Determination of the Weak Phase γ from Rate Measurements in $B^\pm \to \pi K, \pi\pi$ Decays

Matthias Neubert

Theory Division, CERN, CH-1211 Geneva 23, Switzerland

Jonathan L. Rosner

Enrico Fermi Institute and Department of Physics, University of Chicago, Chicago, Illinois 60637
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A method is described which, under the assumption of SU(3) symmetry, allows one to determine the angle $\gamma = \arg(V_{ub}^*)$ of the unitarity triangle from time-independent measurements of the branching ratios for the rare two-body decays $B^+ \to \pi^0 K^+$ and $B^- \to \pi^0 K^-$, as well as of the CP-averaged branching ratios for the decays $B^\pm \to \pi^\pm K^0$ and $B^\pm \to \pi^\pm \pi^0$, all of which are of order 10^{-5}. The effects of electroweak penguin operators are included in a model-independent way, and SU(3)-breaking corrections are accounted for in the factorization approximation. [S0031-9007(98)07845-4]

The study of CP violation in the weak decays of B mesons will provide important tests of the flavor sector of the standard model, which predicts that all CP violation results from the presence of a single complex phase in the quark mixing matrix. The precise determination of the sides and angles of the unitarity triangle, which is a graphical representation of the unitarity relation $V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$, plays a central role in this program [1]. Whereas the angle $\beta = -\arg(V_{td})$ will be accessible at the first-generation B factories through the measurement of CP violation in the decay $B \to J/\psi K_S$, the angle $\gamma = \arg(V_{ub}^*)$ is harder to determine. The sum $(\beta + \gamma)$ can be extracted in a theoretically clean way from measurements of CP violation in the decays $B \to \pi\pi$ (or in the related decays $B \to \pi\rho$ and $\rho\rho$), but because of experimental difficulties such as the detection of the mode $B \to \pi^0\pi^0$ this will be a long-term objective. A method to determine γ proposed by Gronau and Wyler uses rate measurements for six $B \to DK$ decay modes [2], some of which require the reconstruction of the neutral charm-meson CP eigenstate D_+^0. A variant of this approach using $B \to DK^*$ decays has been discussed by Dunietz [3]. Unfortunately, these methods rely either on measurements of some processes with very small branching ratios, posing experimental [4] and theoretical [5] challenges, or on measurements requiring considerable precision (see, e.g., Refs. [6,7], and references therein).

In view of these difficulties, approximate methods to determine the angle γ have received a lot of attention. The simplest of these methods was proposed by Gronau, Rosner, and London (GRL), who suggested a triangle construction involving the amplitudes for the decays $B^+ \to \pi^+ K^0$, $\pi^0 K^+$, and $\pi^+ \pi^0$, as well as for the corresponding CP-conjugated decays [8]. Besides a plausible dynamical assumption this method relies on SU(3) flavor symmetry in relating $B \to \pi\pi$ with $B \to \pi K$ decays. Later, it was argued that the GRL method is spoiled by electroweak penguin contributions, which have

an important impact in $B \to \pi K$ decays and upset the naive SU(3) triangle constructions [9,10]. More sophisticated methods based on quadrangle constructions involving other decay modes such as $B_s^0 \to \pi^0\eta$ [10] or $B^+ \to \eta^{(\prime)} K^+$ [11,12] were invented to circumvent this problem. There have also been proposals for deriving bounds on γ using CP-averaged rate measurements in $B \to \pi K$ decays [13–18], and for combining these measurements with those of rate asymmetries and other decays like $B \to K\bar{K}$ to obtain further information [19,20].

In the present Letter we propose a variant of the original GRL method, which based on the findings of our previous work [18], includes the potentially dangerous electroweak penguin contributions in a model-independent way using Fierz identities and SU(3) symmetry. We thus obtain an approximate method for learning $\cos\gamma$ that is conceptually as simple and uses the same experimental input and theoretical assumptions as the GRL method, though the actual triangle constructions are somewhat more complicated. The main advantage of our approach is that it is based on rare two-body decays that are relatively easy to access experimentally, and that have larger branching ratios than the decays needed for all other methods of measuring γ. Although the accuracy of this extraction may ultimately be limited by theoretical uncertainties, even an approximate value for $\cos\gamma$ will be very useful, if only to help eliminate discrete ambiguities inherent in other determinations [21].

