \ll \Delta_\phi$ , but the number of  $e$ -foldings  $N$  and the amplitude of density fluctuations  $\delta\rho/\rho$  are both  $\chi$  dependent. For  $\alpha \ll 1$ ,  $\lambda \ll 1$ , we find

$$N(\chi) \approx -4\pi \int_{\phi_0}^{\phi_*} \frac{H}{\partial H/\partial\phi} d\phi \approx N_0 + \frac{2\pi\lambda}{\alpha^2} \cos(\beta\chi), \quad (6)$$

and [1]

$$\delta\rho/\rho(\chi) \approx 200m_\phi(\chi). \quad (7)$$

Here,  $N_0$  is the number of  $e$ -foldings (from a specified reference point  $\phi_0$ ) in the  $\chi$ -independent case ( $\lambda = 0$ ), and I have used  $\sin(\alpha\phi_*) \approx 1$ , which is valid for  $\alpha \lesssim 1$  (since  $\phi_* \approx \frac{\pi}{2\alpha} - \frac{1}{6}$ ). The classical and quantum variations of  $\chi$  during the relevant part of the slow roll are, respectively,  $\delta\chi_c/\Delta_\chi \sim (\lambda/4\pi)(\beta/\alpha)^2 \ll 1$  and  $\delta\chi_q/\Delta_\chi \sim V_0^{1/2}\beta/\alpha \ll 1$ , and can be neglected. This justifies the  $\chi = \text{const}$  approximation in Eq. (6) and the use of Eq. (3) for the probability distribution,

$$\mathcal{P}_*(\chi) \propto \exp[6\pi\lambda\alpha^{-2} \cos(\beta\chi)]. \quad (8)$$

The corresponding distribution for  $\delta\rho/\rho$  can easily be written using Eqs. (5) and (7).

As a second example, we consider the spectrum of density perturbations in the standard model of inflation with a single field  $\phi$ . The perturbations are determined by quantum fluctuations  $\delta\phi$  which can be regarded as random Gaussian variables with a dispersion  $\sigma = H/2\pi$ . A perturbation is produced on each comoving scale at the time when that scale crosses the horizon and has a gauge-invariant amplitude [19]:

$$\delta\rho/\rho \sim 8\pi H\delta\phi/H', \quad (9)$$

where  $H' = dH/d\phi$ . Averaging over the Gaussian distribution gives the standard result  $(\delta\rho/\rho)_{\text{rms}} \sim 4H^2/|H'|$ .

Fluctuations of  $\phi$  on different length scales are statistically independent and can be treated separately. We can therefore concentrate on a single scale corresponding to some value  $\phi = \phi_0$ , disregarding all of the rest. The fluctuation  $\delta\phi$  will take different values on different parts of the hypersurface  $\Sigma_0 : \phi = \phi_0$ , so  $\delta\phi$  plays the role of the field  $\chi$  of our previous example. Linde *et al.* [20] have studied the probability distribution for the observed values of  $\delta\phi$  using a constant proper time cutoff and arrived at a surprising conclusion that typical fluctuations can be much greater than the rms value  $H/2\pi$ . We shall see, however, that this large effect disappears in the present approach.

The probability distribution for  $\delta\phi$  on  $\Sigma_*$  can be found directly from Eq. (2) with  $\mathcal{P}_0(\delta\phi)$  being the initial Gaussian distribution on  $\Sigma_0$ . When the field  $\phi$  undergoes a quantum jump  $\delta\phi$ , the number of  $e$ -foldings necessary to complete the classical rollover to  $\phi_*$  is changed by  $\delta N = 4\pi\delta\phi H_0/H'_0$ . Hence, we obtain the distribution

$$\mathcal{P}_* \propto \exp\left[-\frac{2\pi^2}{H_0^2}\left(\delta\phi - \frac{3H_0^3}{\pi H'_0}\right)^2\right], \quad (10)$$

which describes fluctuations with a nonzero mean value,  $\langle\delta\phi\rangle = 3H_0^3/\pi H'_0$ . The inflaton tends to fluctuate in the direction opposite to the classical roll, because backward fluctuations prolong inflation and increase the volume factor. This effect, however, is hopelessly small:  $\langle\delta\phi\rangle/\sigma = 6H_0^2/H'_0 \sim (\delta\rho/\rho)_{\text{rms}} \ll 1$ . We thus see that anthropic considerations do not substantially modify the standard calculations of the density perturbation spectrum [21].

In conclusion, the approach described in this paper allows one to assign unambiguous probabilities to continuously varying constants measured by a typical observer. In models where the inflaton potential is nearly independent of the constants in the quantum diffusion range, the probabilities can be calculated analytically. Otherwise, a numerical simulation is needed to determine the distribution  $\mathcal{P}_0(\chi^{(0)})$  in Eq. (2).

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