Unified Description of Freeze-Out Parameters in Relativistic Heavy Ion Collisions

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It is shown that the chemical freeze-out parameters obtained at CERN/SPS, BNL/AGS, and GSI/SIS energies all correspond to a unique value of 1 GeV for the average energy per hadron in the local rest frame of the system, independent of the beam energy and of the target and beam particles. [S0031-9007(98)07957-5]

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We have found empirically that a unified description of the hadronic abundances produced in heavy ion collisions at the CERN/SPS, the BNL/AGS, and the GSI/SIS accelerators is possible. This description covers a range in beam energies from 200*A* GeV to below 1*A* GeV. As it turns out, the same description can also be applied to the hadronic abundances in the CERN Large Electron-Positron Collider (LEP) and in p-p and $\bar{p}$-p collisions using a slightly different treatment of the strangeness sector which takes into account the strangeness undersaturation in these reactions. The result can be summarized in a surprisingly simple way: the hadronic composition of the final state is determined by an average energy per hadron being approximately 1 GeV in the rest frame of the produced system. This leads to the curve in the T, μ_B plane shown in Fig. 1, where the values of the freeze-out parameters are shown as obtained by various groups (see text below and Table I). The solid line represents an average energy per hadron of 1 GeV; the dashed line represents 0.94 GeV per hadron. This average energy corresponds to the chemical freeze-out stage, namely, before the hadrons decay into stable hadrons.

The curves in Fig. 1 have been calculated in terms of the thermal model which assumes that at freeze-out the particle multiplicities and all other thermodynamic quantities, in this particular case, the average energy $\langle E\rangle$ and the average number of hadrons $\langle N\rangle$, can be calculated from the partition function of a noninteracting system. We have included the contributions to the partition function from all particles and resonances with masses up to $M = 2$ GeV [1]. The baryon number and strangeness conservation is guaranteed by the chemical potentials. The strangeness neutrality condition, $\langle S\rangle = 0$ appropriate in heavy ion collisions, has been used to eliminate the dependence of thermodynamical observables on the strange chemical potential. Consequently, the ratio of $\langle E\rangle/\langle N\rangle$ depends only on two thermal parameters, the temperature T and the baryon chemical potential μ_B. Imposing the condition, $\langle E\rangle/\langle N\rangle = \text{const}$, leads to the curves shown in Fig. 1.

The appearance of the "boundary" curve in the T, μ_B plane has been discussed previously in terms of the extended Bootstrap Model by Hagedorn and Rafelski [2]. The physical meaning of the theoretical curve from Refs. [2,3] and our purely empirical one shown in Fig. 1 is, however, different. In the former case, the curve describes the limiting values of temperature and chemical potential beyond which the usual hadronic matter, composed of interacting hadrons and resonances, ceases to exist. The phenomenological curve calculated in Fig. 1, on the other hand, specifies the value of the thermal parameters at chemical freeze-out where the number-changing inelastic collisions in the system cease and the particle abundances are frozen in. In general, the limiting curve of Hagedorn and Rafelski and the freeze-out curve from our finding do not need to coincide, thus could also correspond to different physical conditions. Indeed, in the former case the limiting curve is described by a fixed energy density, $\epsilon \sim 4B$ (B being the bag constant) while the curve in Fig. 1 corresponds to a

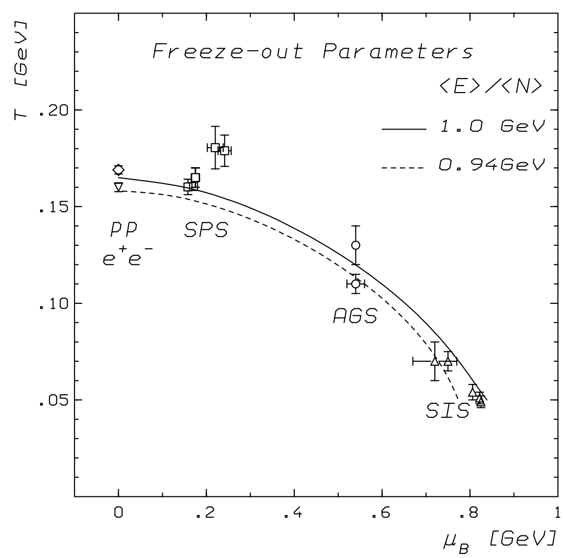

FIG. 1. Freeze-out values obtained from hadronic abundances at CERN/SPS, BNL/AGS, and GSI/SIS. Also indicated are the points obtained from observed hadronic abundances at LEP and in p-p collisions at CERN. The smooth curves correspond to a fixed energy per hadron in the hadronic gas model.

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TABLE I. Freeze-out temperature T and baryon chemical potential μ_B in various collisions.

