Rotational Frequency Shift of a Light Beam

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We explain the rotational frequency shift of a light beam in classical terms and measure it using a mm-wave source. The shift is equal to the total angular momentum per photon multiplied by the angular velocity between the source and observer. This is analogous to the translational Doppler shift, which is equal to the momentum per photon multiplied by the translational velocity. We show that the shifts due to the spin and orbital angular momentum components of the light beam act in an additive way. [S0031-9007(98)07769-2]

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The nonrelativistic, translational Doppler shift is a well-known phenomenon. The relative velocity v between light source and observer leads to a frequency shift $\Delta\omega = vk$, where $\hbar k$ is the linear momentum of each photon. The nonrelativistic rotational frequency shift identified in this paper can be described analogously by $\Delta\omega = \Omega J$, where Ω is the angular velocity between source and observer and $\hbar J$ is the total angular momentum per photon. We show that this result can be understood easily in terms of the rotational symmetry associated with the transverse electric field distribution of circularly polarized Laguerre-Gaussian beams.

It should be emphasized that the rotational frequency shift is completely different from the translational Doppler shift observed for a rotating source that arises from a linear velocity component in the plane of rotation (Fig. 1). The rotational shift is maximal when observed at right angles to the plane of rotation where the translational shift is zero.

It is well known that each photon in a light beam carries a spin angular momentum $\sigma\hbar$, where $\sigma = \pm 1$ corresponds to left- and right-handed circularly polarized light. Less well known is that light beams can also have an orbital angular momentum associated with the phase structure of the beam. In 1992 Allen *et al.* [1] showed for beams with an azimuthal phase term of $\exp(il\phi)$, such as Laguerre-Gaussian laser modes, that the orbital angular momentum is $l\hbar$ per photon. Laguerre-Gaussian modes have helical wave fronts with a phase discontinuity on the axis of the beam and an intensity distribution of one or more concentric rings [2]. These modes form a complete basis set; consequently, any arbitrary light beam can be described by a superposition of Laguerre-Gaussian modes.

For *spin* angular momentum alone, the rotational frequency shift has a number of precedents. In 1981 Garetz [3] attributed phenomena such as rotational Raman spectra and the frequency shift previously observed with a rotating half-wave plate [4] to an "angular Doppler effect." In each case, a circularly polarized light beam rotating with an angular velocity Ω with respect to the observer produces a frequency shift of $\Omega\sigma$. Bialynicki-Birula and Bialynicka-Birula [5] predicted in a quantum mechanical calculation that for an atomic system subject to a rotating potential a polarized transition experiences a frequency shift of ΩJ. We have shown recently [6] that the rotation of beams with an $\exp(il\phi)$ phase term, and so an *orbital* angular momentum of $l\hbar$ per photon, results in a frequency shift of Ωl. This shift can be related to previous predictions for the azimuthal Doppler shift of atomic transition frequencies [7] and to the frequency shift introduced by rotating cylindrical lenses [8]. In this paper we unify previous studies to include both spin and orbital angular momentum simultaneously. We find that the spin and orbital contributions behave in an additive way, such that it is the *total* angular momentum per photon, $\hbar J = \hbar(l + \sigma)$, that gives a rotational frequency shift of $\Delta\omega = \Omega J$.

In our experiment we rotate a circularly polarized beam possessing helical wave fronts and directly measure the corresponding frequency shift. The rotation of a source or detector without the introduction of unwanted off-axis