The basis of our method is the amplitude relation

$$
\begin{aligned}
3A_{3/2} &= \mathcal{A}(B^+ \to \pi^+ K^0) + \sqrt{2}\,\mathcal{A}(B^+ \to \pi^0 K^+) \\
&\approx \sqrt{2}\,\frac{V_{us}}{V_{ud}}\frac{f_K}{f_\pi}\,|\mathcal{A}(B^+ \to \pi^+\pi^0)| \\
&\quad \times e^{i\phi_{3/2}}(\delta_{\text{EW}} - e^{i\gamma}),
\end{aligned} \tag{1}
$$

where $A_{3/2}$ is an isospin amplitude parametrizing the $\Delta I = 1$ transition $B \to (\pi K)_{I=3/2}$, $e^{i\phi_{3/2}}$ is a strong-interaction phase, and $e^{i\gamma}$ is the weak phase associated

with the quark decay $\bar{b} \to \bar{u}u\bar{s}$. The second relation is strictly valid in the SU(3) flavor-symmetry limit; however, the factor $f_K/f_\pi = 1.22 \pm 0.01$ accounts for the leading (i.e., factorizable) corrections to that limit. The crucial new ingredient in (1) with respect to the corresponding relation used in Ref. [8] is the presence of the parameter δ_{EW} accounting for the contributions of electroweak penguin operators. We have recently shown that in the SU(3) limit this parameter is real (i.e., it does not carry a nontrivial strong-interaction phase) and calculable in terms of Wilson coefficients and electroweak parameters [18]. The result is

$$\delta_{EW} = (1 - \kappa) \frac{1.71\alpha}{\lambda^2 R_b} = 0.63 \pm 0.11, \qquad (2)$$

where $\alpha = 1/129$ is the electromagnetic coupling at the weak scale, $\lambda = 0.22$ is the Wolfenstein parameter, $R_b = \lambda^{-1}|V_{ub}/V_{cb}| \approx 0.41 \pm 0.07$ [22], and $\kappa \approx 0.05$ accounts for factorizable SU(3)-breaking corrections. The derivation of this result uses the fact that the relevant electroweak penguin operators are Fierz equivalent to the usual current-current operators Q_1 and Q_2 of the effective weak Hamiltonian for $B \to \pi K$ decays [15], and that in the SU(3) limit the isospin amplitude $A_{3/2}$ receives a contribution only from the combination $(Q_1 + Q_2)$, but not from the difference $(Q_1 - Q_2)$.

As in the original GRL method, we must rely on the dynamical assumptions that $|\mathcal{A}(B^+ \to \pi^+\pi^0)| = |\mathcal{A}(B^- \to \pi^-\pi^0)|$ and $\mathcal{A}(B^+ \to \pi^+K^0) = \mathcal{A}(B^- \to \pi^-\bar{K}^0)$. Whereas the first relation follows from the fact that only the current-current operators contribute to $B^\pm \to \pi^\pm\pi^0$ decays (electroweak penguin contributions can be neglected in this case [23]), the second one assumes that there are only negligible contributions proportional to the weak phase $e^{i\gamma}$ to the amplitude for the decay $B^+ \to \pi^+K^0$, which thus can be taken to have the simple form $\mathcal{A}(B^+ \to \pi^+K^0) = e^{i\pi}e^{i\phi_P}|\mathcal{A}(B^+ \to \pi^+K^0)|$, where $e^{i\pi}$ is the weak phase of the leading top- and charm-penguin amplitudes, and $e^{i\phi_P}$ is a strong-interaction phase. Possible contributions to this amplitude proportional to the weak phase $e^{i\gamma}$ are indeed expected to be very small, because they could come only from up-quark penguins or annihilation topologies [24]. However, this intuitive argument could be invalidated if soft final-state rescattering effects were very important [14–17,19,20]. We stress, therefore, that the assumption $\mathcal{A}(B^+ \to \pi^+K^0) = \mathcal{A}(B^- \to \pi^-\bar{K}^0)$ is a working hypothesis of our method, which must be tested independently. A necessary condition for the validity of this assumption is the absence of a sizable direct CP asymmetry in the decays $B^\pm \to \pi^\pm K^0$. If we write $\mathcal{A}(B^+ \to \pi^+K^0) \propto e^{i\phi_P}(e^{i\pi} + e^{i\gamma}e^{i\eta}\varepsilon_a)$, where $\varepsilon_a \ll 1$ measures the strength of possible rescattering contributions and $e^{i\eta}$ is a strong-interaction phase, then $a_{CP} \approx 2\varepsilon_a \sin\gamma \sin\eta$. Since the global analysis of the unitarity triangle prefers values of γ such that $\sin\gamma = O(1)$, and

