Transfer at Rough Sheared Interfaces

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Transfer of extensities such as heat across sheared rough interfaces is considered, with particular attention to roughnesses characterized by a broad hierarchy of length scales. We analyze as a concrete example the corrections to the Nu-Ra dependence in high Rayleigh number convection cells induced when the thermal layer thickness $\lambda \sim L\,\mathrm{Ra}^{-\beta}$ is of the order of the conducting walls roughnesses ξ. We show that, provided $\frac{\ln(5\,\mathrm{Pr})}{\mathrm{Pr}^{1/2}} \lesssim \frac{L}{\xi}\,\mathrm{Ra}^{1/2-2\beta}$, the increase of exchange surface area due to the covering of the rough surface by the thermal layer may significantly alter the transfer relationship in absolute value, and in law. [S0031-9007(98)07760-6]

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Smoothness is an idealization of nature since any material surface appears rough below a certain scale. Many physical phenomena are controlled by the rate of transfer across an irregular boundary. The examples include turbulent combustion and chemical reactions at distorted fronts [1–3], electrochemical transfers at porous electrodes [4], momentum transfer and friction laws of turbulent flow through rough pipes [5], pollutant transport in urban geometries [6], solute transfer at liquid-gas interfaces [7], diffusion of heat from rough surfaces [8], and turbulent convection in heat exchangers [9] or in high Rayleigh number convection cells [10,11].

In all of these situations, the interface extent, its roughness, and the net flux it absorbs are intimately linked. When the net flux is imposed, the interface area adapts itself for the product of the (mean) local conversion rate at the interface by the surface area to be equal to the net flux [1,3]. When the potential difference, or the local transfer rate across the boundary, is fixed, the interface extent, possibly a function of its roughness, imposes the net flux. This latter case concerns a wide class of applications [4–11] and is particularly relevant to convection over rough surfaces, discussed as a generic example in this paper.

We consider, with no loss of generality, a convection cell as the one sketched in Fig. 1; this configuration has been appreciably well documented recently (see, e.g., Refs. [10–12] and references cited in [13]) and presents, at high Rayleigh number, a coherent, large-scale circulation induced sheared boundary layers at the walls of the convection box. We could equally motivate what follows by considering a wind blowing over the urban canopy [6] or the convection flow over the protrusions in the channels of a heat exchanger [5,9], since the paradigm discussed here is the sheared boundary layer over a rough surface.

The Nusselt number $\mathrm{Nu} = L/2\lambda$ is a measure of the heat flux transferred across the cell and is proportional to the ratio of the linear size of the cell L to the thickness of the thermal boundary layer at the wall λ.

As the Rayleigh number Ra is increased, the thermal gradient steepens at the wall and λ decreases as $\lambda \sim L\,\mathrm{Ra}^{-\beta}$. For effectively flat boundaries (when λ exceeds by far the typical roughness of the conducting plates of the cell), $\beta = \frac{1}{3}$ at moderate Ra, $\beta = \frac{2}{7}$ at higher Ra, and may reach $\frac{1}{2}$ at very high Ra [12,13]. We investigate in this Letter the corrections to the Nu-Ra dependence when λ is of the order of the conducting walls roughness, and we are particularly interested in roughnesses characterized by a broad hierarchy of length scales (Figs. 1 and 2). Our approach is as follows: (i) We assume that the thermal boundary layer thickness λ depends on Ra as it would for a flat and smooth surface according to $\lambda \sim L\,\mathrm{Ra}^{-\beta}$. (ii) we assume that the thermal layer covers the rough surface (Figs. 1 and 2) like a thick sheet, able to fold within every unevenness of the surface provided the ruggedness scale is larger than its own transverse size λ. (iii) We assume that the net heat flux is augmented in proportion to the increase of surface area resulting from the covering process (ii) [1,3].

