Charging Energy of a Chaotic Quantum Dot

S. Möller, H. Buhmann, S. F. Godijn, and L. W. Molenkamp
2. Physikalisches Institut, RWTH-Aachen, Templergraben 55, D-52056 Aachen, Germany
(Received 30 June 1998)

The scaling behavior of the charging energy of a quantum dot with asymmetrically adjusted tunnel barriers is measured through the amplitude of the Coulomb oscillations in the thermovoltage. For weak coupling between the dot and the reservoirs, we observe a linear scaling of the effective charging energy when the transmission probability of one tunnel barrier is increased. At higher transmission probabilities, we find a deviation from the linear scaling and a crossover to a constant value. This behavior is caused by the chaotic nature of the electron trajectories within the dot. [S0031-9007(98)07846-6]

PACS numbers: 72.20.Pa, 73.20.Dx

The low temperature transport properties of a quantum dot, weakly coupled through tunnel barriers to electron reservoirs, are dominated by the Coulomb blockade. This effect leads to oscillations of several transport quantities as a function of the voltage applied to a capacitively coupled gate electrode. The most well known are the Coulomb-blockade oscillations of the conductance [1]. Another quantity that exhibits an oscillatory behavior is the thermopower, which is defined as $S = -\lim_{\Delta T \to 0}(\Delta V_{\mathrm{th}}/\Delta T)$, with ΔV_{th} the potential difference across the dot caused by a temperature difference ΔT. As a function of gate voltage, the thermovoltage V_{th} exhibits sawtoothlike oscillations [2,3].

For finite transmission of the barriers between dot and reservoirs (G in the order of $2e^2/h$), quantum fluctuations are expected to influence the occurrence of the Coulomb blockade in such a way that for increasing transmission probability, t, the Coulomb oscillations become less distinct. This behavior has been studied in two different limits, i.e., in the limit where the tunnel barriers contain many conducting channels [4], as applicable for metallic quantum dots, as well as in the limit for split-gate defined semiconductor quantum dots [5,6], where only one conducting channel is present. In the latter case, which is of interest here, the one-dimensional character of the tunnel barriers is often taken into account using techniques which are based on a Tomonaga-Luttinger formalism [7]. A mapping of these results to the problem of two coupled dots [8] was successfully used to explain experimentally found splittings in the Coulomb peak due to molecular electron states [9].

Reference [5] describes the effect of quantum fluctuations in terms of a renormalization of the bare charging energy $E_c = e^2/2C$, where C is the self-capacitance of the dot. A result is the scaling of the effective charging energy E_c^* with the transmission probabilities of the coupling barriers according to

$$E_c^* = E_c(1 - t)^{N_c}. \quad (1)$$

Here, N_c is the total number of tunnel barriers with a transmission probability t leading to the dot. ($t = 1$ corresponds to a conductance of the tunnel barrier equal to the conductance quantum of $2e^2/h$). Although Eq. (1) was derived under the assumption that t is close to one, the predicted scaling behavior has been experimentally observed for a range of $0 < t < 0.3$, using an electrometer device consisting of two coupled quantum dots [10]. This experiment was performed by monitoring the conductance of one, well-defined dot as a function of the gate voltage applied to the other while the transmission probabilities of two quantum point contacts leading to the dot were varied. A quadratic scaling behavior of the charging energy was found in agreement with Eq. (1). The sensitivity of the experiment did not allow for a thorough check of the scaling behavior near $t = 1$.

Recently, Aleiner and Glazman [11] pointed out that for asymmetrical quantum dots, where the classical electron trajectories are chaotic, a residual oscillatory dependence of the transport properties should remain even if the conductance of *one* of the leads becomes close to $2e^2/h$. This implies a finite effective charging energy at $t \approx 1$ which contradicts Eq. (1) as well as other published scaling theories [6]. To test the prediction of Ref. [11], a technique is needed that allows for sensitive measurements of the charging energy of a quantum dot even if the transmission probability of a coupling point contact is close to one.

According to the theory of Ref. [2], the amplitude and line shape of the thermopower oscillations depend strongly on the ratio k_BT/E_c. Thus measuring the thermovoltage V_{th} of a quantum dot at a fixed temperature for different transmission probabilities of a coupling point contact should provide information about the scaling behavior of the charging energy. Here, we present experimental results on the scaling behavior of the charging energy through changes of the amplitude and line shape of the thermovoltage oscillations.

