Nucleon Decay from Deformed Nuclei

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The absolute decay width of a single-particle level in a deformed potential is evaluated exactly. Experimental data are analyzed by using the formalism developed here. It is found that proton decay is a powerful tool to determine the deformation of nuclei as well as to probe small components of the wave functions of the decaying states. This will allow one to study the behavior of single-particle resonances in nuclei close to the drip line, and may guide future experiments. [S0031-9007(98)06649-6]

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There is at present an intense experimental activity to reach and study nuclei far from stability. However, the interpretation of the coming experimental data requires an adequate theoretical framework to analyze the unstable nuclei to be measured. This is also essential to guide future experiments. The theory should take into account that the processes to be analyzed may occur in the continuum part of the spectrum. This is a difficult undertaking because the most relevant of those processes are time dependent with the additional complication that the initial states may not be well defined. These difficulties notwithstanding, the possibilities opened by the various phenomena related to the single-particle resonances, which will now be measured, are very interesting. One of the important features of these measurements is that the resonances will have a single-particle character. In contrast, resonances in stable nuclei lie very high in the spectrum and are a mixture of many degrees of freedom. For instance, in ^{209}Bi the proton strength of the state $0j_{15/2}$ was found to be spread in an energy range from 4 up to 13 MeV, with a centroid at 8 MeV [1]. This is a general feature of nuclei in regions of the chart of nuclides close to the stability line. In these nuclei only a fraction of the single-particle strength is concentrated in the "single-particle" resonance [2,3]. In nuclei on the drip line, instead, the Fermi level is very close to, or even immersed in, the continuum. Thus, in odd systems the lowest excited levels, which, as usual, are single-particle excitations, lie in the continuum and one expects that other degrees of freedom would not play an important role.

Single-particle resonances may have rather unusual properties. In particular, it was recently shown [4] that the noncrossing rule for Nilsson levels may be violated. Properties such as this and the corresponding formalism can be tested experimentally by means of nucleon-decay experiments by measuring, for example, the position and the corresponding decay width of the single-particle resonances. In this simple case, where the cluster is a nucleon, the exact quantum mechanics evaluation of the decay width is a simple task if the nucleus is spherical. This is an important point which is worthwhile to develop in detail. We will thus derive the decay width corresponding to a pure single-particle state, for example, the one obtained from a diagonalization of a Woods-Saxon potential. That is, we will consider a single-particle model and therefore the parent nucleus will be described by a nucleon moving in a deformed potential.

In order to compare the theoretical and experimental single-particle quantities, one may need to include renormalization effects due to the fact that the outgoing nucleon may leave the daughter nucleus in an excited state. This can be taken into account by multiplying this "bare" single-particle width by a spectroscopic factor. In the case of an odd decaying nucleus, described by the BCS model, this factor is simply u^2.

At large distances the bare single-particle radial wave function $r\psi_{lj}^{\rm out}(r)$ corresponding to the outgoing nucleon in a state (lj) has the form

$$r\psi_{lj}^{\rm out}(r) = N_{lj}[G_{lj}(r) + iF_{lj}(r)], \tag{1}$$

where N is a normalization constant and F and G are the regular and irregular Coulomb functions, respectively. The probability rate per second that the particle goes through a surface element $dS = r^2 \sin\theta d\theta d\varphi$ is given by $\mathcal{F}_{lj} = |\psi_{lj}^{\rm out}(\vec{r})|^2 v dS$, where $v = \hbar k/\mu$ is the velocity of the particle. Since

$$\lim_{r\to\infty} |r\psi_{lj}^{\rm out}(r)|^2 = |N_{lj}|^2, \tag{2}$$

the decay probability per second, i.e., the inverse of the half-life T, obtained by integrating $\mathcal{F}$ over the angles, is $1/T = |N_{lj}|^2 v$, noticing that $|N_{lj}|^2$ has a dimension of 1/fm. The normalization N is determined by matching at a certain distance R the wave function $\psi_{lj}^{\rm out}(R)$ to the solution $\psi_{lj}(R)$ of the Schrödinger equation, which is regular at the origin and has outgoing boundary conditions, i.e., the Gamow state. Therefore, the

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width is

$$\Gamma_{lj}(R) = \hbar/T = \frac{\hbar^2 k}{\mu} \frac{R^2 |\psi_{lj}(R)|^2}{F_{lj}^2(R) + G_{lj}^2(R)}, \quad (3)$$

which is independent of R for distances outside the range of the nuclear potential as can be seen from Eq. (1).

