Edge States in the Fractional Quantum Hall Effect Regime Investigated by Magnetocapacitance

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The magnetocapacitance between a two-dimensional electron system (2DES) and the gate electrode on it has been measured in the fractional quantum Hall effect regime. By measuring the edge-length dependence of the magnetocapacitance, we show that the conducting region (the edge state) is formed along the sample boundary. The width of edge state (W_e) and the bulk conductivity of the 2DES are estimated from the frequency dependence of the magnetocapacitance. The estimated W_e is much wider than that expected from the one-electron approximation picture in the integer quantum Hall regime. [S0031-9007(98)07725-4]

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The concept of edge state in the integer quantum Hall effect (IQHE) [1,2] has been adopted by many experimental researchers, since this concept can explain not only the IQHE intuitively but also a lot of the apparent curious phenomena observed in the IQHE regime such as the nonscaling resistance with sample size [3], the nonlocal resistance [4–6], the influence of extra probes on resistance [7,8], and the gate voltage dependence of Hall resistance [9–11], etc. When an ac capacitance between a two-dimensional electron system (2DES) and the gate electrode on the sample is measured in the IQHE regime, the magnetocapacitance shows quantum oscillations with minima at the quantum Hall plateaus [12]. Until now, we have investigated the magnetocapacitance in the IQHE regime [13]. It was revealed that the minima are determined mainly by the conductive region along the sample edge, which should be attributed to the edge state. The width of the edge state estimated from the capacitance is much wider than the order of the magnetic length [$\lambda = (\hbar/eB)^{1/2}$], as expected from the one-electron approximation picture.

It is also expected that the edge state exists in the fractional quantum Hall effect (FQHE) regime. Theoretically, there have been mainly two pictures on the edge state in the FQHE regime. One is the compressible-incompressible model proposed by Beenakker [14]. The other is the many-branches model by MacDonald [15]. The FQHE comes from the many-body effect of electrons and cannot be explained by the one-electron picture. Since the edge state in the IQHE regime is regarded as the cross line between the Fermi level and the lifted Landau level due to the confinement potential, this picture cannot be simply adopted in the FQHE regime. The picture of the edge state in the FQHE regime is not as clear as that in the IQHE regime.

There have also been some calculations [16,17] and experimental investigation on the edge state in the FQHE regime. The existence of the edge state in the FQHE regime was evidenced by the nonlocal resistance [18], the quantized conductance through a point contact [19], the breakdown of diagonal resistance minima [20] and the multiterminal capacitance [21], etc. However, the experiments on the width of the edge state are few [22,23]. We have investigated the edge state in the FQHE regime by the magnetocapacitance measurement and estimated the width of the edge state.

The samples used in the magnetocapacitance measurement were made from the GaAs/AlGaAs heterostructure wafer. The nondoped AlGaAs, doped AlGaAs, and the GaAs cap layer were 60, 60, and 5 nm in thickness, respectively. The electron carrier concentration and mobility of 2DES in the wafer were $N_S = 1.3 \times 10^{11}$ cm^{-2} and $\mu = 2.2 \times 10^6$ cm^2/Vs at 70 mK, respectively. The samples were fabricated by the wet chemical etching and UV-lithography techniques. The capacitance was measured by an ac capacitance meter. The details of sample fabrication and the measurement procedure were described in Ref. [13]. The magnetocapacitance measurement was performed in a dilution refrigerator at temperatures down to 35 mK placed in magnetic fields up to 16 T.

In order to confirm the existence of a conductive region in the FQHE regime, that is, the edge state along the sample boundary, we measured the magnetocapacitance of the samples with different edge lengths ($L_e = 2.8$ mm, $4L_e$, $8L_e$) caused by the different types of fins, as shown in the inset of Fig. 1, as similar to the previous study in the IQHE regime [24]. The real parts of magnetocapacitance of these samples are shown in Fig. 1, where ν is the filling factor of the Landau level. The slight capacitance difference at zero magnetic field is due to the increment of the gated area by the extra fins. On the other hand, the minimum capacitance values become larger by more than several times with the edge length. The minimum values at $\nu = 1$ and 2 (IQHE regime) are almost linearly proportional to the edge length, since the bulk conductivity of 2DES (σ_{xx}) is extremely small at

