Nonlinear Diffusion of the Magnetic Field in Weakly Ionized Plasmas

A. I. Smolyakov and I. Khabibrakhmanov

Department of Physics and Engineering Physics, University of Saskatchewan, Saskatoon, Saskatchewan, Canada S7N 5E2

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Enhanced nonlinear penetration of the external periodic magnetic field into a weakly ionized plasma is investigated. The penetration is enhanced by the convection of the magnetic field due to the Lorentz force. The nonlinearity leads to the wavelike propagation of the magnetic field or to the nonlinear diffusion similar to the diffusion in porous medium depending on plasma collisionality. It is shown that the periodic magnetic field experiences much slower decay inside the plasma compared to that of the standard skin effect. [S0031-9007(98)07748-5]

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In this paper we study nonlinear effects that significantly enhance the penetration of the magnetic field into the weakly ionized plasma. The penetration is enhanced by the nonlinear convection due to the Lorentz force associated with induced magnetic field. It is also accompanied by the nonlinear generation of higher harmonics leading to the steepening of the wave front and the formation of a shock wave. Low frequency components of the external perturbation form a quasistationary long lived magnetic field inside the plasma. The nonlinear model and behavior of the magnetic field in the weakly ionized plasma described in this paper have direct applications to a wide variety of laboratory [1–4] and space [5–10] plasmas. It is also similar to the nonlinear diffusion of the magnetic flux in the superconductors [11].

The effects studied in this paper are associated with the nonlinear modification of the Ohm's law due to the induced magnetic field $\tilde{B}$. The regime when the Lorentz force (Hall term) is important in the electron momentum balance equation and the ion motion can be neglected is commonly referred to as electron (Hall) magnetohydrodynamic (EMH) [3]. Applicability conditions for EMH are typically satisfied in the ionospheric plasma perturbed by electrodynamic tethers [5] and in the magnetosphere plasma [5] perturbed by short term variations of the solar wind, as well as in laboratory pulsed plasmas, in particular, plasma opening switch [4]. For an unmagnetized weakly ionized plasma the approach of the electron magnetohydrodynamic is applied when the electron cyclotron frequency $\omega_c = e\tilde{B}/m_e c$ in the induced magnetic field $\tilde{B}$ becomes comparable to the electron-neutral collisional frequency ν_{en} or to the characteristic frequency of the oscillations $\omega \simeq \tilde{B}^{-1}\partial\tilde{B}/\partial t$. The weakly ionized plasma typically used in low density inductively coupled plasma reactors for materials processing are often in the regimes with $\omega_c \sim 50{-}180$, $\nu_{en} \sim 5{-}20$, and $\omega \sim 1{-}85$ [1] (in units of 10^6 s^{-1}). Thus the low temperature plasma in inductive plasma reactors represents another example of plasma that is described by the EMH. In this paper we consider a situation when, in general, ion motion is important, so that the EMH condition will be slightly modified below. Nonlinear effects considered in this work are different from phenomena studied in [3] (see also Refs. [4,12], and references therein). The latter are related to inhomogeneities of plasma density and/or to the curvature of the magnetic field. In particular, the nonlinear diffusion considered in the present paper is operative for a homogenous plasma density and in a slab geometry (we consider cylindrical geometry here to make connection to experimental conditions in Ref. [1]).

The electron and ion components are described by the following moment equations:

$$\frac{d_\alpha \mathbf{v}_\alpha}{dt} = -\frac{e_\alpha}{m_\alpha}\left[\mathbf{E} + \frac{1}{c}\mathbf{v}_\alpha \times \mathbf{B}\right] - \nu_{\alpha n}\mathbf{v}_\alpha\,, \tag{1}$$

where $\nu_{\alpha n}$ is the collisional frequencies due to particle-neutral interaction, $\alpha = (e, i)$, $d_\alpha/dt = \partial/\partial t + \mathbf{v}_\alpha \cdot \nabla$. These equations are closed with the Ampere's law

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} = \frac{4\pi}{c}en(Z_i\mathbf{v}_i - \mathbf{v}_e)\,. \tag{2}$$

We assume plasma quasineutrality, $n_i = n_e = n$, and neglect all perturbations of plasma density. The plasma displacement current is also neglected in (2) assuming that the characteristic oscillation frequency is sufficiently low. The electron equation of motion (1) is conveniently written in the form of conservation of the generalized momentum [3]

$$\frac{\partial}{\partial t}\nabla \times \mathbf{p}_e - \nabla \times (\mathbf{v}_e \times \nabla \times \mathbf{p}_e) = -\nu_{en}m_e\nabla \times \mathbf{v}_e\,, \tag{3}$$

where $\mathbf{p}_e = m_e\mathbf{v}_e - e\mathbf{A}/c$, and $\mathbf{A}$ is the magnetic vector potential, $\mathbf{B} = \nabla \times \mathbf{A}$.