since $\sin\eta$ is unlikely to be small because without sizable strong phases there would not be a rescattering contribution in the first place, a small experimental value for the asymmetry would be a strong indication that our working hypothesis is justified.

Let us define the amplitude ratios

$$\varepsilon_{3/2} = \frac{V_{us}}{V_{ud}} \frac{f_K}{f_\pi} \frac{\sqrt{2}|\mathcal{A}(B^+ \to \pi^+\pi^0)|}{|\mathcal{A}(B^+ \to \pi^+K^0)|},$$

$$r_\pm = \frac{\sqrt{2}|\mathcal{A}(B^\pm \to \pi^0K^\pm)|}{|\mathcal{A}(B^+ \to \pi^+K^0)|}, \qquad (3)$$

which under the assumptions stated above can be determined experimentally through time-independent rate measurements via

$$\varepsilon_{3/2} = \sqrt{2}\frac{V_{us}}{V_{ud}} \frac{f_K}{f_\pi}$$

$$\times \left[\frac{\mathcal{B}(B^+ \to \pi^+\pi^0) + \mathcal{B}(B^- \to \pi^-\pi^0)}{\mathcal{B}(B^+ \to \pi^+K^0) + \mathcal{B}(B^- \to \pi^-\bar{K}^0)} \right]^{1/2},$$

$$r_\pm = 2\left[\frac{\mathcal{B}(B^\pm \to \pi^0K^\pm)}{\mathcal{B}(B^+ \to \pi^+K^0) + \mathcal{B}(B^- \to \pi^-\bar{K}^0)} \right]^{1/2}. \qquad (4)$$

A future measurement of $r_+ \neq r_-$ would signal direct CP violation in the decays $B^\pm \to \pi^0K^\pm$. At present, preliminary data reported by the CLEO Collaboration [25] imply $[\frac{1}{2}(r_+^2 + r_-^2)]^{1/2} = 1.46 \pm 0.37$ and, combined with some theoretical guidance, $\varepsilon_{3/2} = 0.24 \pm 0.06$ [18]. Moreover, we define

$$\delta_{EW} - e^{i\gamma} \equiv \varrho(z)e^{-i\psi} \qquad (5)$$

with $z = \cos\gamma$, so that

$$\varrho(z) = \sqrt{1 - 2z\delta_{EW} + \delta_{EW}^2}, \qquad \sin\psi = \frac{\sin\gamma}{\varrho(z)}. \qquad (6)$$

In terms of these quantities, the triangle relation (1) and its CP conjugate take the form

$$1 + \varepsilon_{3/2}\varrho(z)e^{i(\Delta\phi \mp \psi)} = r_\pm e^{i\xi_\pm}, \qquad (7)$$

where $\Delta\phi = \phi_{3/2} - \phi_P$ is an unknown strong-interaction phase difference, while the phases $\xi_\pm$ contain both strong and weak contributions. It follows that

$$\cos(\psi \mp \Delta\phi) = \frac{r_\pm^2 - 1 - \varepsilon_{3/2}^2\varrho^2(z)}{2\varepsilon_{3/2}\varrho(z)} \equiv x_\pm(z),$$

