Metal-Insulator Transition of Disordered Interacting Electrons

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We calculate the corrections to the conductivity and compressibility of a disordered metal when the mean free path is smaller than the screening length. Such a condition is shown to be realized for low densities and large disorder. Analysis of the stability of the metallic state reveals a transition to the insulating state in two dimensions. [S0031-9007(98)07734-5]

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The discovery of a metal-insulator transition in the two-dimensional electron gas, in a Si-MOSFET [1,2] and subsequently observed in other systems [3–6], suggests that there remains much to be understood in this classic problem. The available theory ignoring interactions predicts an insulating state for any disorder at all densities [7]. The most systematic theory, including effects of both disorder and interactions, due to Finkelstein [8], followed the discovery of singularities in the problem by Altshuler and Aronov [9]. It predicts a metallic state at all densities. The new experiments have also generated much recent theoretical activity [10].

Finkelstein's theory has, however, a remarkable prediction with which experiments are consistent. For a magnetic field coupling to spins an insulating state appears to occur at all densities at low enough temperature [2].

Finkelstein's theory is based on the existence of two scaling variables—effective disorder, parametrized by the conductance g, and an effective dimensionless spin-spin interaction parameter γ_t. The reduction to these two parameters is largely based on the assumption that compressibility must be continuous across a metal-insulator transition. This ensures that the electron-electron interactions γ_s in the singlet channel are irrelevant. Indeed, existing explicit calculations on the metallic side show no singular correction to the compressibility in leading order in disorder [8,9,11].

Here we seek to add an important physical feature to the theory of interacting disordered fermions so that the modified theory has a metal-insulator transition. It might be argued that Finkelstein's theory scales at low temperatures to a metallic state but with strong coupling in the spin-spin interaction channel where the analysis breaks down. Could a strong-coupling analysis of the same theory lead to a new low energy scale below which an insulating state emerges? We believe the experimentally observed metal-insulator transition is not due to the emergence of a new energy scale primarily because such a transition is apparent already at temperatures of $O(E_F)$ [1–6].

We have been motivated to reexamine the question of the renormalization of the compressibility by two arguments: First, the metal-insulator transition in the pure limit, i.e., the Wigner transition. As r_s is increased, either in two or three dimensions, a first-order transition to the Wigner crystal is expected to occur due to the long-range nature of the Coulomb interaction even for spinless electrons. The Wigner transition appears to satisfy conditions in which disorder turns a first-order transition to a continuous transition [12]. It seems surprising then that the singlet interaction would become irrelevant in the disordered problem. Second, if the insulating state has unscreened interactions, as commonly assumed, for instance, in the Efros-Shklovskii [13,14] derivation, the screening length on the metallic side must diverge as the metal-insulator transition is approached. The screening length is generally proportional to the inverse compressibility [see Eq. (5) below].

The correction to the compressibility can be calculated from the correction to the exchange and correlation contribution to the ground state energy (per unit area) from the so-called ring diagrams, with disorder, shown in Fig. 1:

$$E_{\text{ring}} = \frac{i}{2} \int d\omega \int d^2q \, \ln[1 + U(q)\pi(q,\omega)]. \quad (1)$$

Here $U(q) = 2\pi e^2/\epsilon q$ in two dimensions. The *proper* polarization in the diffusive regime is

$$\pi(q,\omega) = \nu \frac{Dq^2}{i\omega + Dq^2}, \qquad \text{for } q \ll \ell^{-1}, \omega \ll \tau^{-1}. \quad (2)$$

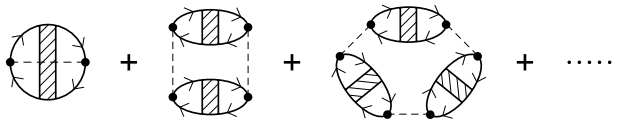

FIG. 1. Series for the ring contribution to the ground state energy with disorder. The hatched lines represent the t matrix for impurity scattering, and the dashed lines represent the Coulomb interaction.

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For other q and ω which we refer to as the ballistic regime, the polarizability is given by the generalization of the usual form [15] to include the leading order contribution of impurity scattering, $\omega \to \omega + i/\tau$, where $1/\tau$ is the single-particle scattering rate [16]. In Eq. (2) D is the diffusion constant, ℓ the mean free path, and ν the density of states. The compressibility at fixed density κ is calculated by $\kappa^{-1} = d^2E/dn^2$, where n is the density.