The sample, schematically shown in Fig. 1, is electrostatically defined by TiAu Schottky-gates, labeled $A, B, \ldots, F$, in a two-dimensional electron gas (2DEG), which is formed within a (Al,Ga)As modulation doped heterostructure. The 2DEG has an electron density

 © 1998 The American Physical Society

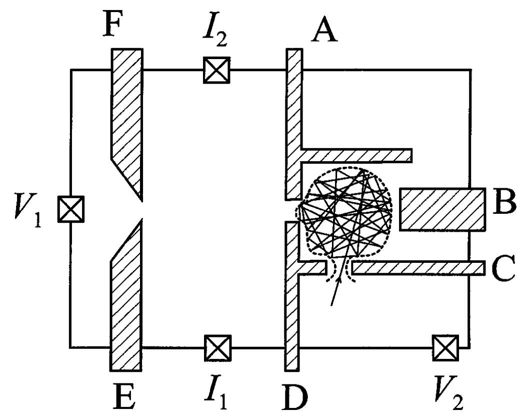

FIG. 1. Schematic top view of the sample structure. The hatched areas show the structure of the Schottky gates, the crosses denote Ohmic contacts. The heating current is passed between I_1 and I_2. The thermovoltage V_{th} is measured between V_1 and V_2. The chaotic electron trajectories are indicated in the gate-defined dot region (dashed line). An arrow points at the varied quantum-dot lead.

$n_s \approx 3.4 \times 10^{11}$ cm^{-2} and an electron mobility $\mu \approx 10^6$ cm^2(V s)$^{-1}$. The gates define the following main features of the sample: (1) the quantum dot (gates A, B, C and D) with a lithographical size of 700 nm × 800 nm, coupled to the reservoirs by two adjustable barriers [quantum point contacts AD and CD, defined by gates A (C) and D (D), respectively]; (2) a 2 μm wide and 20 μm long channel, $ADEF$, between the gates A, D, E, and F and a quantum point contact EF, defined by gates E and F. The electrochemical potential of the dot can be varied by changing the applied voltage to gate B, V_g^B, while the tunnel barriers are kept constant. During the scaling experiments, the sample was kept at a base temperature of 40 mK.

A temperature difference ΔT between the two reservoirs adjacent to the dot is created by current-heating techniques [12]: A low-frequency (13 Hz) current I is passed through the electron channel $ADEF$, increasing the temperature of the electron system in the channel by $\Delta T \propto I^2$. Measuring the potential difference between the voltage contacts V_1 and V_2 [cf. Fig. 1] by phase sensitive lock-in techniques at twice the frequency of the heating current gives a thermovoltage

$$V_{th} := V_1 - V_2 = (S_{ref} - S_{dot})\Delta T . \tag{2}$$

Thus, assuming a constant thermopower S_{ref} of the reference point contact EF and a constant averaged temperature difference ΔT between the electron channel and the reservoirs, variations of V_{th} reflect directly changes in the thermopower of the dot.

The scaling behavior of the charging energy is determined by measuring V_{th} as a function of V_g^B for various values of the conductance of point contact CD, which is changed by adjusting V_g^C, the voltage on gate C. The transmission of point contact AD is kept constant at a value of ≈0.06. Some of the resulting curves are shown in Fig. 2(a) for a gate-voltage range of -938 mV $< V_g^B < -925$ mV and six different transmission probabilities of point contact CD, $t = 0.06$, 0.19, 0.29, 0.38, 0.43, and 0.82 (top to bottom). The thermovoltage oscillations decrease with increasing point-contact conductance and become more symmetric. Figure 2(b) shows the behavior of the calculated thermopower of a quantum dot (solid line) as a function of the Fermi energy in the reservoirs, i.e., the electrochemical potential in the dot, according to the theoretical model given by Ref. [2]. Here, the *only*