$$\cos(2\psi) = 1 - \frac{2(1 - z^2)}{\varrho^2(z)}. \qquad (8)$$

Combining these results, we find that the allowed solutions for $z = \cos\gamma$ can be obtained from the real zeros of the equation

$$\frac{(r_+^2 - r_-^2)^2}{16\varepsilon_{3/2}^2} + \frac{(1 - z^2)^2}{\varrho^2(z)} = (1 - z^2)[1 - x_+(z)x_-(z)], \qquad (9)$$

which, taking into account the z dependence of $\varrho(z)$ and $x_\pm(z)$, correspond to the zeros of a fourth-order polynomial in z.

A simplified analysis can be performed if the phase difference $\Delta\phi$ turns out to be small or close to $180°$—a possibility that can be tested for experimentally. To this end, one exploits the following exact relations:

$$\cos\gamma = \delta_{\text{EW}} - \frac{\frac{1}{2}(r_+^2 + r_-^2) - 1 - \varepsilon_{3/2}^2(1 - \delta_{\text{EW}}^2)}{2\varepsilon_{3/2}(\cos\Delta\phi + \varepsilon_{3/2}\delta_{\text{EW}})},$$
$$r_+^2 - r_-^2 = 4\varepsilon_{3/2}\sin\gamma\sin\Delta\phi. \tag{10}$$

The global analysis of the unitarity triangle prefers values of γ in the range $47° < \gamma < 105°$ [26], which would imply $\sin\gamma > 0.73$. Then the second relation can be used to obtain a reasonable estimate and upper limit for $\sin\Delta\phi$. If it turns out that $\sin\Delta\phi$ is small, corresponding to a situation where $|\Delta\phi| \approx 0°$ or $180°$, one can set $\cos\Delta\phi = \pm1$ in the first relation to obtain

$$\cos\gamma \approx \frac{(1 \pm \varepsilon_{3/2}\delta_{\text{EW}})^2 - \frac{1}{2}(r_+^2 + r_-^2) + \varepsilon_{3/2}^2}{2\varepsilon_{3/2}(\pm1 + \varepsilon_{3/2}\delta_{\text{EW}})}, \tag{11}$$

which determines $\cos\gamma$ up to a possible twofold ambiguity. From (10), it follows that a criterion for the validity of this approximation is that the deviation of $\cos\Delta\phi$ from ±1 be less than the uncertainty in the product $\varepsilon_{3/2}\delta_{\text{EW}}$, i.e., $\min(|\Delta\phi|, |\Delta\phi - \pi|) < \sqrt{2\Delta(\varepsilon_{3/2}\delta_{\text{EW}})}$. With present uncertainties on the parameters $\varepsilon_{3/2}$ and δ_{EW}, which are unlikely to be improved much in the near future, this implies $\min(|\Delta\phi|, |\Delta\phi - 180°|) < 17°$. With the current experimental values for the various parameters, and in the absence of independent experimental results for r_+ and r_-, the relations (10) do not yet provide for a useful estimate of $\cos\gamma$; however, they may become valuable with more precise measurements. It is remarkable that even in the case $r_+ = r_-$, i.e., in the absence of direct CP violation in $B^\pm \to \pi^0 K^\pm$ decays, $\cos\gamma$ can be determined using relation (11), which becomes exact in that limit.

In practice, the determination of γ using (9) or (10) is limited by experimental as well as theoretical uncertainties in the extraction of the parameters $r_\pm$, $\varepsilon_{3/2}$, and δ_{EW}. Let us illustrate the situation with a realistic example. Assume that the true values of the parameters are $\gamma = 76°$ (the center of the region preferred by the global analysis), $\varepsilon_{3/2} = 0.24$, and $\delta_{\text{EW}} = 0.63$ (the current central values), and that the strong phase difference takes the value $\Delta\phi = 20°$. It then follows that $r_+ \approx 1.18$ and $r_- \approx 1.04$. Let us assume that we can measure the values of these parameters with some errors given by $\Delta\varepsilon_{3/2} = 0.04$, $\Delta\delta_{\text{EW}} = 0.09$, and $\Delta r_\pm = 0.05$. We do not anticipate that $\varepsilon_{3/2}$ and δ_{EW} will soon be known with an accuracy much better than today, because these quantities are affected by theoretical uncertainties such as the estimate

