Comment on "Nonuniversal Exponents in Interface Growth"

In a recent Letter, Newman and Swift [1] made an interesting suggestion that the strong-coupling exponents of the Kardar-Parisi-Zhang [2] (KPZ) equation reported previously [3,4] may not be universal, but rather depend on the precise form of the noise distribution. The purpose of this Comment is to show that their numerical findings can be attributed to a percolative effect instead of a portentous breakdown of universality.

The interface growth model of Ref. [1] can be equivalently formulated as

$$\tilde{h}_i(t) = h_i(t) + \xi_i(t), \tag{1a}$$

$$h_i(t+1) = \min_{j \in S_i} \{\tilde{h}_j(t)\}, \tag{1b}$$

where the minimum in (1b) is taken over the set S_i which includes site i and all its nearest neighbors on a d-dimensional hypercubic lattice. In this form, the height variable $h_i(t)$ can be readily taken as the ground state energy of a directed polymer on a $(d+1)$-dimensional lattice with the upper end fixed at (i, t) [4].

In Ref. [1], the noise (or random potential) term $\xi_i(t)$ in Eq. (1a) was drawn from a distribution,

$$p_\alpha(\xi) = \frac{1+\alpha}{2}(1 - |\xi|)^\alpha, \qquad -1 \le \xi \le 1. \tag{2}$$

It was observed numerically that, as the parameter α decreases towards its limiting value -1, the exponent β which characterizes the scaling of the interface width $W(t) \equiv (\langle h^2 \rangle - \langle h \rangle^2)^{1/2} \sim t^\beta$, appears to decrease dramatically, particularly in high dimensions ($d = 3, 4$). This behavior led Newman and Swift to suggest that the universality hypothesis is broken in the KPZ equation.

Here we give an alternative interpretation of the slow growth of $W(t)$ over the time interval they investigated. In the limit $\alpha = -1$, the distribution (2) reduces to a discrete one where $\xi = \pm 1$ with equal probability. For $d > 1$, we found [5] that the $\xi = -1$ sites percolate in the sense defined by (1b). Consequently, $W(t)$ saturates to a constant after an initial transient, yielding $\beta = 0$. (The directed percolation threshold p_c equals 0.539 and 0.265 for $d = 1$ and 2, respectively, which explains why the "nonuniversal" behavior is absent in $d = 1$.) In the directed polymer language, this corresponds to a finite density of paths going through the minimum energy sites $\xi = -1$ and they all yield the same ground-state energy $h_i(t) = -t$. The ground-state degeneracy is lifted when $\alpha > -1$. However, for α close to -1, the effectively competing paths are those which utilize only the bottom part of the distribution (2), i.e., those going through sites with ξ close to -1. Therefore, the effective noise fluctuation is much weaker than what appears to be from (2). This, plus an "intrinsic width" effect (i.e., W remains finite even for $\alpha = -1$), is the origin of the observed behavior.

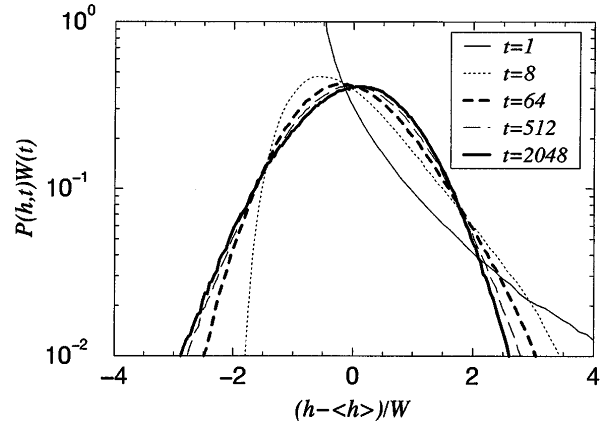

FIG. 1. Normalized distribution of surface height at different times for $d = 2$ with 2000^2 sites. Here $\alpha = -1/2$.

A quantitative analysis of the crossover to the asymptotic scaling can be carried out by examining the evolution of the surface-height distribution [5]. In the scaling regime, one expects the normalized distribution to converge to a limiting form. From Fig. 1 we see that such convergence is rather slow for $d = 2$ and $\alpha = -0.5$. The slow convergence to the asymptotic distribution (which we believe to be universal) is more pronounced for smaller values of α (weaker effective noise) or in higher dimensions (diminishing directed percolation threshold p_c). Details of our analysis, including calculations on the Berker lattice, will be published elsewhere [5].

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