The ion and electron equations (1) are combined in the momentum balance equation

$$\frac{\partial}{\partial t}\mathbf{u} = \frac{1}{cnm}\mathbf{J} \times \mathbf{B} - \nu\mathbf{u} + \frac{\nu_{en}c}{4\pi en}\nabla \times \mathbf{B}\,, \tag{4}$$

where $m\mathbf{u} \equiv m_e\mathbf{v}_e + m_i\mathbf{v}_i$, $m \equiv m_i + m_e$, and $\nu = (\nu_{en}m_e + \nu_{in}m_{in})/m$. We have used the ordering $u \sim v_i \sim m_e v_e/m_i$ and neglected the small terms of the order of m_e/m_i.

When the collisionless skin depth is small compared to the characteristic wavelength, $k^2c^2/\omega_{pe}^2 < 1$, the field component of the generalized momentum is dominant, $m_e v_e \ll eA/c$. Then Eq. (3) reduces to

$$\frac{\partial}{\partial t}\mathbf{B} - \nabla \times (\mathbf{u} \times \mathbf{B}) + \nabla \times \left(\frac{c}{4\pi en}(\nabla \times \mathbf{B}) \times \mathbf{B}\right) = \frac{\nu_{en} m_e c}{e} \nabla \times \left(\mathbf{u} - \frac{c}{4\pi en} \nabla \times \mathbf{B}\right). \qquad (5)$$

Nonlinear Eqs. (4) and (5) describe complex coupled dynamics of the magnetic field and plasma velocity. In particular, the last term on the left hand side of this equation describes the nonlinear Hall effects associated with gradient of plasma density and curvature of the magnetic field [3,4,12]. In this work we study a simplest one-dimensional version of (4) and (5) where the nonlinear convection of the magnetic field by the plasma flow $\mathbf{u}$ is not masked by the Hall effect. We consider an infinite plasma cylinder along the z axis and a uniform plasma density, $n = \text{const}$. The azimuthally symmetric magnetic field is in the z direction and varies in radial direction r, $\mathbf{B}(r) = B(r)\hat{\mathbf{z}}$. Then the Hall term in (5) vanishes identically and the magnetic induction equation (5) takes the form

$$\frac{\partial}{\partial t} b + \frac{1}{r}\frac{\partial}{\partial r}(r u_r b) = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial}{\partial r} b\right). \qquad (6)$$

The radial plasma velocity u_r is directed toward the axis of the cylinder. Here, the time is normalized to the frequency of the external perturbation, $t' = \omega t$, the dimensionless length scale $r'^2 \to r^2/\delta^2$, $u_r' \to u_r/\omega\delta$, $\delta^2 = D_m/\omega$. The magnetic diffusion coefficient is $D_m = \nu_{en} m_e c^2/4\pi e^2 n$, and the dimensionless amplitude b is introduced by the relation

$$b^2 = \frac{e^2 B^2}{(\nu_{en} m_e + \nu_{in} m_{in})\nu_{en} m_e c^2}. \qquad (7)$$

The parameter b is a ratio of the cyclotron frequency in the induced magnetic field to the effective collisional frequency $\nu_{\text{eff}} = [(\nu_{en} + \nu_{in} m_i/m_e)\nu_{en}]^{1/2}$ and characterizes the nonlinearity of the problem. Note that according to Ref. [1] this parameter can be large in the inductively coupled plasma reactors. The second term in (6) describes the convection of the magnetic field by the radial plasma flow u_r. The equation for u_r decouples from (4) and takes the form

$$\alpha \frac{\partial}{\partial t} u_r = -\frac{\partial}{\partial r}\frac{b^2}{2} - u_r, \qquad (8)$$

where $\alpha = \omega/\nu$. The radial plasma velocity u_r is driven by the gradient of the magnetic field pressure. Note the analogy to Darcy's law for the fluid flow in porous medium. Equations (6) and (8) constitute our main equations for the nonlinear evolution of the magnetic field. For an axisymmetric infinite plasma cylinder, a singularity of the radial plasma velocity u_r may occur at the boundary $r = R$ and at the center of the cylinder $r = 0$. Practically, the continuity of the plasma flow can be maintained by ionization and recombination processes.