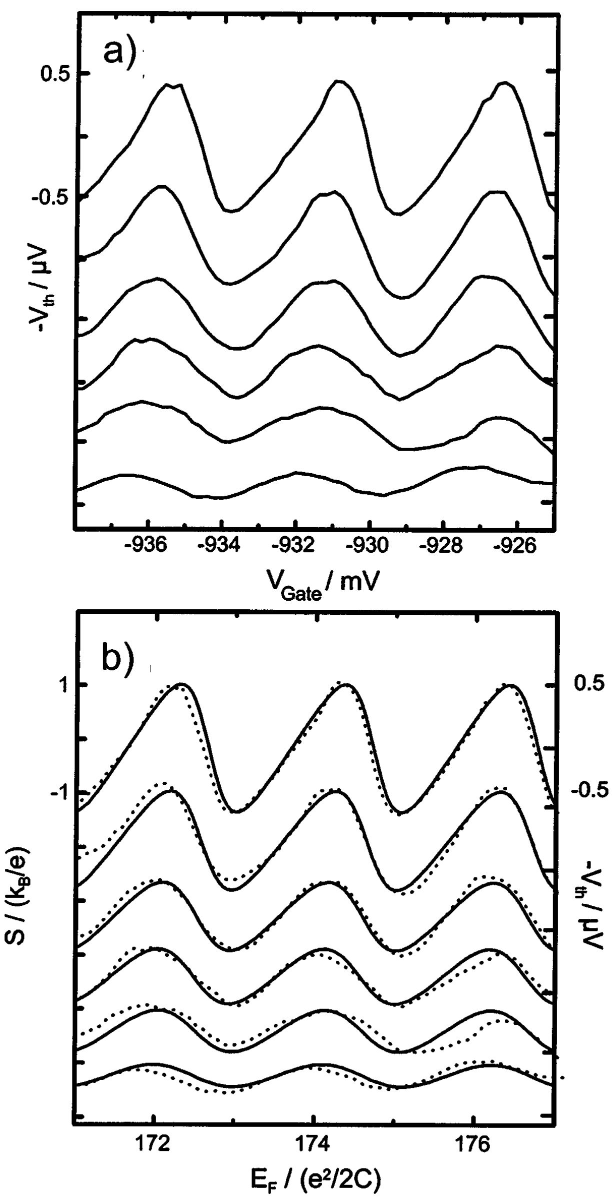

FIG. 2. (a) Experimental traces of the thermovoltage of the quantum dot for a heating current of 40 nA. The transmission probability of point contact CD was 0.06, 0.19, 0.29, 0.38, 0.43, and 0.82 from top to bottom. (b) Calculated curves of the thermopower of a quantum dot. The values of k_BT/E_c are 0.22, 0.25, 0.30, 0.33, 0.37, and 0.45 from top to bottom (solid line). The experimental thermovoltage measurements from (a) are added as dashed lines.

variable parameter is the ratio between thermal energy and charging energy, $k_B T / E_c$ to fit the line shape of the measured thermovoltage [dashed lines in Fig. 2(b)]. Thus, assuming a constant temperature, the measured decrease and change in line shape of the thermovoltage can be interpreted as a renormalization of the charging energy, E_c^*. The fit parameter $k_B T / E_c$ for the curves (solid line) displayed in Fig. 2(b) are 0.22, 0.25, 0.30, 0.33, 0.37, and 0.45 (top to bottom).

Using the bare charging energy of the quantum dot, $E_c \approx 100 \pm 20$ μeV, which was determined from thermal activation studies of the conductance in the Coulomb-blockade regime, it is possible to evaluate the effective charging energy quantitatively. As a result, the temperature difference across the dot, ΔT, can be calculated from the ratio of the measured thermovoltage and the calculated thermopower [cf. Eq. (2)], yielding $\Delta T \approx 8$ mK.

In order to check the prediction of Eq. (1), the transmission probabilities of point contact CD are needed. These are obtained from conductance measurements of the bare point contact as a function of V_g^C, which are corrected for the electrostatic influences of the other active gates by subtracting an experimentally determined offset. In Fig. 3 E_c^*/E_c is plotted versus the reflection probability $1 - t$ of point contact CD, obtained by the fitting procedure discussed above. According to Eq. (1), one expects a linear behavior to account for $N_c = 1$, the number of varied point contacts. For values of $0.5 < (1 - t) < 1$ we indeed find a linear scaling, which establishes the validity of Eq. (1) in this regime. However, for smaller reflectivity values ($1 - t < 0.5$) we find a change of slope which appears to approach an asymptotic value of ≈ 0.45 E_c^*/E_c for $t \rightarrow 1$, which contradicts the predicted scaling behavior of Eq. (1).

As mentioned above, deviations from the scaling behavior are expected if the electrons inside the quantum dot are scattered randomly. If the averaged dwell time τ_d of the electrons in the dot exceeds the ergodic time τ_{erg}, the electron motion in the dot is chaotic. We now need to decide whether the electron motion in the investigated system is chaotic or not. A description of a chaotic system is possible in the framework of the random matrix theory (RMT) [13]. This theory gives the following expression for the average magneto conductance of a chaotic quantum dot with *open* leads ($N \gg 1$) in the ballistic regime around $B = 0$ T:

$$\langle \delta G(\Phi) \rangle = \frac{\delta G_0}{1 + (\Phi/\Phi_c)^2}, \tag{3}$$

with

$$\Phi_c = \frac{h}{e} \left(\frac{\tau_{\mathrm{erg}}}{\tau_d} \right)^{1/2},$$

and $\Phi = BA$, where A is the area of the dot. By measuring the conductance of the dot with open leads as a function of an external magnetic field for a large number of different dot configurations, i.e., a large number of different gate voltages V_g^B, it is possible to obtain the statistical ensemble necessary to verify the chaotic behavior of the electron motion in the dot. The inset of Fig. 3 displays the averaged conductance of the open dot ($N = 4e^2/h$) as a function

