Comment on "Quantized Thermal Conductance of Dielectric Quantum Wires"

By applying the Landauer formalism, Rego and Kirczenow [1] have recently predicted a thermal conductance of a dielectric quantum wire due to phonon transport in the ballistic regime, surprisingly to be quantized with the same fundamental unit $\pi^2 k_B^2 T/3h$ as found for a 1D electron system in the ballistic regime by Greiner *et al.* [2].

The description of the phenomena within a correlation function formalism as given in [3] and applied in [2] is able to explain the result that under the given conditions ballistic degenerate electron and phonon systems show the same thermal conductance behavior even if they obey different statistics. This Comment will detail such an explanation.

For the ballistic case the main feature is that fluctuations may neither be created nor destroyed inside the wire. Therefore particle and energy currents for a single one-dimensional channel (i.e., electronic subband or vibrational mode) follow

$$I_\mu(t) = \frac{1}{L} \int j_\mu(z,t)\, dz\,, \tag{1}$$

where $j_\mu(z,t)$ is the current density, L is the length of the system, and $\mu = 1, 2$ denotes the particle or energy current-density, respectively.

The correlation functions (as given for the electronic system in [2]) for systems following either Bose or Fermi statistics with arbitrary dispersion read

$$C_{\mu\nu}(t) = \frac{N_\alpha}{2\pi L} \int_{-\infty}^{\infty} dk\, v_g^2(k) E(k)^{\mu+\nu-2} f(k)[1 \pm f(k)] \times \left(1 - \frac{|v_g(k)|}{L}\, t\right) \Theta\left(1 - \frac{|v_g(k)|}{L}\, t\right), \tag{2}$$

where $v_g(k) = 1/\hbar\, dE(k)/dk$ is the group velocity, $\Theta(x)$ is the unit step function, and N_α is the number of channels with the lowest energy minimum. The plus sign corresponds to the bosonic case, while the minus sign corresponds to the fermionic case.

Integrating over time and transforming the integration to energy space, the group velocities cancel because of the 1D density of states. A general equation for the spectral densities at zero frequency is found

$$S_{\mu\nu}(0) = \frac{N_\alpha}{h} \int_0^\infty dE E^{\mu+\nu-2} f(E)[1 \pm f(E)], \tag{3}$$

which is independent of the dispersion relation. The kinetic coefficients are calculated according to

$$L_{\mu\nu}(\omega) = \frac{L S_{\mu\nu}(\omega)}{k_B T}\,, \tag{4}$$

where L denotes the length of the system. In the case of massless bosons there is no gradient of the chemical potential ζ, thus L_{11} and $L_{12} = L_{21}$ have no meaning. A temperature gradient is the only driving force and the thermal conductance is given by $K = L_{22}/LT$, which indeed yields

$$K = \frac{N_\alpha \pi^2 k_B^2 T}{3h}\,, \tag{5}$$

as found for fermions [2] where $N_\alpha = 2$ due to the spin degeneracy.

The theory is readily extended to the case of massive bosons, where in general $\zeta \neq 0$. Thus, besides the temperature gradient the gradient of the chemical potential occurs as a generalized driving force. For the kinetic coefficients we then have

$$L_{11} = \frac{N_\alpha L}{h} \frac{1}{e^{-\zeta/k_B T} - 1}\,, \tag{6}$$

$$L_{12} = L_{21} = \frac{N_\alpha L k_B T}{h} \int_{-\zeta/k_B T}^{\infty} \frac{dx}{e^x - 1}\,, \tag{7}$$

$$L_{22} = \frac{N_\alpha L k_B^2 T^2}{h} \int_{-\zeta/k_B T}^{\infty} \frac{(x + \frac{\zeta}{k_B T})\, dx}{e^x - 1}\,. \tag{8}$$

(Note that for charged particles, if the electrical current instead of the particle current is used L_{11} has to be multiplied by e^2 and $L_{12} = L_{21}$ by e, where e is the unit charge.) For an ideal Bose gas with $\zeta \to 0$ we find that $L_{11} \to \infty$ and $L_{12} = L_{21} \to \infty$, while $L_{12}^2/L_{11} \to 0$. Thus, also in this case Eq. (5) for the thermal conductance holds.

At first sight the results found in [1] seem to be surprising. Application of the correlation function formalism to those ballistic 1D systems shows, on the contrary, that they reflect the statistical properties of the contacts, provided those contacts are ideal, i.e., completely absorbing and in thermal equilibrium.

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