We investigate the inward penetration of the harmonic magnetic field along the z axis applied at the surface of the plasma cylinder $r = R$. In order to illustrate the properties of nonlinear diffusion we solved the system (6)–(8) for $\alpha = 1$, $R = 10$ with vanishing derivatives $\partial u/\partial r$ and $\partial b/\partial r$ at $r = 0$, $u(r) = b(r) = 0$ at $t = 0$, and $b(t, r = R) = b_0 \sin(2\pi t)$. The pseudospectral Chebyshev polynomial approximation in space and Crank-Nicholson discretization in time were used.

For a small amplitude of the external field $b_0(t) \lesssim 1$ the magnetic pressure gradient force is small so that nonlinear convective velocity u_r in (6) can be neglected. Then the magnetic induction equation (6) reduces to the linear diffusion equation, and the penetration depth is independent of the amplitude and given by the skin depth δ. The characteristic length scale (defined as the distance where the amplitude decreases in e-fold times) increases with the amplitude of the external field. This behavior is shown in Fig. 1 where the averaged over the period T profiles of the mean square of the magnetic field $\langle b \rangle = (\oint \tilde{B}^2 \, dt/T)^{1/2}$ are shown for different amplitudes. The position where the amplitude decreases in e-fold times for a given amplitude is marked on each curve;

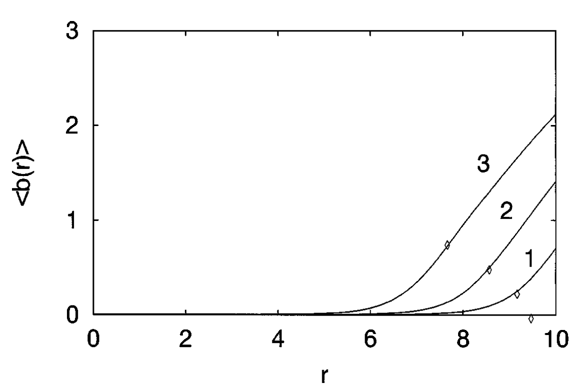

FIG. 1. The averaged magnetic field $\langle b \rangle$ as a function of distance for different amplitudes of the external perturbation: $b_0 = 1$, 2, and 3.

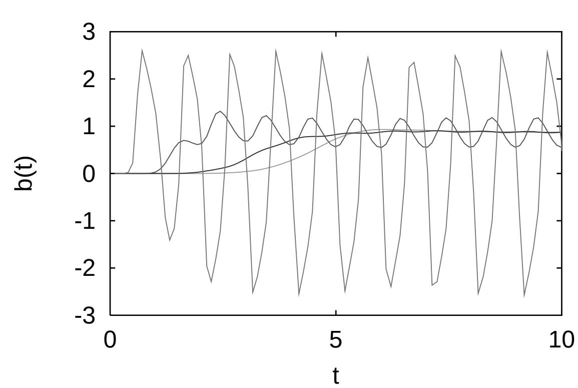

FIG. 2. The time history of the magnetic field at different positions ($r = 3R/4$, $R/2$, $R/4$, and 0) inside the plasma for $b_0 = 4.2\sin(2\pi t)$.

the characteristic length scale for the linear diffusion is marked on the x, axis. It should be noted also that the magnetic field decay changes from the exponential in the linear regime to a much slower pattern with a noticeable trend of changing the sign of the curvature of the profile in the region where the amplitude is large (Fig. 1).