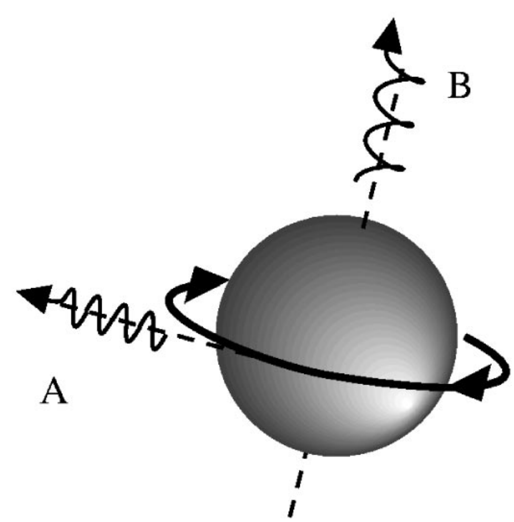

FIG. 1. Both the rotational frequency shift and the translational Doppler shift can be observed in the same rotating source. The frequency of light beam *A* is shifted due to the translational Doppler shift; light beam *B* is frequency shifted due to the rotational frequency shift.

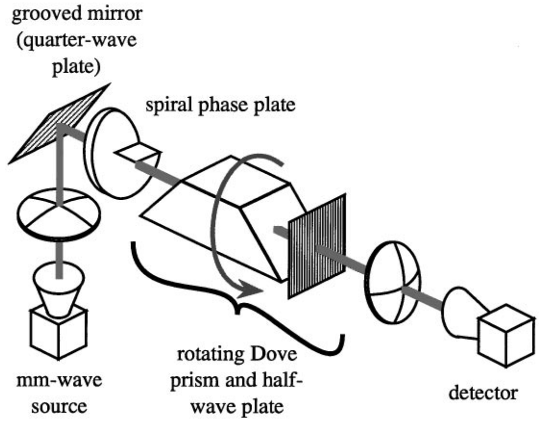

FIG. 2. Schematic diagram of the experimental apparatus for the observation of the rotational frequency shift.

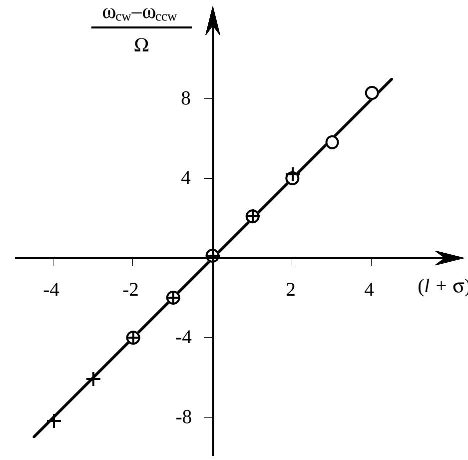

FIG. 3. Observed rotational frequency shift for various values of $(l + \sigma)$. The crosses and circles correspond to σ of -1 and $+1$, respectively. The beam angular velocity Ω was 1.25 Hz. The error associated with each measurement is 0.2 Hz.

movement is difficult to achieve. As an alternative, we use a Dove prism as an image rotator; the transmitted image and associated phase structure rotate at twice the angular velocity of the prism. Similarly, a half-wave plate rotates the electric field vector of a transmitted beam at twice the angular velocity of the wave plate. The combination of a Dove prism and a half-wave plate therefore rotates the whole beam while leaving the source stationary. To minimize difficulties in alignment of the prism with the beam axis, our experiment is performed in the mm-wave regime, using a Gunn diode source oscillating at 94 GHz with a corresponding wavelength of approximately 3 mm. Figure 2 shows the experimental arrangement. The source frequency is phase locked to a high harmonic of a crystal oscillator giving a short-term frequency stability of approximately 1 Hz. A corrugated feed horn couples approximately 10 mW into a linearly polarized fundamental Gaussian beam. This beam is collimated using a polyethylene lens and converted into a beam with helical wave fronts by transmission through a polyethylene phase plate whose thickness increases with azimuthal angle [9]. The beam is circularly polarized by reflection from a surface that has $\lambda/8$ deep grooves aligned at 45° to the incident linear polarization [10]. The frequency of the beam after transmission through the rotating polyethylene Dove prism and half-wave plate [10] is measured directly using a mm-wave frequency counter which is phase referenced to the source. The rotation of the beam introduces a shift in frequency. Reversing either the rotation direction or the sign of J changes the sense of the shift. Consequently, we expect the frequency difference between clockwise and counterclockwise rotation to be given by $\omega_{\mathrm{cw}} - \omega_{\mathrm{ccw}} = 2\Omega J = 2\Omega(l + \sigma)$. Figure 3 shows the observed frequency difference for various combinations of l and σ. The experimental observations agree with our predictions.