The expression of Eq. (3) gives the exact quantum mechanics value of the width. It coincides with the one obtained by Thomas [5] as the residues of the S matrix, although the Thomas expression is valid for the decay of any cluster and, therefore, $\psi(R)$ is the formation amplitude of the cluster; this is just the overlap between the mother wave function and the antisymmetrized tensor product of the daughter and cluster wave functions at a distance R between the corresponding centers of mass, including in this way the spectroscopic factor.

An alternative and equivalent method to obtain the decay width is discussed in detail in Ref. [6]. The Schrödinger equation is solved with outgoing wave boundary conditions [7] and the complex energies $\mathcal{E}_n = E_n - i\Gamma_n/2$ are obtained, which correspond to resonances with energy E_n and decaying width Γ_n. For the case of spherical nuclei there are standard computer codes to do this [8,9], although often approximations to Eq. (3) are used to evaluate those widths [10,11]. These calculations were used to determine the spin and parity of the resonances. A similar study for deformed nuclei is important, for instance, in the drip line region where nuclei are expected to have significant deformations. The problem of evaluating complex energy eigenvalues in nonspherical systems was solved only recently [4], but for the present study one needs to evaluate the partial decay widths for angular momentum projected states.

We will assume that the mother nucleus is odd. Using standard notation, the angular momentum projected wave function is given by

$$\Psi_m^{J_i M_i, K_i} = \left(\frac{\hat{J}_i}{16\pi^2}\right)^{1/2} \{\mathcal{D}_{M_i K_i}^{J_i} \chi_{K_i} + (-1)^{J_i + K_i} \mathcal{D}_{M_i - K_i}^{J_i} \chi_{\bar{K}_i}\}, \quad (4)$$

where $\hat{J}_i = 2J_i + 1$, $\mathcal{D}$ are the rotation matrices, and χ is the intrinsic single-particle wave function that can be expanded in spherical components [4] as

$$\chi_{K_i}(\vec{r}) = \sum_{j \geq K_i} \alpha_{lj}(r) [Y_l(\hat{r}) \chi_{1/2}]_{jK_i}, \quad (5)$$

where the orbital angular momentum l is determined by the parity of the state.

Since electromagnetic transitions are usually faster than the decay by proton emission, the decay is more probable when the nucleus is in the lowest energy state with $J_i = K_i$.

The exit channel wave function will be the tensorial product of the internal wave functions of the daughter nucleus Ψ_d times the wave function corresponding to the relative motion of the proton with respect to the mother nucleus. The daughter wave function is

$$\Psi_d^{J_d M_d K_d} = \left(\frac{\hat{J}_d}{8\pi^2}\right)^{1/2} \mathcal{D}_{M_d K_d}^{J_d}. \quad (6)$$

The most probable decay is usually the one that leaves the daughter nucleus in the ground state, since in this way the proton has the largest possible energy. Therefore the relation $J_d = M_d = K_d = 0$ can be imposed. The generalization to decays to other members of the ground band in the daughter nucleus is straightforward.

Angular momentum conservation implies that the angular momentum of the outgoing proton j_p is such that $j_p = J_i = K_i$. A spectroscopic factor of $1/(j_p + 1/2)$ has to be included since we require that the angular momentum of the daughter nucleus must be equal to zero. For the purpose of the discussion we will neglect this multiplicative factor that will be taken into account in the comparison with the experimental data.

At a large distance R the nuclear potential vanishes and the Coulomb potential is spherically symmetric. Therefore the wave function of Eq. (5) at R has, as in the spherical case, the form

$$R\chi_{K_i}^{\text{out}}(\vec{R}) = \sum_{lj} N_{lj}[G_{lj}(R) + iF_{lj}(R)][Y_l(\hat{R})\chi_{1/2}]_{jK_i}, \quad (7)$$

where N_{lj} are normalization constants which are determined by matching the outgoing solution χ^{out} with the internal solution of Eq. (5). As before, the probability rate of decay to the channel lj is given by $1/T_{lj} = N_{lj}^2 v$.

