Calculation of Proton-Deuteron Elastic Scattering at 10 MeV with a Realistic Potential

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We present the first results of a calculation of the differential cross section and of polarization observables for proton-deuteron elastic scattering at 10 MeV proton laboratory energy, for the Paris potential. The method used is the "screening and renormalization approach" which allows one to correctly take into account the Coulomb repulsion between the two protons. Comparison is made with the precise experimental data of Sagara *et al.* [Phys. Rev. C **50**, 576 (1994)] and of Sperison *et al.* [Nucl. Phys. **A422**, 81 (1984)]. [S0031-9007(98)07756-4]

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During the past decade, the theoretical calculation of neutron-deuteron (nd) scattering observables has become feasible using the most up-to-date models of the nucleon-nucleon (NN) interaction including three-nucleon forces [1]. The goal behind this endeavor is to get a better understanding of interesting physical phenomena such as three-nucleon forces and off-shell behavior of the nuclear interaction; but also signatures of the quark substructure may be obtainable. Furthermore, extraction of NN on-shell information which is difficult to deduce from NN scattering experiments could be possible. Such expectations are, however, to be confronted with the experimental situation which for neutrons as projectiles is rather unsatisfactory: despite great efforts, the available data are sparse and of an accuracy which as yet does not allow one to meaningfully differentiate between all of the various theoretical assumptions entering the calculations.

For the proton-induced reaction (pd), on the other hand, a rich body of accurate data is available and is still being continuously expanded. There, however, the theoretical situation has been unsatisfactory until now. Either, when being of similar sophistication as in the nd case, reliable calculations had to be confined to energies below the deuteron breakup threshold [2,3] or, when performed at positive energies, had to resort to a simple ansatz for the nuclear interaction [4]. But the few cases where experiments have been performed for the nd and the pd reaction, for the same observable, at the same energy, clearly show that the presence of the Coulomb force in the latter case, in general, modifies the observables appreciably. Hence, the standard procedure of comparing "realistic" nd calculations with pd data must be considered unsatisfactory in that the expected (smaller) effects of the above-mentioned interesting topics might be more or less veiled by the neglected Coulomb effects.

Thus, in order to make adequate use of the high precision pd data it is of foremost importance to reliably estimate the influence of the Coulomb repulsion between the two protons, for all energies, with realistic nuclear forces. In this paper, we present the first results obtained along these lines for various pd elastic scattering observables, at an energy above the deuteron breakup threshold. For our calculations we use the screening and renormalization approach [5,6] (for a recent review, see [7]) as formulated for momentum space integral equations. Here, we only sketch the basic idea. It consists of separating the long-range from the shorter-range Coulomb effects and treating the former separately. The (technically convenient, but not mandatory) starting point is a separable representation of the nuclear interaction for each partial wave taken into account. For this purpose any of the well-tested separable expansion methods can be used. Then the T matrices describing scattering in the np subsystem are purely separable while the ones for the pp subsystem contain the additional nonseparable Coulomb amplitude T^R, as calculated from a screened Coulomb potential. R denotes the screening radius. Clearly, by switching off the Coulomb interaction, the amplitudes for the neutron-induced reaction are recovered.

We enumerate the identical particles [the two protons (neutrons) in the proton-(neutron-)induced reaction] by 1 and 2, and the odd particle by 3. Thus the α-subsystem T matrix is assumed to be given as

$$T_\alpha^{(R)} = \sum_{m,n} |\tilde{\chi}_{\alpha m}^{(R)}\rangle \Delta_{\alpha,mn}^{(R)} \langle \tilde{\chi}_{\alpha n}^{(R)}| + \delta_{\alpha 3} T^R, \qquad \alpha = 1,2,3. \tag{1}$$

The element $\Delta_{\alpha,dd}^{(R)}$, for $\alpha = 1,2$, is to contain the deuteron pole which is guaranteed if at the pole the deuteron "form factor" $|\tilde{\chi}_{\alpha d}^{(R)}\rangle$ is related to the deuteron bound state wave function $|\psi_d\rangle$ in the standard manner (note that only the pp-subsystem form factors $|\tilde{\chi}_{3n}^{(R)}\rangle$ contain Coulomb distortions). Here and in the following, the energy dependence of the various operators is suppressed. Since for finite R all potentials are of short range, conventional three-body scattering theory is applicable.