The rotational frequency shift could be interpreted in terms of an energy exchange with the rotating optical element [8,11]. However, it is more illuminating to explain the shift in terms of the field distribution of circularly polarized Laguerre-Gaussian beams. As the electric field evolves in time, its distribution rotates. In each case the field distribution has $(l + \sigma)$-fold rotational symmetry (Fig. 4) and the electric field undergoes a change of $l + \sigma$ cycles for each rotation of the beam.

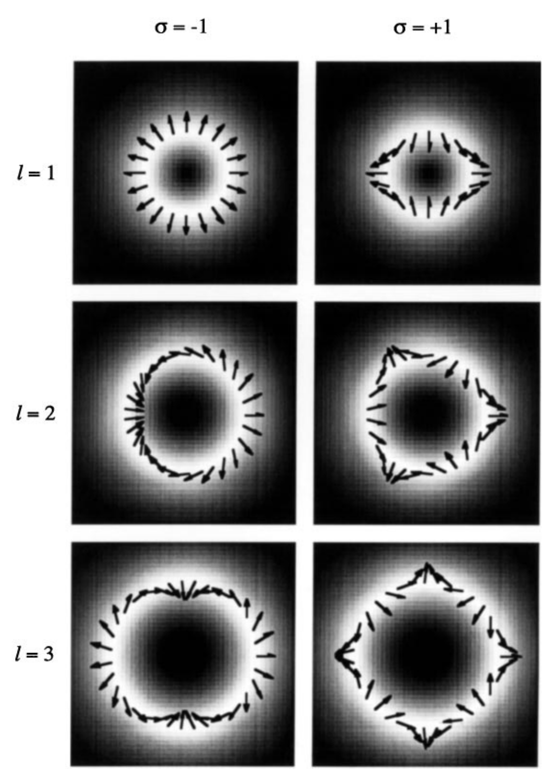

FIG. 4. Vector plots of the transverse electric field for circularly polarized Laguerre-Gaussian beams, showing the $(l + \sigma)$-fold rotational symmetry.

For circularly polarized light the electric field rotates at the optical frequency, and therefore rotation of the beam results in a frequency shift. The direction of this shift depends on whether the sense of the beam rotation adds or subtracts to the rotation of the electric field. This is completely analogous to the translational Doppler shift which also depends upon a change in the number of cycles observed.

The translational Doppler shift is proportional both to the relative velocity and to the unshifted frequency. Although the rotational frequency shift is proportional to the angular velocity, it is independent of the unshifted frequency. This allowed us to perform our experiments in the mm-wave regime without reducing the magnitude of the shift. Unlike the translational Doppler shift, the rotational frequency shift depends on the distribution of the transverse electric field of the beam. Only for a pure circularly polarized Laguerre-Gaussian beam is there a single frequency shift and not a splitting. However, any arbitrary field distribution can be decomposed into a superposition of Laguerre-Gaussian beams; the resulting spectrum will consequently consist of sidebands at shifts proportional to the J value of each component.

We have described classically the rotational frequency shift, analogous to the translational Doppler shift, which arises from an angular velocity Ω between source and observer. For circularly polarized Laguerre-Gaussian beams the total angular momentum per photon is $\hbar J = \hbar(l + \sigma)$. The resulting rotational frequency shift is given by $\Delta\omega = \Omega J$. Any field distribution can be expressed as a superposition of circularly polarized Laguerre-Gaussian beams and, if rotated, results in a frequency spectrum consisting of sidebands about the unshifted frequency. Specific combinations of Laguerre-Gaussian beams allow the generation of particular frequency distributions, which may have applications for ranging, communication systems or optical guiding of atoms.

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