From the angular momentum conservation discussed above and the orthogonality of the different partial waves one finds that the partial decay width corresponding to the decay to the channel $l_p j_p$ is given by

$$\Gamma_{l_p j_p}(R) = \frac{\hbar^2 k}{\mu} \frac{R^2 \alpha_{l_p j_p}^2(R)}{F_{l_p j_p}^2(R) + G_{l_p j_p}^2(R)}, \quad (8)$$

which has the same structure of Eq. (3) for the spherical system.

In all of these derivations it was assumed that the deformation of the daughter nucleus is the same as in the mother one.

We have checked numerically that the total width calculated by summing $\Gamma_{lj}(R)$ of Eq. (8) over all of the channels labeled by lj at large R is minus twice the imaginary part of the energy for the bare single-particle state. The partial decay width is then defined as

$$\Gamma_{lj}(R) = -2\mathcal{E}_i \mathcal{P}_{lj}, \quad (9)$$

where $\mathcal{E}_i$ is the imaginary part of the energy and $\mathcal{P}_{lj}$ is the probability of finding the particle in the channel lj normalized to the outgoing wave. Defining the quantity $\beta_i^2(R) = \alpha_i^2/[F_i^2(R) + G_i^2(R)]$ that probability becomes

$$\mathcal{P}_{lj} = \beta_{lj}^2(R) \Big/ \sum_{l'j'} \beta_{l'j'}^2(R), \quad (10)$$

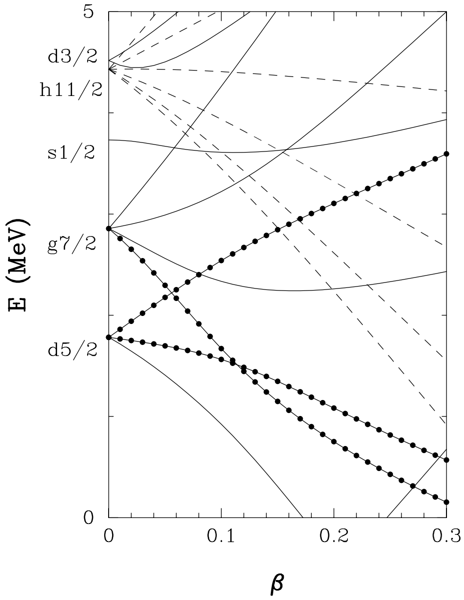

FIG. 1. Proton Nilsson levels corresponding to ${}^{113}_{55}$Cs. The dotted lines are the levels around the Fermi surface. They correspond to the state $K = 1/2^+$ (coming from the shell $g_{7/2}$), $K = 3/2^+$, and $K = 5/2^+$ (these two come from the shell $d_{5/2}$).

with R as a large distance beyond the range of the nuclear mean field and beyond the distance where the Coulomb potential can be considered spherically symmetric.

In order to compare the results of the exact formalism presented here with other calculations, we will analyze the proton decay of the nucleus ^{113}Cs (968 $\pm$ 6 keV) [12], with a half-life of 16.7 $\pm$ 0.7 μs [13]. This decay was studied in Ref. [14] within the framework of perturbation theory. To probe this approximated treatment we analyzed the dependence of the half-life evaluated from the exact Eq. (8) as a function of the deformation parameter β_2 and found a very peculiar feature which is missing in Ref. [14]. The Nilsson levels corresponding to this nucleus, shown in Fig. 1, were obtained by using a Woods-Saxon potential with the so-called universal parameters [15]. The dotted lines are the states which lie close to the Fermi level, for which we evaluate the decay width.

Figure 2 shows the dependence of the half-life on β_2 mentioned above. While the $1/(j_p + 1/2)$ factor has been included, the part of the spectroscopic factor coming from the BCS, u^2, was not.