Conditions (i) and (ii) are fulfilled provided crests of the surface irregularities having the same height are

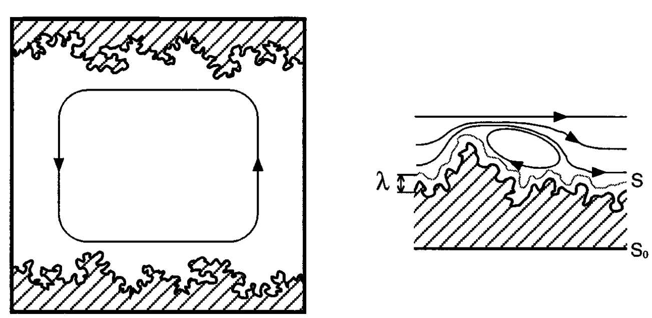

FIG. 1. Sketch of the convection cell in a Rayleigh number regime, where the boundary layers at the walls are sheared by a large recirculation flow. When the diffusion time λ^2/κ limits the transfer rate, the thermal layer covers all of the unevenness of the surface larger than its own thickness, thereby increasing the exchange surface $S(\lambda)$ with respect to the reference smooth surface S_0.

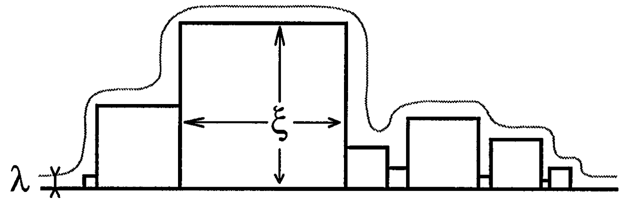

FIG. 2. A rough surface made as a collection of adjacent cubes with a given distribution of sizes $P(\xi)$.

sufficiently distant from each other. In that case, the separated boundary layer in the wake of a given crest forms a turbulent recirculating eddy before reaching the next crest. In this limit, valid as soon as the distance between two consecutive crests is larger than about their height [6,7], it is easy to see that the resistance to the transfer remains located in the thermal sublayer, independently of the rugosity: Let ξ be the height of any corrugation of the surface (Fig. 1). The recirculating eddy formed downstream of a protrusion of height ξ has a linear size proportional to ξ; estimating the mixing time in this recirculating eddy as [14] $t_m \sim (\xi/u)\ln(5\,\mathrm{Pr})$ for $\mathrm{Pr} > 1$ ($t_m \sim \xi/u$ for $\mathrm{Pr} < 1$), where u is taken here to be proportional to the mean circulating velocity, the diffusion time λ^2/κ, with $\lambda \sim L\,\mathrm{Ra}^{-\beta}$ is larger than the uniformization time of temperature in the eddy as long as $\lambda^2/\kappa > t_m$, that is, $\ln(5\,\mathrm{Pr})/\mathrm{Pr}^{1/2} \lesssim (L/\xi)\mathrm{Ra}^{1/2-2\beta}$. we have made use of $\mathrm{Pr} = \nu/\kappa$ and $\mathrm{Re} = (\mathrm{Ra}/\mathrm{Pr})^{1/2}$. This condition is reached because of the factor L/ξ, even at a high Rayleigh number for $\beta = \frac{2}{7}$, and thick rugosities. Taking, for instance, $\mathrm{Ra} = 10^{10}$ and $\mathrm{Pr} = 7$, the diffusion time λ^2/κ actually limits the transfer rate as soon as $L/\xi \gtrsim 4$, a condition always fulfilled in practice. The relevant thickness which sets the intensity of the transfer rate is thus λ in all cases (rough or not) and the thermal layer lies on the rough interface, visiting all available unevennesses larger than its own size.

In the other limit, when crests of a given height are spaced by a distance much shorter than their height, the flow essentially skims over the crests and over the stable vortices in the grooves through which heat diffuses slowly, realizing a "quasismooth" situation for which rugosity plays no role [7].