The remarkable feature of the nonlinear dynamics described by Eqs. (6) and (8) is the fast penetration (on the time scale of several periods) of the mean magnetic field deep inside the plasma. We define the mean field $\overline{b}$ as the average in time over the period, $\overline{b} = \oint b\, dt/T$. As the magnetic field penetrates into the plasma an amplitude of oscillations decreases while the amplitude of the mean component grows. This is illustrated in Fig. 2 where the time dependent magnetic field is plotted at several positions inside the plasma column. It can clearly be seen that while the mean component of the magnetic field is absent at the plasma boundary it appears at a finite distance inside the plasma. It reaches its maximum on a time scale of the order of several periods. It should be noted that this mean magnetic field is not steady state but rather experiences a slow decay on a very long time scale of the order of several thousands of periods. When amplitude b_0 is increased characteristic decay time increases. The decay becomes faster again for larger amplitudes, when the oscillating components penetrate to the plasma interior. In our simulations we observe a significant amplitude of oscillations in the center for $b_0 = 6$ as shown in Fig. 3. Very few oscillations are present for $b_0 = 4.2$, and practically no oscillations in the center are observed for $b_0 = 3$.

The absolute value of the mean component of the magnetic field inside the plasma depends on the initial conditions of the applied field. Applying at time $t = 0$ the perturbation $b = b_0 \sin(2\pi t + \phi)$ with initial $b = 0$, $u = 0$ we found the amplitude of the mean field as a function of the initial phase ϕ. This dependence is shown in Fig. 4. Though partially it is determined by the low frequency part of the spectrum of the harmonic wave steplike switched on at $t = 0$, it is further modified by nonlinear effects. A similar behavior of the mean field was observed also for nonlinear penetration of the magnetic flux into the superconductors [11].

In the presence of a mean magnetic field the penetration of the external field is further enhanced due to appearance of wave solutions to Eqs. (6)–(8) that are associated with magnetosonic-type waves $\omega = k_r v_A$; k_r is the radial wave vector, $v_A^2 = B_0^2/4\pi nm$ is the Alfvén velocity, and B_0 is the permanent magnetic field. Wavelike propagation of the pulses of the magnetic field and reflection from the center of the plasma cylinder was also observed in our simulations.

The penetrating mean and oscillating components of the magnetic field create the finite averaged field $\langle b \rangle$ in the interior region far deeper than the linear skin depth as shown in Fig. 5. Such finite amplitude fields were observed experimentally in Ref. [1]. The averaged field $\langle b \rangle$ in Fig. 5 are given for the external harmonic perturbation $b = b_0 \sin(2\pi t)$ started at $t = 0$. For large amplitudes of the external perturbation, oscillations of the magnetic field penetrate plasma interior and their contribution to $\langle b \rangle$ becomes essential.

In the inertialess limit, when the characteristic frequency is small compared to the collisional frequencies, $\omega \ll \nu$, the α parameter is small and the time derivative term in (8) can be neglected so that we obtain a nonlinear evolution equation for the magnetic field [9]

$$\frac{\partial}{\partial t} b = \frac{1}{r} \frac{\partial}{\partial r} \left(r(1 + b^2) \frac{\partial}{\partial r} b \right). \tag{9}$$

This equation is similar to the porous medium equation and describes the diffusion of the magnetic field in a plasma with a strong plasma-neutral friction [9]. In general, numerical solutions of this equation show properties

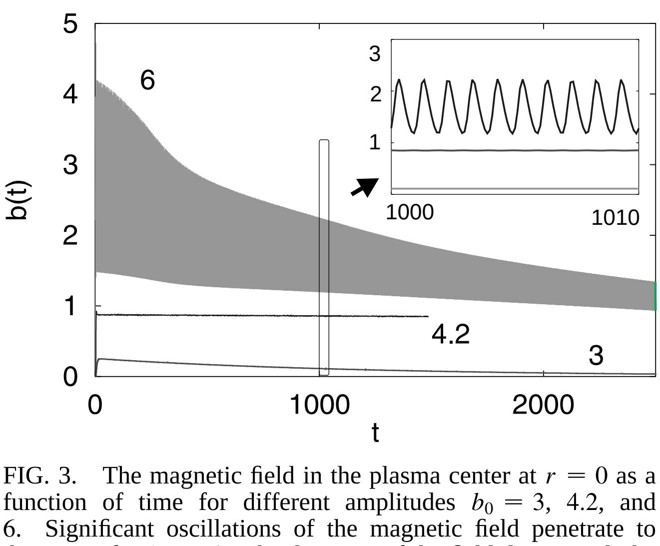

FIG. 3. The magnetic field in the plasma center at $r = 0$ as a function of time for different amplitudes $b_0 = 3$, 4.2, and 6. Significant oscillations of the magnetic field penetrate to the center for $b_0 = 6$. The damping of the field driven with the amplitude around $b_0 = 4.2$ is negligible. The inset shows the magnetic field for the time period from $t = 1000$ to $t = 1010$.