We use the effective-two-body formulation of the three-body theory proposed in [8] in which the two-fragment

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amplitudes $\mathcal{T}^{(R)}_{\beta n,\alpha m}$ are obtained as solutions of multichannel Lippmann-Schwinger-type equations which in operator matrix form read as

$$\mathcal{T}^{(R)} = \mathcal{V}^{(R)} + \mathcal{V}^{(R)}\mathcal{G}^{(R)}_0\mathcal{T}^{(R)}. \quad (2)$$

Denoting the incoming and the outgoing center-of-mass (c.m.) momenta by $\mathbf{q}_\alpha$ and $\mathbf{q}'_\beta$ and the corresponding bound state quantum numbers by m and n, respectively, the physical transition amplitudes from channel (αm) to channel (βn) are then given as $\mathcal{T}^{(R)}_{\beta n,\alpha m}(\mathbf{q}'_\beta, \mathbf{q}_\alpha) = \langle \mathbf{q}'_\beta | \mathcal{T}^{(R)}_{\beta n,\alpha m} | \mathbf{q}_\alpha \rangle$. Here,

$$\mathcal{V}^{(R)}_{\beta n,\alpha m} = \langle \tilde{\chi}^{(R)}_{\beta n} | [\overline{\delta}_{\beta\alpha} G_0 + \overline{\delta}_{\beta 3}\overline{\delta}_{\alpha 3} G_0 T^R G_0] | \tilde{\chi}^{(R)}_{\alpha m} \rangle \quad (3)$$

is the effective potential and $\mathcal{G}^{(R)}_{0;\beta n,\alpha m} = \delta_{\beta\alpha}\Delta^{(R)}_{\alpha,nm}$ is the effective free Green function. G_0 is the free resolvent and $\overline{\delta}_{\beta\alpha} = 1 - \delta_{\beta\alpha}$. For more details, see [5,7]. Note that, for the case considered here, Eqs. (2) and (3) are exact.

The important point is that the diagonal part $\mathcal{V}^{(R)}_{\alpha m,\alpha m}$ contains, for $\alpha \neq 3$ and with the α subsystem being in the deuteron state, as its longest-ranged part (in the limit $R \to \infty$), the so-called c.m. Coulomb potential v^R_α. It describes the Coulomb scattering of proton α (= 1 or 2) off the total charge of the deuteron concentrated in its center of mass and is the only part which requires application of the renormalization procedure. As shown in [5,6], multiplication of the on-shell solution of (2) by appropriate, explicitly known renormalization factors $Z_{\alpha,R}(q_\alpha)$ and $Z_{\beta,R}(q'_\beta)$ guarantees the existence of

$$\lim_{R\to\infty} Z^{-1/2}_{\beta,R}(q'_\beta)\mathcal{T}^{(R)}_{\beta n,\alpha m}(\mathbf{q}'_\beta, \mathbf{q}_\alpha; E_+)Z^{-1/2}_{\alpha,R}(q_\alpha) = \delta_{\beta\alpha}\delta_{nm}t^C_\alpha(\mathbf{q}'_\alpha, \mathbf{q}_\alpha) + \langle \mathbf{q}^{\prime(-)}_{\beta,C} | \mathcal{T}^{SC}_{\beta n,\alpha m}(E_+) | \mathbf{q}^{(+)}_{\alpha,C} \rangle, \quad (4)$$

where $E_+ = E + i0$. Here, t^C_α is the amplitude, and $|\mathbf{q}^{(\pm)}_{\alpha,C}\rangle$ is the scattering wave function (for an asymptotic momentum $\mathbf{q}_\alpha$ and energy $3q^2_\alpha/4M$, M being the nucleon mass) belonging to the unscreened Coulomb potential $v^C_\alpha := \lim_{R\to\infty} v^R_\alpha$. Note that the definition (4) of the charged-composite particle amplitude coincides with the one following from time-dependent scattering theory [7,9].