Under the assumptions (i) and (ii), if ϕ_0 is the flux measured at the reference flat surface S_0 (Fig. 1), condition (iii), expressing flux conservation, writes

$$\phi_0 S_0 = \phi(\lambda) S(\lambda), \tag{1}$$

with $\phi(\lambda) \sim L/\lambda$ being the flux imposed by the thickness of the thermal gradient λ and $S(\lambda)$ is the net area of the interface measured at scale λ. The apparent Nusselt number is thus

$$\mathrm{Nu} \sim \frac{L}{\lambda}\,\frac{S(\lambda)}{S_0}. \tag{2}$$

The effect of a multiple-scale roughness on the structure of the transfer law (2) is readily demonstrated for a fractal or self-affine surface, a property exhibited by most of natural, fractured, unpolished surfaces for scales smaller that about a fraction of a millimeter [15]. Assume that between ξ_M and ξ_m, respectively, the maximal and minimal scales of the surface rugosity, the surface of exchange $S(\lambda)$ follows a power law $S(\lambda)/S_0 \sim (\lambda/\xi_M)^{2-d_f}$, with d_f the fractal dimension of the surface ($d_f = 3 - H$ if H is the Hurst exponent of the roughness distribution for a self-affine surface [15]). Then, with $\lambda \sim L\,\mathrm{Ra}^{-\beta}$, Eq. (2) reads

$$\mathrm{Nu} \sim \mathrm{Ra}^{\beta+\beta(d_f-2)}. \tag{3}$$

The apparent exponent $\beta + \beta(d_f - 2)$ is larger than β because the net surface of exchange $S(\lambda)$ increases when λ decreases. For $d_f = 2.1$ (a typical value, see [15]), the apparent exponent of the transfer law (2) is thus increased by 10% with respect to the smooth case.

Irregular surfaces are more likely to be described by the histogram of their fluctuation height ξ above a given reference plane [15]. The histogram of heights ξ can be adjusted at will by constructing a rough surface as a set of adjacent objects of aspect ratio unity (cubes or spherical balls, for instance), with variable sizes ξ, these being chosen according to a given probability density function $P(\xi)$, as shown in Fig. 2.

This setup is easily realized in the laboratory and presents, as for the fractal surface, an enhanced heat flux with respect to the smooth case, but with *non-power-law* corrections to the Nusselt-Rayleigh dependence.

The distribution of object sizes $P(\xi)$ is bounded by maximal and minimal sizes ξ_M and ξ_m, and the reference surface S_0 is the sum of the projected areas on a plane of all of the N objects in the distribution

$$S_0 = N \int_{\xi_m}^{\xi_M} \xi^2 P(\xi)\, d\xi\,. \tag{4}$$

We cover entirely the smooth surface of area S_0 with roughness elements so that, once $P(\xi)$, ξ_m, and ξ_M are given, the number of objects N has to be adjusted so that (4) is satisfied. Let the objects have a cubic shape. When projected on the reference plane, a cube of size ξ has a projected area ξ^2. It contributes to the increase of the exchange surface by its lateral sides as soon as $\lambda < \xi$, and the corresponding increase of area, relative to ξ^2, is proportional to $\xi(\xi - \lambda)$. We do not consider any screening effect due to the presence of adjacent cubes on the surface increase so that our estimate is actually an upper bound. Thus the net increase of area is the sum of all of the contributions of cubes whose height is larger than λ,

$$\frac{S(\lambda) - S_0}{S_0} = \frac{\int_{\lambda}^{\xi_M} \xi(\xi - \lambda) P(\xi)\, d\xi}{\int_{\xi_m}^{\xi_M} \xi^2 P(\xi)\, d\xi}\,. \tag{5}$$

If the roughness distribution is peaked around a single value ξ_0 [i.e., $P(\xi) \sim \delta(\xi - \xi_0)$] as in the experiments of Shen *et al.* [11], then (5) leads to $[S(\lambda) - S_0]/S_0 = 1 - \lambda/\xi_0$, for $\lambda < \xi_0$. The surface of exchange, independent of Ra for $\lambda \gg \xi_0$, increases sharply when λ matches ξ_0, and then gets constant again for $\lambda \ll \xi_0$ as the Rayleigh number is further increased, augmented by a constant factor. The Nusselt number thus experiences a jump, and the transition occurs at the Rayleigh number such that $\lambda \approx \xi_0$. Before and after the transition, Nu and Ra are thus linked by the same power law ($\mathrm{Nu} \sim \mathrm{Ra}^{\beta}$), but with different prefactors, consistently with the observations of Shen *et al.* [11].