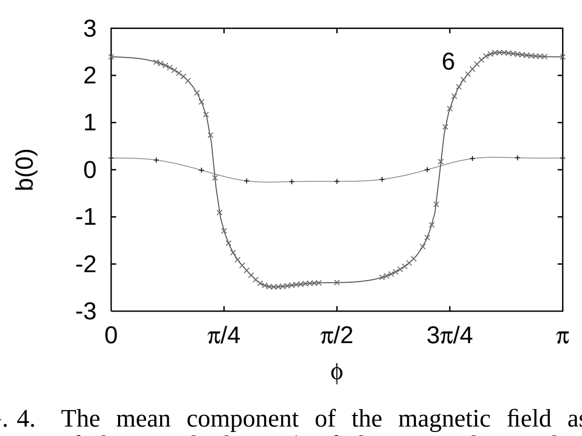

FIG. 4. The mean component of the magnetic field as a function of the initial phase ϕ of the external perturbation $b = b_0 \sin(2\pi t + \phi)$ for $b_0 = 6$ and $b_0 = 3$ (initial $b = 0$, $u = 0$ are used).

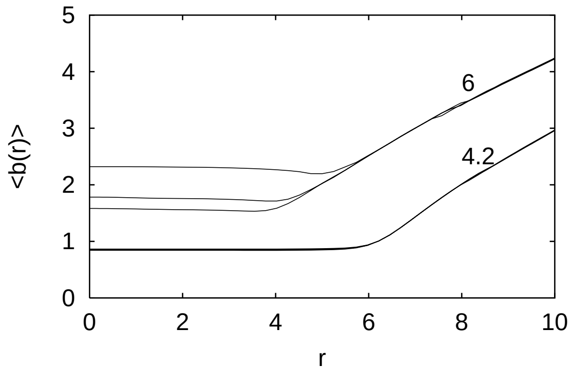

FIG. 5. The averaged magnetic field $\langle b \rangle$ as a function of distance for $b_0 = 4.2$ and $b_0 = 6$ at $t = 200T$, $800T$, and $1200T$. A weak decay of the mean magnetic field inside is noticeable for $b_0 = 6$; it is practically negligible for $b_0 = 4.2$ where the profiles at $t = 200T$, $800T$, and $1200T$ are indistinguishable.

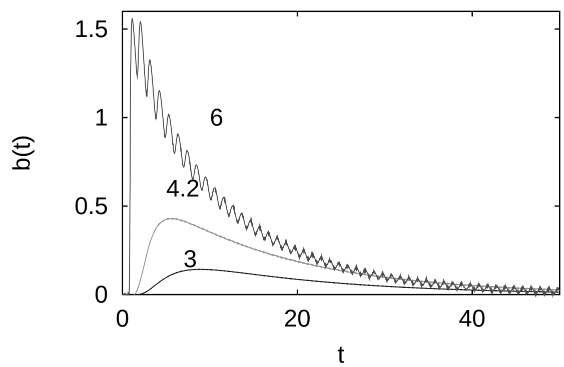

FIG. 6. The magnetic field in the plasma center at $r = 0$ as a function of time in the inertialess regime described by Eq. (9).

similar to those observed for Eqs. (6)–(8). Notice, however, that the characteristic time scale for the decay of the mean field is much shorter for (9) as illustrated in Fig. 6; compare with Fig. 3 for Eqs. (6)–(8).

The importance of the effects of the induced magnetic field on the heating and transport in the inductive plasmas was emphasized in Refs. [2,13–16]. In the present paper we have developed a self-consistent nonlinear model for the magnetic field penetration into such a plasma. Note that the detailed comparison with the experimental data on low temperature inductive discharges require consideration of the thermal dispersion effects [17] which are beyond the scope of this paper. The nonlinear mechanism analyzed in this paper can possibly explain a number of other phenomena in laboratory and space plasmas such as extremely long propagation of the magnetic pulses generated by lightning in the ionosphere [18] and formation of quasisteady magnetic structures in the ionosphere of Venus due to dynamic interaction with the magnetic field of the solar wind [6].

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