of SU(3)-breaking effects. We thus assign a 15% error to them [27]. The assumed error on the amplitude ratios $r_\pm$ corresponds to a measurement of the corresponding ratios of branching ratios with a precision of about 10%. In this example, the approximate value for $\cos\gamma$ obtained by setting $\cos\Delta\phi = 1$ in (10) is $\cos\gamma \approx 0.26 \pm 0.14(r_\pm) \pm 0.09(\delta_{\text{EW}}) \pm 0.09(\varepsilon_{3/2})$, which is close to the correct value $\cos\gamma \approx 0.242$. We have quoted the various sources of errors separately. It is apparent that the precision in the measurements of the ratios $r_\pm$ is the limiting factor of our method. The approximate solution obtained with $\cos\Delta\phi = -1$ is $\cos\gamma \approx 1.13 \pm 0.19(r_\pm) \pm 0.12(\delta_{\text{EW}}) \pm 0.10(\varepsilon_{3/2})$, which is excluded by the global analysis of the unitarity triangle. In Fig. 1, we show the distribution of the exact real solutions of Eq. (9) for 1000 random choices of the Gaussian errors in the various input quantities. The solutions where $\cos\gamma \approx \pm1$ can again be excluded based on the global analysis of the unitarity triangle. From the central peak, we obtain $\cos\gamma = 0.24 \pm 0.18$, implying at 1 standard deviation $|\gamma| = (76 \pm 11)°$.

To conclude, we have shown that the weak phase $\gamma = \arg(V_{ub}^*)$ can be determined using time-independent measurements of the branching ratios for the decays $B^+ \to \pi^0 K^+$ and $B^- \to \pi^0 K^-$, as well as of the CP-averaged branching ratios for the decays $B^\pm \to \pi^\pm K^0$ and $B^\pm \to \pi^\pm \pi^0$. The new development that makes this method practical is the observation that the strong phases of the $I = \frac{3}{2}$ electroweak penguin and tree amplitudes are related to one another by Fierz identities and SU(3) flavor symmetry. SU(3)-breaking corrections can be accounted for in the factorization approximation. On the other hand, like many earlier proposals our method relies on the dynamical assumption that final-state rescatterings do not induce a sizable contribution proportional to the weak phase $e^{i\gamma}$ in the amplitude for the process $B^+ \to \pi^+ K^0$. The
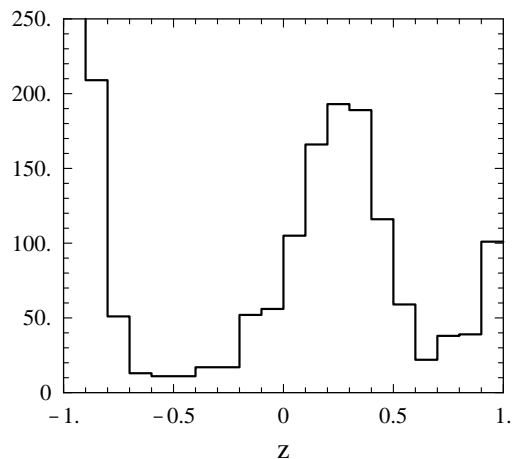

FIG. 1. Real solutions for $z = \cos\gamma$ obtained from (9) in a simulation of 1000 experiments with Gaussian errors as specified in the text. The correct value is $z = \cos 76° \approx 0.242$.

validity of this assumption can be tested for experimentally by searching for direct *CP* violation in this decay.

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[27] One may be able to reduce the error on $\varepsilon_{3/2}$ somewhat by studies of factorization in $B \to \pi^+\pi^-$ and $B \to \pi\ell\bar{\nu}_\ell$ decays; see the first paper in Ref. [19], where an ultimate error of 10% was projected for a related quantity without the color-suppressed contribution.