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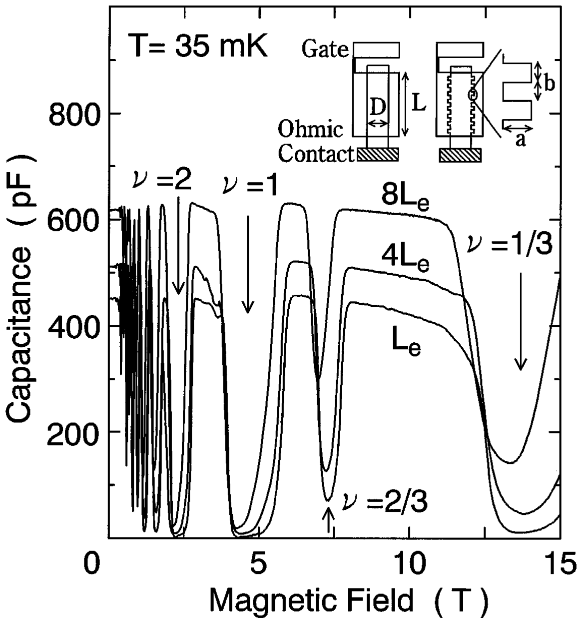

FIG. 1. Magnetocapacitance of the samples with various edge lengths ($2nL$; $n = 1$, 4, and 8) due to different types of fins, as shown schematically in the inset. Main sample length L, width W, and fin width b are 1.4 mm, 0.4 mm, and 20 μm, respectively. Fin length a is $(n - 1)b$ with $n = 4$ and 8. Modulation frequency and gate voltage in ac capacitance measurement are 1 kHz and 1 mV, respectively.

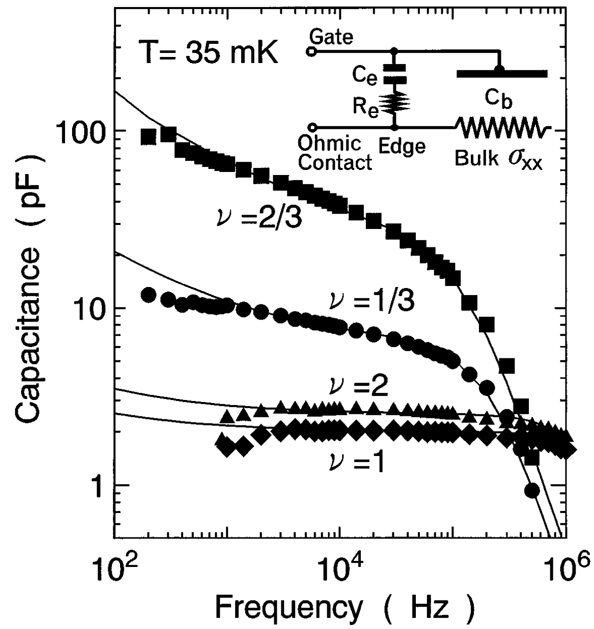

FIG. 2. Frequency dependence of the capacitance minina in the sample with L_e at various ν. Modulation gate voltage is 1 mV. Solid curves are calculated (see text). Inset shows the equivalent circuit model of the sample.

$\nu = 1$ and 2, and the capacitance contribution from the bulk state can be neglected. The contribution from the bulk state at $\nu = \frac{1}{3}$ and $\frac{2}{3}$ (FQHE regime) cannot be neglected. We estimate the contribution from the edge and bulk states separately, from the frequency dependence of capacitance minima in the following way.

The equivalent circuit of the sample used for the analysis of frequency dependence [13,23] is shown in the inset of Fig. 2. In this equivalent circuit model, the capacitance from the bulk state is treated as a distributed circuit of the resistive plate with conductivity σ_{xx} and becomes complex. The capacitance due to the edge state (C_e) is assumed to be purely capacitive (real) and independent of frequency, since the conductivity of the edge state is high enough and there is almost no energy loss in the measured frequency range. We add a series resistance (R_e) relating to the energy dissipation at the boundary between the edge state and the Ohmic contact. The measured effective capacitance (C) at frequency (f) is expressed as

$$C = C_b(\sigma_{xx}, f) + C_e^*(R_e, f), \tag{1}$$

where $1/C_e^* = 1/C_e + i\omega R_e$ and $\omega = 2\pi f$.