We recall two of the main advantages of this approach: (i) In Eq. (2) only the amplitudes for all *binary* processes are coupled, and (ii) in Eq. (3) the pp Coulomb amplitude is taken into account in three-dimensional form. Thus, no problems arise from lack of convergence of Coulomb partial wave series. In contrast, for methods based on (integro-)differential equations for wave functions in coordinate space two (of several) as yet unresolved difficulties originate in this context (cf. the discussion in [10]).

When solving Eq. (2) we have made only the so-called Coulomb-Born approximation (CBA) which consists of replacing everywhere the Coulomb T operator by the potential. In the Coulomb parts of the effective potential (3) this has been shown (for $R \to \infty$) in [11,12] to be accurate to better than 10%; and, since the latter are known to be of the order of at most 10% of the purely nuclear parts, this approximation is estimated to lead to inaccuracies in the pd amplitudes of less than 1%. In fact, the sole reason for using the CBA is that even then the required CPU time increases by approximately a factor of 25 over a calculation without the Coulomb interaction.

We have used the Paris potential, in fact a separable representation (PEST1-6) thereof which is known to provide an excellent approximation to the original local potential [13]. This choice was motivated by the availability of numerical below-threshold pd phase parameters [2] against which we could check our code. The interaction is taken into account in the states $^3S_1 - {}^3D_1$, $^1S_0$, and in all P waves. Isospin is not introduced. To reach the unscreening limit for cross sections, a value $R = 100$ fm of the screening radius was found to be sufficient. For polarization observables, however, for c.m. scattering angles larger than 70° the same goal was achieved only with amplitudes calculated with $R \gtrsim 300$ fm, while for smaller angles even screening radii larger than $R = 625$ fm would be needed; such calculations were presently not attempted because of expected excessive increase of necessary computer time. We finally mention that convergence with respect to the number of total angular momenta was reached for $J_{\max} = 19/2$.

In Figs. 1–6, we present a set of scattering observables calculated for a proton laboratory energy of 10 MeV. Shown are the results using the solution of Eq. (2) with the Coulomb interaction switched on and off; in the latter case, we arrive at the corresponding observables for the nd reaction. For illustrative purposes, we also include the observables obtained by the standard procedure for approximately taking into account Coulomb effects which

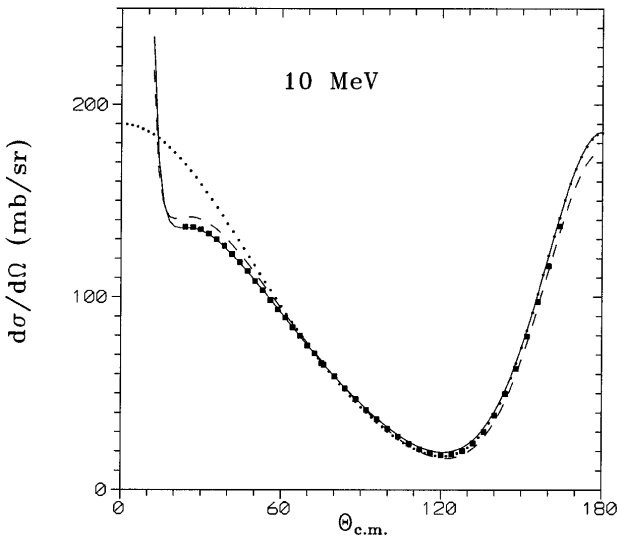

FIG. 1. Differential cross section at 10 MeV proton laboratory energy vs c.m. scattering angle. Experiment (■): Ref. [14]. Solid line: Full calculation; dashed line: only "external" Coulomb corrections; dotted line: nd cross section.