First consider the contribution to $E_{\rm ring}$ from the diffusive part. For $\ell \gg s_0$, where $s_0 \equiv (2\pi e^2\nu/\epsilon)^{-1}$ is the screening length in the Thomas-Fermi approximation, the leading contribution to $E_{\rm ring}$ is $\sim D/\ell^4$. This yields a nonsingular correction to the inverse compressibility $(\frac{1}{\kappa}) \sim (k_F\ell)^{-3}$, which decreases the compressibility with increasing disorder. For $\ell \ll s_0$, the contribution to $(E_{\rm ring})$ is $\sim D/s_0^4$. This is proportional to the density, so it produces no correction to the compressibility.

Consider next the contribution of the ballistic part. This is similar to the classic calculation of Gell-Mann and Brueckner [17,18] and others, but with the lower cutoff in the q integral given by ℓ^{-1}. Upon evaluation, the energy per particle, to the leading order in disorder, may be written in units of a Rydberg as

$$\mathcal{E}_{\rm ring} = \mathcal{E}^0_{\rm ring} - A\,\frac{s_0}{\ell}\,. \tag{3}$$

Here $\mathcal{E}^0_{\rm ring}$ is the contribution for $\ell \to \infty$, and the constant $A \approx 4/\pi$. The disorder correction comes mostly from a correction to the zero point energy of the plasmons. The corresponding additive correction to the inverse compressibility is

$$\left(\frac{\kappa_0}{\kappa}\right)_{\rm ring} \approx 0.16 r_s^2\,\frac{s_0}{\ell}\,, \tag{4}$$

where $\kappa_0 = \nu$ is the contribution to the compressibility of the kinetic energy. Here we have taken τ to be independent of the density, as is appropriate for the experimental systems in the immediate vicinity of the metal-insulator transition [19].

The actual screening length s is related to the compressibility through

$$s/s_0 = \kappa_0/\kappa\,. \tag{5}$$

Thus the screening length increases as ℓ^{-1}. We look for the condition that $s \gg \ell_0$, the value of ℓ at temperatures of the order of the Fermi energy. Equation (4) provides the dominant contribution for $r_s \gg 1$ and gives the condition

$$r_s \gtrsim 3(\omega_0\tau_0)^{1/2}, \tag{6}$$

where τ_0 is the scattering time and $\omega_0 = \hbar/ma_0^2$.

So far we have focused on the ring-diagram contribution to the ground state energy. The ring diagrams take into account direct processes and are sufficient for small momentum transfers even when r_s is not small. For large momentum transfers, processes beyond ring diagrams, representing exchange corrections at short distances, become important. We have calculated the contributions from these additional processes following the Hubbard interpolation scheme [20], in which the bare Coulomb interaction in the susceptibility is multiplied by a factor $[1 - F(q)]$, where $F(q) = q^2/2(q^2 + k_F^2)$. The leading order-disorder correction to the ground state energy is essentially unchanged from that given in Eq. (3).

We next consider the problem in a finite box of size L much larger than $\ell(L)$ (or equivalently a temperature $T = DL^{-2}$). For our considerations to be meaningful it is necessary that $\ell(L) \ll s(L)$ for L of order a few times ℓ, i.e., condition (6), be satisfied and remain consistently so as L is increased. To test the latter, we must first calculate the correction to ℓ as a function of L through the calculation of the conductance $g(L)$. In calculating the corrections to the conductance we assume that the condition $s(L) \gg L \gg \ell(L)$ is satisfied and check later for its consistency.

In this limit, the bare Coulomb interaction appears in the exchange correction to the conductivity. Consequently, the infrared singularity in $d = 2$ (and 3) is stronger than in the opposite limit. For $s \ll \ell$, the perturbative correction is proportional to $\ln(L/\ell)$, with a universal (and negative) coefficient [9]. The same processes with unscreened Coulomb interactions give

$$\frac{\delta g}{g} \simeq -\frac{2^{1/2}}{\pi^2}\,\frac{r_s L}{\ell}, \qquad \text{for } s \gg L \gg \ell\,. \tag{7}$$

whose coefficient is r_s dependent. This singular contribution arises from the contribution of momenta less than ℓ^{-1} and is related to the diffusion poles.

Next consider the Hartree corrections to the conductivity, which tend to enhance the conductivity in the limit $s \ll \ell$. The interactions appearing in Hartree corrections do not depend on the total momentum of the particle-hole pair carrying the current. They involve characteristic momenta of $O(k_F)$. The only effect of the interactions at small momentum transfers is to produce a $\ln(s)$ enhancement to the triplet amplitude. The Hartree terms provide the same logarithmic corrections for $s \gg \ell$ as in the opposite limit, and so may be neglected compared to the contribution of Eq. (7).