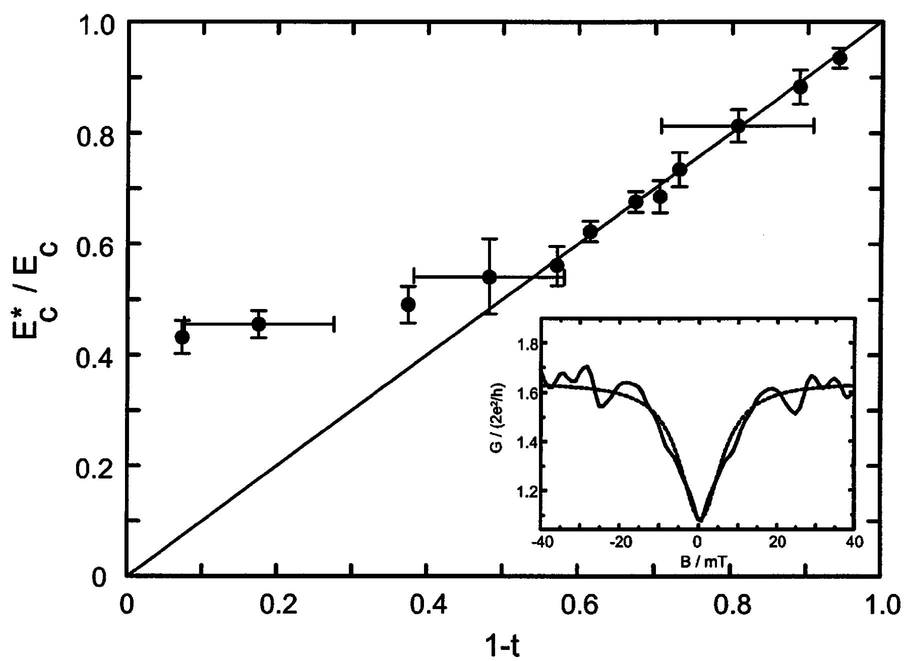

FIG. 3. Plot of the measured ratio E_c^*/E_c as a function of reflection probability $1 - t$ of point contact CD. The solid line shows the trace of linear scaling according to Eq. (1). Inset: Averaged conductance of over 30 individual curves for the open dot ($N_c = 4$) taken with slightly different gate voltages V_g^B (solid line). The x axis is expanded around $B = 0$ T and a Lorentzian fit of the weak- localization peak added (dashed line).

of a magnetic field for over 30 individual measurements [14]. Similar experiments on chaotic dots have been reported, e.g., in Ref. [15,16]. The observed dip around $B = 0$ T is due to the weak-localization effect. The ratio $\tau_d/\tau_{\mathrm{erg}}$ is obtained by fitting the line shape according to Eq. (3) which yields a value of $\tau_d/\tau_{\mathrm{erg}} \approx 5.3$. This result implies that the motion of the electron in the dot is chaotic and Aleiner and Glazman's conjecture [11] for the observed deviation from the predicted scaling behavior of the effective charging energy is appropriate. We have obtained additional evidence for the chaotic nature of the electron trajectories from thermopower measurements in the ballistic regime. These experiments, which exhibit characteristic non-Gaussian fluctuation distributions, will be discussed elsewhere.

Aleiner and Glazman [11] used RMT in connection with the Tomonaga-Luttinger formalism to compute the behavior of the transport properties of a quantum dot connected to a reservoir by a fully transmitted mode, predicting a residual Coulomb blockade, as long as the other contact(s) remain fully in the tunneling regime. Because of the occurrence of chaos, an electron that has been reflected by (one of) the tunneling barrier(s) will not immediately leave the dot through the open contact. In terms of a renormalized charging energy this can be expressed by making the following substitution to Eq. (1) (for $N_c = 1$):

$$(1 - t) \rightarrow \frac{\Delta E}{E_c} \ln^2\!\left(\frac{E_c}{\Delta E}\right). \tag{4}$$

Here, ΔE is the mean spacing of the electronic energy levels in the dot. For the actual structure ΔE can be estimated from the 2DEG density of states and the size of the quantum dot, resulting in $\Delta E \approx 23$ μeV. The value of the chaos-induced effective charging energy at $t = 1$ thus is predicted to be

$$E_c^*(t = 1) = 0.49 \pm 0.03 E_c\,. \tag{5}$$

Extrapolating the data of Fig. 3 to $(1 - t) = 0$ gives

$$E_c^*(t = 1) \approx 0.45 E_c\,. \tag{6}$$

The remarkable agreement between these two values indicates indeed the existence of a residual chaos-induced Coulomb blockade at a fully transmitted channel, in agreement with the theory of Ref. [11].