The increase of surface area is, however, continuous as Ra is varied if $P(\xi)$ is broadly distributed with, for instance, a power-law shape of the form $P(\xi) = A\xi^{-\alpha}$. A is a normalizing constant. Figure 3(a) shows how the exchange surface relative to the smooth surface S_0 depends on λ for several integer values of the roughness exponent α. As λ becomes smaller than the maximal corrugation scale ξ_M, the whole hierarchy of ever smaller and smaller cubes corrugates progressively the thermal layer and increases the surface $S(\lambda)$, as soon as λ compares to their size ξ. Ultimately, when $\lambda < \xi_m$, the interface area has been augmented by a factor proportional to S_0.

$S(\lambda)$ is not a power law of λ but rather a continuous crossover. For a given thermal thickness λ/ξ_M, the value of the surface area is smaller for larger α, but the width of the crossover and the variation rate $dS(\lambda)/d\lambda$ are more large because the cube size distribution $P(\xi)$ is steep, i.e., α is large.

The crossover is stretched and spread over a broad range of Rayleigh numbers by the transformation $\lambda \sim L\,\mathrm{Ra}^{-\beta}$, since $\beta < 1$. By the conservation relation (2), the Nusselt-Rayleigh law is found to exhibit, first, larger Nusselt numbers in absolute value with respect to the smooth case and, second, a steeper dependence very close to a power law (although it is not a strict power law but a broad crossover) on a Rayleigh number range limited to a few decades.

Taking, for instance, $\alpha = 2$ and assuming $\xi_m/\xi_M \ll 1$, Eqs. (2) and (5) give

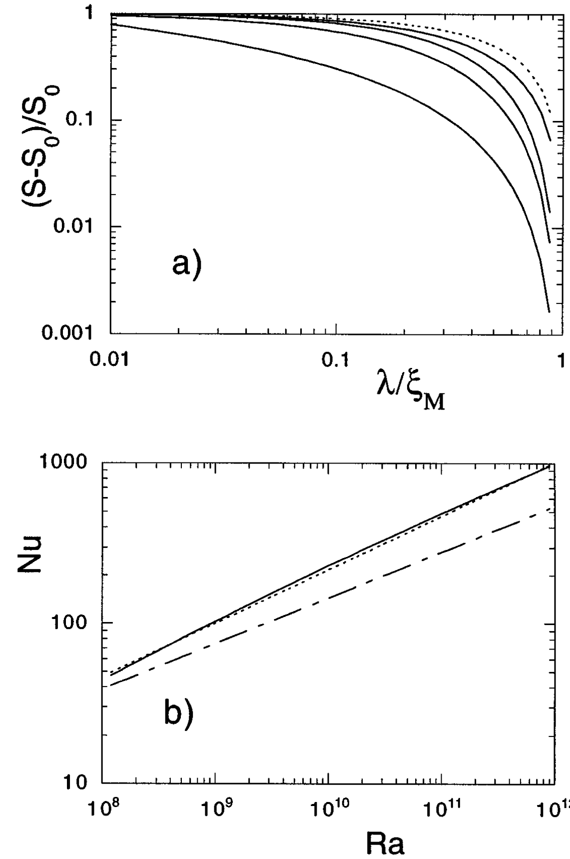

FIG. 3. (a) Relative increase of exchange areas $S(\lambda) - S_0$ with respect to the smooth surface area S_0 as a function of the thermal layer thickness λ for the following: (dotted line) a unique roughness size; (solid lines) a power law distribution of the sizes $P(\xi) \sim \xi^{-\alpha}$; and, from top to bottom, $\alpha = 0, 1, 2, 3$. (b) Nusselt-Rayleigh dependence for the following: (dashed-dotted line) the reference smooth transfer law $\mathrm{Nu} \sim \mathrm{Ra}^{2/7}$; (solid line) at a rough surface with $\alpha = 2$ and $L/\xi_M = 40$; (dotted line) a power law fit of the form $\mathrm{Nu} \sim \mathrm{Ra}^{1/3}$.