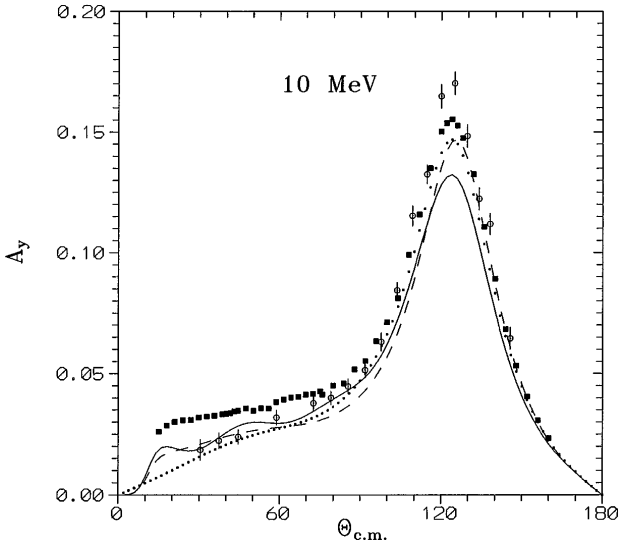

FIG. 2. Proton analyzing power. Notation as in Fig. 1. *pd* data (■) from Ref. [14]; *nd* (○) data from Ref. [16].

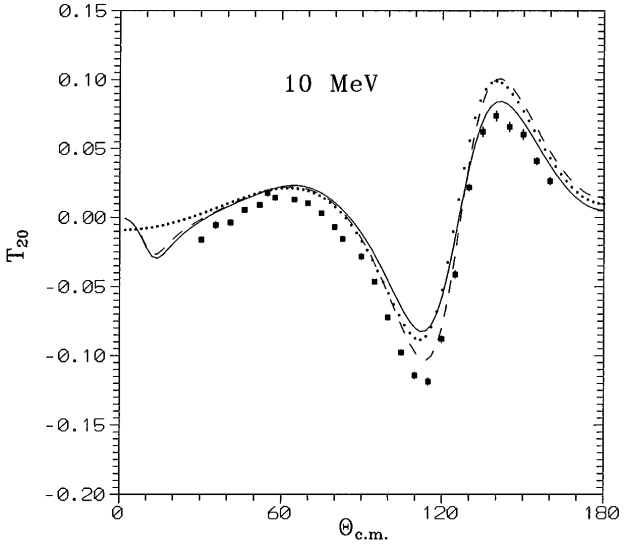

FIG. 4. Deuteron tensor polarization T_{20}. Notation as in Fig. 1. Data (■) from Ref. [17].

consists of keeping only the "external," and neglecting all "internal," Coulomb corrections. Explicitly, the partial-wave amplitudes for the *nd* reaction are multiplied by the c.m. Coulomb partial-wave *S* matrices to account for the asymptotic distortion, before summing them up to the three-dimensional amplitudes to which then the c.m. Rutherford amplitude is added.

Inspection of Fig. 1 shows that the differential cross section data of [14] are well reproduced. However, the calculated proton analyzing power A_y, displayed in Fig. 2, lies well below experiment [14]. For the *nd* reaction this underestimation of the region around the maximum is known as the "A_y puzzle" the resolution of which appears not to be attainable with standard nuclear potential models (including standard three-nucleon forces) but seems to require readjustment of some input *NN* phase parameters [15]. It clearly shows up also when comparing our *nd* result with the *nd* data of [16]. Not surprisingly, this underestimation problem continues to persist when going over to the *pd* reaction. A similar "puzzle" exists for iT_{11} as can be inferred from Fig. 3. For energies below the deuteron breakup threshold these facts have already been pointed out in [3]. However, the experimental difference $A_y(nd) - A_y(pd)$ seems to be rather well reproduced by our calculation. For the various deuteron tensor polarizations shown in Figs. 4–6, we achieve a reasonable reproduction of the data of [17]. It will be important to

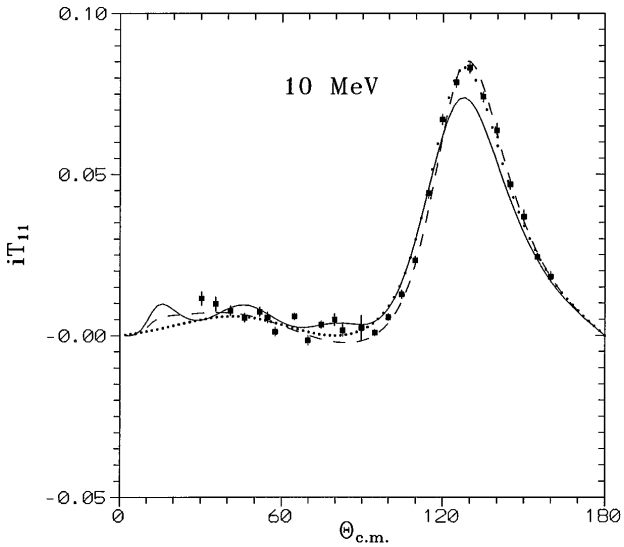

FIG. 3. Deuteron vector polarization iT_{11}. Notation as in Fig. 1. Data (■) from Ref. [17].