We have presented thermopower measurements of a chaotic quantum dot in order to determine the scaling behavior of the charging energy as a function of the transmission probability of the coupling point-contact barriers in the range of $0 < t < 1$. For transmission probabilities $0 < t < 0.5$ the scaling follows a power-law behavior according to theoretical predictions. For barrier transmission probabilities above $t = 0.5$, the measured behavior seems to contradict the expected vanishing of the Coulomb blockade for $t \rightarrow 1$. However, the observation of a finite effective charging energy for high transmission probabilities can be explained by taking into account classical chaotic trajectories of the electrons inside the dot. The experimental value for an effective charging energy at $(1 - t) \rightarrow 0$ is in good agreement with theoretically expected values for a chaotic quantum dot of the relevant dimensions.

We would like to thank K. Flensberg, C.M. Marcus, and L.I. Glazman for fruitful discussions. Part of this work was supported by the DFG, SFB-341 and DFG, MO 711/3-1.

[1] For a review of Coulomb blockade, see H. van Houten, C. W. J. Beenakker, and A. A. M. Staring, in *Single Sharge Tunneling*, edited by H. Grabert and M. Devoret, NATO ASI, Ser. B, Vol. **294** (Plenum, New York, 1992).

[2] C. W. J. Beenakker and A. A. M. Staring, Phys. Rev. B **46**, 9667 (1992).

[3] A. A. M. Staring, L. W. Molenkamp, B. W. Alphenaar, H. van Houten, O. J. A. Buyk, M. A. A. Mabesoone, C. W. J. Beenakker, and C. T. Foxon, Europhys. Lett. **22**, 57 (1993).

[4] See, e.g., G. Falci, G. Schön, and G. T. Zimanyi, Phys. Rev. Lett. **74**, 3257 (1995); P. Joyez, V. Bouchiat, D. Esteve, C. Urbina, and M. H. Devoret, Phys. Rev. Lett. **79**, 1349 (1997).

[5] K. Flensberg, Phys. Rev. B **48**, 11 156 (1993).

[6] K. A. Matveev, Phys. Rev. B **51**, 1743 (1995).

[7] The Tomonaga-Luttinger formalism was first applied to electronic transport through weak links by C. L. Kane and M. P. A. Fisher, Phys. Rev. Lett. **68**, 1220 (1992).

[8] J. M. Golden and B. I. Halperin, Phys. Rev. B **53**, 3893 (1996); K. A. Matveev, L. I. Glazman, and H. U. Baranger, Phys. Rev. B **53**, 1034 (1996).

[9] E.g., F. R. Waugh, M. J. Berry, D. J. Mar, R. M. Westervelt, K. L. Campman, and A. C. Gossard, Phys. Rev. Lett. **75**, 705 (1995).

[10] L. W. Molenkamp, K. Flensberg, and M. Kemerink, Phys. Rev. Lett. **75**, 4282 (1995).

[11] I. L. Aleiner and L. I. Glazman, Phys. Rev. B **57**, 9608 (1998); L. I. Glazman (private communication).

[12] For a detailed description of the current-heating technique, see, e.g., L. W. Molenkamp, H. van Houten, C. W. J. Beenakker, R. Eppenga, and C. T. Foxon, Phys. Rev. Lett. **65**, 1052 (1990).

[13] For a review of the RMT, see C. W. J. Beenakker, Rev. Mod. Phys. **69**, 731 (1997).

[14] Note that because of the limited gate-voltage interval, available in this device, the chaotic conductance fluctuations are not totally averaged out. This does not, however, affect the fit to the line shape of the localization peak.

[15] C. M. Marcus, A. J. Rimberg, R. M. Westervelt, P. F. Hopkins, and A. C. Gossard, Phys. Rev. Lett. **69**, 506 (1992).

[16] I. H. Chan, R. M. Clarke, C. M. Marcus, K. Campman, and A. C. Gossard, Phys. Rev. Lett. **74**, 3876 (1995).