$$\mathrm{Nu} = 0.2\,\mathrm{Ra}^{2/7}\left[2 - 2.5\,\frac{L}{\xi_M}\,\mathrm{Ra}^{-2/7} + 2.5\,\frac{L}{\xi_M}\,\mathrm{Ra}^{-2/7}\ln\!\left(2.5\,\frac{L}{\xi_M}\,\mathrm{Ra}^{-2/7}\right)\right], \tag{6}$$

where $\mathrm{Nu} = 0.2\,\mathrm{Ra}^{2/7}$ has been chosen for the smooth transfer law [10–13]. One sees in Fig. 3(b) how the Nu-Ra relationship has been altered from the smooth to the rough case for $L/\xi_M = 40$ (a maximal rugosity of a few millimeters in a container about ten centimeters large). The effect is dramatic. The progressive surface increase transforms in that particular case a $\frac{2}{7}$ dependence into a dependence close to $\frac{1}{3}$.

The rate of dissipation of vorticity at a sheared rough surface is fixed by the maximal roughness only because viscous dissipation takes place in the wakes and in the recirculating eddies downstream of the largest surface protrusions. The size of the eddies and the mean shearing velocity set the viscous dissipation rate and therefore the drag coefficient [5]. Unlike vorticity, and provided the mixing time in the eddies is smaller than the diffusion time in the thermal sublayer, that is, as long as $\ln(5\,\mathrm{Pr})/\mathrm{Pr}^{1/2} \lesssim (L/\xi)\mathrm{Ra}^{1/2-2\beta}$ for $\mathrm{Pr} > 1$, heat transfer is sensitive to the details of the roughness distribution. This effect should be considered with caution in the investigation of transfer laws and "universal ultimate regimes" at real, and therefore rough, surfaces in the high Rayleigh or Reynolds number limit.

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[1] D. Damköhler, Z. Elektrochem. **46**(11), 601–652 (1940).

[2] W. R. Hawthorne, D. S. Wendell, and H. C. Hottel, *Third Symposium on Combustion and Flame and Explosion Phenomena* (Williams & Wilkins, Baltimore, 1949), pp. 266–288.
[3] E. Villermaux, in *Advances in Turbulence*, edited by V. R. Benzi (Kluwer, Dordrecht, 1995), pp. 529–533.
[4] B. Sapoval, J. N. Chazalviel, and J. Peyrière, Phys. Rev. A **38**, 5867–5887 (1988).
[5] H. Schlichting, *Boundary Layer Theory* (McGraw-Hill, New York, 1987).
[6] T. R. Oke, Energy Build. **11**, 103–113 (1988); J. F. Sini, S. Anquetin, and P. G. Mestayer, Atmos. Environ. **30**(15), 2659–2677 (1996).
[7] J. T. Davies, *Turbulence Phenomena* (Academic, New York, 1972).
[8] D. Vandembroucq, A. C. Boccara, and S. Roux, Europhys. Lett. **30**(4), 209–214 (1995).
[9] W. H. McAdams, *Heat Transmission* (McGraw-Hill, New York, 1954).
[10] S. Ciliberto, S. Cioni, and C. Laroche, Phys. Rev. E **54**, 5901–5905 (1996).
[11] Y. Shen, P. Tong, and K. Q. Xia, Phys. Rev. Lett. **76**, 908–911 (1996); see also Y.-B. Du and P. Tong, Phys. Rev. Lett. **81**, 987–990 (1998).
[12] X. Chavanne *et al.*, Phys. Rev. Lett. **79**, 3648–3651 (1997).
[13] E. D. Siggia, Annu. Rev. Fluid Mech. **26**, 137–168 (1994).
[14] E. Villermaux, C. Innocenti, and J. Duplat, C. R. Acad. Sci. Paris **326**, IIb, 21–26 (1998).
[15] E. Bouchaud, G. Lapasset, and J. Planès, Europhys. Lett. **13**, 73–79 (1990); J. Feder, *Fractals* (Plenum, New York, 1988).