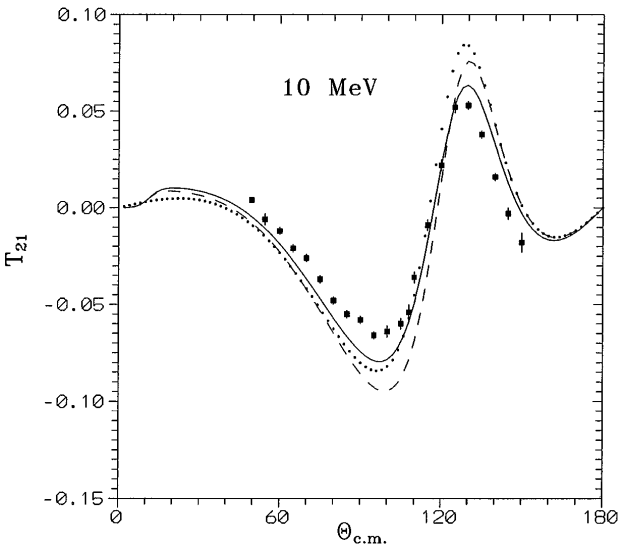

FIG. 5. Deuteron tensor polarization T_{21}. Notation as in Fig. 1. Data (■) from Ref. [17].

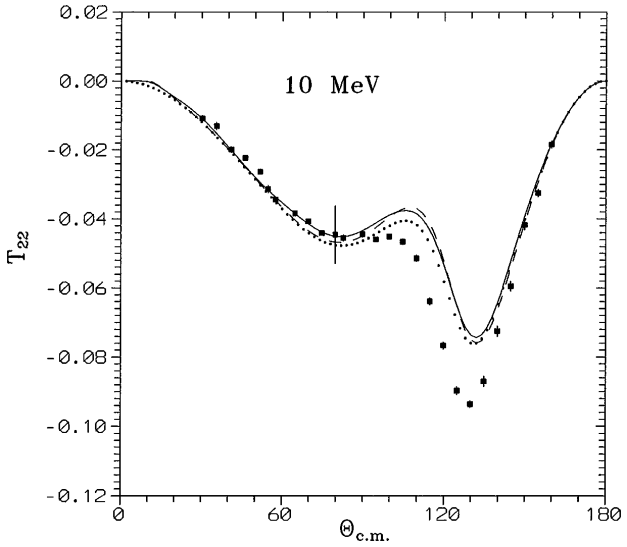

FIG. 6. Deuteron tensor polarization T_{22}. Notation as in Fig. 1. Data (■) from Ref. [17].

investigate the energy dependence of the Coulomb effects in the various observables.

From this the following conclusions can be drawn. (i) Coulomb effects play a minor role for the differential cross section (with the exception of the small-angle region) but lead to sizable corrections in several polarization observables, in the maximum, minimum, and small-angle regions. This feature has already been observed for energies below the breakup threshold in [2,3]. (ii) The above-mentioned standard approximation for taking into account Coulomb effects, while being of acceptable accuracy for the differential cross section, fails for the more sensitive polarization observables.

Clearly, the numerical results presented here are still lacking the full sophistication achieved for the *nd* reaction. Indeed there exist several possibilities for improvement. Most important is the use of more modern *NN* potentials for which we need low-rank separable expansions of high quality. This should considerably improve the agreement with the *nd* data, and, hence, also lead to more realistic *pd* results. Moreover, as is known from *nd* calculations, still higher *NN* partial waves are expected to contribute even at this relatively low energy. Finally, increasing the screening radius even further while maintaining an acceptable numerical accuracy of our results will quite generally improve on the polarizations in the small-angle region and, in particular for A_y, will reduce spurious oscillations.

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