We can now check for the consistency of the assumption of unscreened interactions over the length scale L. The compressibility given by Eq. (4) is L dependent through $\ell(L)$. First we note that Eq. (7) leads to a linear decrease of the mean free path as L is increased. It introduces a length scale L^* at which the mean free path decreases to the atomic scale,

$$L^* \approx \ell_0\left(1 + \frac{\pi^2}{2^{1/2} r_s}\right). \tag{8}$$

The initial increase of the screening length, as L is increased, is given by

$$s(L) \approx s(\ell_0) + \frac{\sqrt{2}\, r_s}{\pi^2}\,\frac{s(\ell_0)}{\ell_0}\,(L - \ell_0)\,. \tag{9}$$

It is then easily seen that $s(L) \gg L \gg \ell(L)$, provided that $s(\ell_0)/\ell_0 \gg \max(1, \pi^2/\sqrt{2}\, r_s)$.

We have also calculated the correction to the single-particle density of states for the case that $s \gg \ell$,

$$\frac{\delta \nu}{\nu} \simeq -\frac{1}{\sqrt{2}\,\pi} \frac{r_s L}{\ell}. \tag{10}$$

Equation (10) implies that the leading correction to the single-particle density of states at zero energy is proportional to $-T^{-1/2}$. Equation (10) also implies that the single-particle self-energy is momentum dependent. The single-particle scattering time is then singularly modified, in a form similar to that of the transport lifetime.

It ought to be stressed that the results of this paper give only the leading high temperature corrections to the quantities calculated. However, Eq. (7) implies that the scale for the low temperature phenomena is of the order of the Fermi energy. The leading correction suggests that, provided the condition (6) is fulfilled, the screening length is consistently much longer than the mean free path and the relevant size of the system L so that the Coulomb interaction is unscreened inside L. The conductivity in that case rapidly tends to zero. In the opposite regime $s(\ell_0) \ll \ell_0$, the singularities found here are absent. The problem then is dominated by the diffusion processes, and the Finkelstein scaling equations, which scale towards the metallic state, are valid. In the transition regime, $s(L) \approx \ell(L)$, processes considered in both theories must be included. We hope to pursue such an analysis. But since the initial flow downwards of the conductance in Eq. (8) is much faster than the behavior in Finkelstein's theory, $s(\ell_0) \approx \ell_0$ may be taken as a good approximate condition for the metal-insulator transition.

The insulating state with disorder and Coulomb interactions is most likely a glass exhibiting the Efros-Shklovskii [13] phenomena. The precise behavior in the critical regime of the transition to such a glassy state is a difficult question which needs further study.

The major new result here is the demonstration of a route to a metal-insulator transition in two dimensions as density is decreased, as is found in experiments [1–6]. The metal-insulator transition is evident in the theory at temperatures of $O(E_F)$, also as in experiments. The theory preserves Finkelstein's prediction that a magnetic field coupling to spins turns the metallic state insulating, since γ_t becomes irrelevant. But the approach to the insulating state is likely to be different.

The most important prediction of the theory is the vanishing of the compressibility as the transition is approached from the metallic side. Some existing observations [21] in n-GaAs are consistent with the compressibility approaching zero as density is decreased towards $r_s \approx 6.8$. But the metal-insulator transition was not monitored in this experiment. We urge simultaneous compressibility and transport measurements to check Eq. (6). Frequency-dependent transport and susceptibility experiments are also suggested in the critical regime to test the idea that the transition is to a glassy state.

Another prediction of the theory is the condition (6) for the metal-insulator transition. There is not enough data to test this condition systematically. What exists is consistent with it in the τ dependence and approximately in magnitude of the critical r_s. In the reported results [1–6], the metal-insulator transition occurs at $\omega_0\tau_0$ of about 100 and r_s of about 20 with r_s at the transition showing slight increases as sample quality is improved. Equation (6) also implies that $k_F\ell$ at the transition point is of order unity, as is seen in the experiments [1–6].

While considerations of the variation of compressibility in a problem with Coulomb interactions lead to a metal-insulator transition in two dimensions, such a transition is already present in Finkelstein's theory in three dimensions. However, such considerations change the nature of the transition. We urge a study of the variation in compressibility as well as a study of frequency-dependent transport as well as magnetic susceptibilities near and across the metal-insulator transition, in three dimensions as well.

The basic ideas of this paper are of interest to several other electronic transitions, for example, the superconductor to insulator transitions [22] and the quantum-Hall transitions [23].

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