There are several interesting conclusions that one can draw from Fig. 2. First, it is not possible to reproduce the experimental data by using a spherical (i.e., $\beta_2 = 0$) scheme with a reasonable value for the occupation probability u^2. Allowing for deformations instead, the experimental half-life can be fitted by the state $1/2^+$, coming from the orbital $0g_{7/2}$ at deformation $\beta_2 \approx 0.12$, by the state $3/2^+$, coming from the orbital $1d_{5/2}$ at deformation $\beta_2 \approx 0.15$–0.20, or by the state $5/2^+$, coming from the orbital $1d_{5/2}$ at deformation $\beta_2 \approx 0.22$–0.27, using a value of $u^2 \approx 0.5$. This last possibility can be easily discarded, since the state is too far from the Fermi surface at that deformation. Of the two remaining possibilities, only the second was noticed in Ref. [14]. But perhaps more intriguing is the sharp increase of the curve corresponding to the state $1/2^+$ at $\beta_2 \approx 0.08$ indicating that a divergence exists at that point. In a further analysis we found that this is indeed the case. To understand this point we show in Fig. 3 the spherical components $\alpha_{lj}(R)$ as a function of the deformation for the states $K = 1/2^+$, $K = 3/2^+$, and $K = 5/2^+$ at $R = 13$ fm. These are the three states marked with dots in Fig. 1. One notices that for

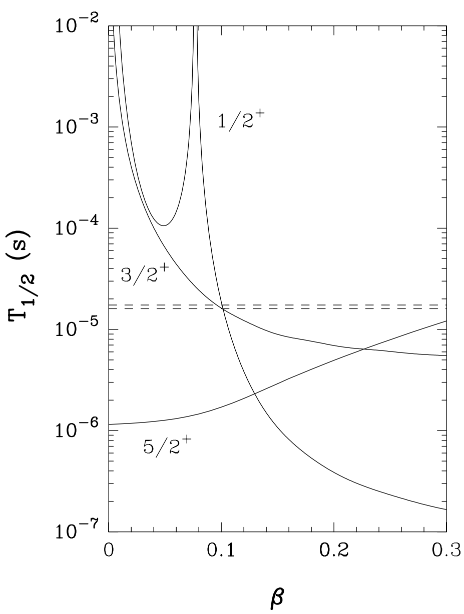

FIG. 2. Half-life of the resonance ^{113}Cs (968 $\pm$ 6 keV) as a function of the deformation β_2 for the states $K = 1/2^+$, $3/2^+$, and $5/2^+$ that in Fig. 1 were dotted. The corresponding experimental value [13] is within the dashed lines.

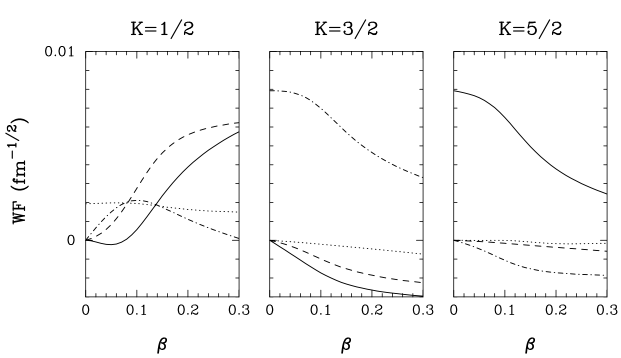

FIG. 3. Spherical components α_{lj} corresponding to the wave function of the resonance ^{113}Cs (968 $\pm$ 6 keV) as a function of β_2. The solid line corresponds to $j = K$, the dash-dotted line to $j = K + 1$, the dashed line to $j = K + 2$, and the dotted line to $j = K + 3$.

$K = 1/2^+$ the component corresponding to the state $s_{1/2}$, which is responsible for the decay, changes sign just at the value of β_2 that produces the divergence in Fig. 2. This divergence is due to the fact that the half-life is inversely proportional to the square of the wave function. If the nucleus had this particular deformation, then the decay to the 2^+ state could compete with the decay to the ground state. We would like to stress that, in order to be able to distinguish such a possibility, it would be essential to measure the angular momenta of the daughter and mother nuclei. This is a suggestion for further experiments in this field, since this would become a powerful tool to probe small components of the single-particle wave functions.

In conclusion, we have presented in this paper a formalism and the corresponding equations to calculate the width corresponding to the decay of a nucleon moving in a single-particle level in a deformed potential. By comparing the calculation with experimental data, we have shown that these processes provide a powerful tool to investigate the properties of single-particle resonances in nuclei close to the drip lines. We also found that proton decay can be used to probe small components of the deformed wave function in the mother nucleus, which would otherwise be very difficult, if at all possible, to measure.

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