Comment on "Generalization of a Fermi Liquid to a Liquid with Fractional Exclusion Statistics in Arbitrary Dimensions: Theory of a Haldane Liquid"

In a recent Letter [1], Iguchi compares the concepts of Landau's Fermi liquid and Haldane's Luttinger liquid [2], introduces the concept of a Haldane liquid "as a generalization of Landau's Fermi liquid theory," and seeks to "generalize Luttinger's theorem for a Fermi liquid to Haldane liquids." Further, he asserts that quasiparticles with Haldane statistics [3] have not been previously studied. In fact, many of Iguchi's results have been previously obtained, as we shall show below.

Iguchi's concept of the Haldane liquid is in essence identical to the generalized Landau liquid introduced in Refs. [4,5], and Iguchi's generalized quasiparticles are the pseudoparticles of Ref. [4]. Several subsequent papers [5–10] studied the implications of this novel approach to Haldane's Luttinger liquids, focusing, in particular, on the 1D case. The equivalence of Iguchi's approach to this prior work is somewhat masked by his use of a slightly different representation. Specifically, the Bethe-ansatz (BA) solutions can be associated with two equivalent quasiparticle representations, which we call here (A) and (B). In representation (A), the quasiparticle discrete momenta are $q_j = \frac{2\pi}{L} I_j$, where L and I_j are the system length and the integer or half-odd integer BA quantum numbers, respectively. In representation (B), the rapidity numbers (or their real parts) k_j are chosen as the quasiparticle discrete momenta. Our earlier work used representation (A), whereas Iguchi uses (B). Since, however, the BA equations provide a one-to-one relation between the $[q_j]$ and the $[k_j]$ for each Hamiltonian eigenstate, these alternative representations are directly related and contain the same information.

In Ref. [5], five criteria were introduced to define the new generalized quantum liquids. Both the usual Fermi liquids and the 1D Landau liquids meet these five criteria. We emphasize that criteria (i) and (ii) of Ref. [5] *are equivalent* to the conjecture of Iguchi that there exists a class of interacting quasiparticle systems *in any dimension* such that Eq. (12) of Ref. [1] becomes exact.

Moreover, we have evaluated the exact form of the f functions for one-dimensional quantum liquids, shown that they can be expressed in terms of two-body scattering amplitudes between pseudoparticles [4,5,8–10], and related those to microscopic electronic quantities [9]. In addition, and in agreement with the suggestion of Iguchi, we have already used our theory to study many properties of the 1D Landau liquids along the lines of Fermi-liquid theory [5,7,8].

Finally, let us remark on two key distinctions: (1) Type (A) pseudoparticles have rational statistical parameters [see Eqs. (54)–(56) of Ref. [10]], as in Haldane's original theory [3], whereas Iguchi's quasiparticles have often momentum-dependent, nonrational statistical parameters; and (2) representation (A) is more suitable then representation (B) for exploring the relation to Landau's Fermi-liquid quasiparticles. For instance, in contrast to Eqs. (1), (6), and (8) of Ref. [1] and to Iguchi's $p_{F,0}$ expression, which are specific to representation (B), the α pseudoparticles and pseudoholes (with $\alpha = c, s$) of Refs. [4,5,7–9] have momentum distributions such that $N_\alpha(q) + N_\alpha^h(q) = 1$ and the total momentum, $P = \sum_{\alpha,q} q N_\alpha(q)$, and pseudo-Fermi momentum, $q_{F\alpha} = \pi N_\alpha / N_a$, are independent of the statistical parameters. This is similar to the Fermi-liquid quasiparticles and in contrast to the quasiparticle representation (B). In summary, the main universal features of 1D Luttinger liquids and 3D Fermi liquids have been studied extensively in our prior work, and several of Iguchi's properties concerning fractional statistical distributions and interactions are specific to his choice of representation (B).

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[1] Kazumoto Iguchi, Phys. Rev. Lett. **80**, 1698 (1998).
[2] F. D. M. Haldane, J. Phys. C **14**, 2585 (1981).
[3] F. D. M. Haldane, Phys. Rev. Lett. **67**, 937 (1991).
[4] J. Carmelo and A. A. Ovchinnikov, J. Phys. Condens. Matter **3**, 757 (1991).
[5] J. M. P. Carmelo, P. Horsch, and A. A. Ovchinnikov, Phys. Rev. B **46**, 14 728 (1992); see p. 14 762.
[6] J. M. P. Carmelo *et al.*, Phys. Rev. Lett. **68**, 871 (1992); **70**, 1904 (1993); **73**, 926 (1994); **74**(E), 3089 (1995).
[7] J. Carmelo, P. Horsch, P.-A. Bares, and A. A. Ovchinnikov, Phys. Rev. B **44**, 9967 (1991).
[8] J. M. P. Carmelo, P. Horsch, and A. A. Ovchinnikov, Phys. Rev. B **45**, 7899 (1992).
[9] J. M. P. Carmelo and A. H. Castro Neto, Phys. Rev. B **54**, 11 230 (1996).
[10] J. M. P. Carmelo and N. M. R. Peres, Phys. Rev. B **56**, 3717 (1997).