Energy	T [MeV]	μ_B [MeV]	Ref.
SPS			
S + S 200*A* GeV	180.5 ± 10.9	220.2 ± 18.0	[7]
S + Ag 200*A* GeV	178.9 ± 8.1	241.5 ± 14.5	[7]
S + S 200*A* GeV	160.04	171.0	[10]
S + S 200*A* GeV	165.0 ± 5.0	175.0 ± 5.0	[8]
S + Au 200*A* GeV	160.2 ± 3.5	158.0 ± 4.0	[9]
AGS			
Si + Au 14.6 GeV	130 ± 10	540	[11]
Si + Au 14.6 GeV	110 ± 5	540 ± 20	[12]
SIS			
Ni + Ni 1.9*A* GeV	70 ± 10	720 ± 50	[13]
Ni + Ni 1.8*A* GeV	70 ± 5	750 ± 8	[15]
Ni + Ni 1.0*A* GeV	54 ± 4	806 ± 5	[15]
Ni + Ni 0.8*A* GeV	49 ± 3	825 ± 5	[15]
Au + Au 1.0*A* GeV	51 ± 3	822 ± 3	[15]

fixed $\langle E\rangle/\langle N\rangle \sim 1$ GeV. The freeze-out curve, without the observation that it corresponds to a constant average energy per hadron, has also been discussed recently by Braun-Munzinger and Stachel [4].

It is well known that various effects (e.g., flow) severely distort the momentum spectra of the particles produced in heavy ion collisions. It has been repeatedly pointed out, however, that many of these effects cancel out for ratios of fully integrated particle multiplicities (see, e.g., [5,6]). The analysis of measured particle ratios is therefore the best method to obtain reliable information on the chemical freeze-out parameters of the hadronic final state in heavy ion collisions. Such an analysis, relying as much as possible on fully integrated particle multiplicities, was carried out for BNL/AGS, for CERN/SPS, and for GSI/SIS data.

In Fig. 1, the SPS points are indicated by open squares while the AGS points are indicated by open circles. A description of the data points is given in the table. The two SPS points coming from Ref. [7] correspond to $S+S$ and $S+\mathrm{Ag}$ collisions, hence, the slightly different results. Another SPS point comes from the analysis of Ref. [8], where different excluded volume corrections have been used and complete chemical equilibrium of strangeness has been assumed. The result for S-Au collisions are from Ref. [9] and corresponds to the fit of the thermal model to midrapidity data. The result of Ref. [10] uses a time-dependent, nonequilibrium hadronization of a quark-gluon plasma droplet; the parameters quoted correspond to the collective chemical freeze-out. We have not included any points from the Pb beam since these results are not yet final. An exhaustive list of references to the thermal analysis of particle ratios in relativistic heavy ion collisions can be found in Ref. [9].

The two points denoted by AGS are based on the particle abundances given in Ref. [11]. The analysis of Ref. [12], using somewhat different excluded volume corrections, is consistent with these results but favors a slightly lower temperature.

Data using Ni and Au beams at energies between 0.8 and 1.9*A* GeV have become available recently from the GSI/SIS accelerator. These data have attracted considerable interest due to the surprisingly large number of K^- mesons being produced below threshold. A very detailed and extensive discussion of these results in the framework of thermal models has been presented in [13–15]. The result for the freeze-out parameters for Ni-Ni at 1.9*A* GeV is shown as an open triangle in Fig. 1. The points with the lowest temperature correspond to Ni-Ni collisions at 0.8 and 1.0 and Au-Au at 1.0*A* GeV. The authors of Ref. [15] emphasize the fact that due to the low temperatures involved in the GSI/SIS data it is essential to use the canonical ensemble in describing the strange particle sector since most of the kaons are produced below threshold. However, due to the negligible overall contribution of the strangeness sector to the average total energy $\langle E\rangle$ and particle number $\langle N\rangle$ at freeze-out, the grand canonical treatment can be used with confidence, to calculate the freeze-out curve in Fig. 1.

A similar analysis has been performed in [16] for e^+e^- annihilation into hadrons at LEP. Since no baryons are involved here, this corresponds, in the grand canonical ensemble, to zero baryon chemical potential, $\mu_B = 0$. We point out, however, that the application of the thermal model for particle productions in elementary collisions required canonical formulation of all conservation laws. This is simply because we are dealing here with a small amount of matter closed in rather small volume [17]. An impressive fit of the thermal model in the canonical formulation has been obtained in Ref. [16] since no less than 29 different hadronic abundances can be reproduced. This analysis was subsequently extended [18] to p-p and $\bar{p}$-p reactions at CERN. It must be emphasized that the role of the strange particle sector in these collisions is different from that in relativistic heavy ion collisions since there is a considerable suppression (or undersaturation) of strange particles in elementary collisions.

In the underlying hadronic gas model, all of the freeze-out points can be described by a single curve corresponding to a fixed average energy per particle, $\langle E\rangle/\langle N\rangle$, which has approximately the value of 1 GeV per particle in the hadronic gas. *This value characterizes all the final states produced by beams having 1A GeV all the way up to 200A GeV.* This empirical observation leads to a considerable unification in the description of the hadronic final states produced in high energy collisions.

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