The bulk capacitance (C_b) can be calculated by the one-dimensional resistive plate model [13] and is given as

$$C_b(\sigma_{xx}, f) = (C_0 - C_e)\,\frac{\tanh(Z)}{Z}, \tag{2}$$

where $Z = [i\omega(C_0 - C_e)(D/4L)/\sigma_{xx}]^{1/2}$, and D and L are the sample width and length, respectively. The C_0 is the capacitance at $f = 0$ or $\sigma_{xx} = \infty$ and $R_e = 0$, which is substituted for the zero magnetic field capacitance, since σ_{xx} is large enough and C_e is zero at zero field.

In Fig. 2, the frequency dependence of capacitance minima of the sample with L_e is shown at $\nu = \frac{1}{3}$, $\frac{2}{3}$, 1, and 2. The frequency dependence of capacitance in the low frequency region ($f < 100$ kHz) is determined mainly by the σ_{xx}. The decreasing rate of capacitance with f increases with larger σ_{xx}. The rapid decrease of capacitance above 100 kHz is due to the series resistance. The solid curves are calculated from Eqs. (1) and (2) by using three parameters (C_e, σ_{xx}, R_e). The obtained parameters are listed in Table I in the sample with L_e. We determined C_e of the samples with various edge lengths from such fits of the frequency dependence of the measured capacitance. In Fig. 3, thus determined C_e at various ν is plotted versus sample edge length. The edge capacitance (C_e) is linearly proportional to the edge length, except for the C_e at $\nu = \frac{1}{3}$ in the sample with $8L_e$. It clearly confirmed that the C_e at the FQHE regime comes from the sample edge, as observed in the previous experiment of the IQHE regime.

The C_e at $\nu = \frac{1}{3}$ in the sample with $8L_e$ deviates from a linear dependence on the edge length. The reason for the deviation is not clear. One of the speculations on the deviation is as follows. As the magnetic field values of capacitance minima in the $8L_e$ sample are lower than those in the other samples, the electron concentration of

TABLE I. Obtained parameters of the sample with L_e at 35 mK. The modulation gate voltage in the ac capacitance measurement is 1 mV, except for the data in parentheses (0.5 mV).

Filling factor (ν)	$\frac{1}{3}$	$\frac{2}{3}$	1	2
C_e (pF)	6.4	26	2.0	2.5
W_e (μm)	2.8 (1.9)	12 (8.0)	0.89	1.1
σ_{xx} (S)	1.1×10^{-11}	9.3×10^{-10}	1×10^{-14}	5×10^{-14}
R_e (kΩ)	115	46	33	40

the $8L_e$ sample is smaller than that of the other samples which may be caused by the accidental fluctuation of 2DES in the wafer. This lower electron concentration may enhance the inhomogeneity of 2DES, and the edge state becomes a percolative conductive channel, which spreads into the bulk area in the fin. The C_e is enlarged at $\nu = \frac{1}{3}$. On the other hand, the C_e at $\nu = \frac{2}{3}$ does not deviate appreciably from the linear dependence. Since the width of edge state at $\nu = \frac{2}{3}$ estimated from the C_e is comparable to half of the fin width ($b/2 = 10$ μm), as shown in Table I, the edge state almost fills in the fins of the sample. In such a case, the slope of the line is uniquely determined by the fins' area and the deviation does not occur.

The width of edge state (W_e) is calculated on the assumption that C_e is a capacitance of a parallel capacitor with an area of edge state under the gate ($L_e W_e$). The obtained widths at the IQHE regime ($\nu = 1$ and 2) are almost the same as those determined by the previous experiment between 0.4 and 4.2 K [13]. It is also confirmed that W_e and σ_{xx} are practically independent of temperatures (35–100 mK) in this measurement. From these results, it is considered that the edge state at the IQHE regime ($\nu = 1$ and 2) does not change appreciably below 4.2 K. The obtained values of R_e are of the order of the quantized resistance $h/(\nu e^2) = 25.8/\nu$ kΩ which is observed between the edge state and the Ohmic contact in both FQHE and IQHE regimes.

The temperature dependence of W_e and σ_{xx} at $\nu = \frac{1}{3}$ and $\frac{2}{3}$ is shown in Fig. 4 between 35 and 100 mK. With increasing temperatures, the W_e does not change appreciably though the σ_{xx} increases an order of magnitude. From these results, the W_e is not influenced by the σ_{xx} and the edge state is clearly separated from the bulk state.

The width of edge state at the FQHE regime is much wider than the order of the magnetic length, which is expected from the one-electron approximation picture. Such a wide W_e was also observed in the IQHE regime [13,24] and was explained by the compressible-incompressible

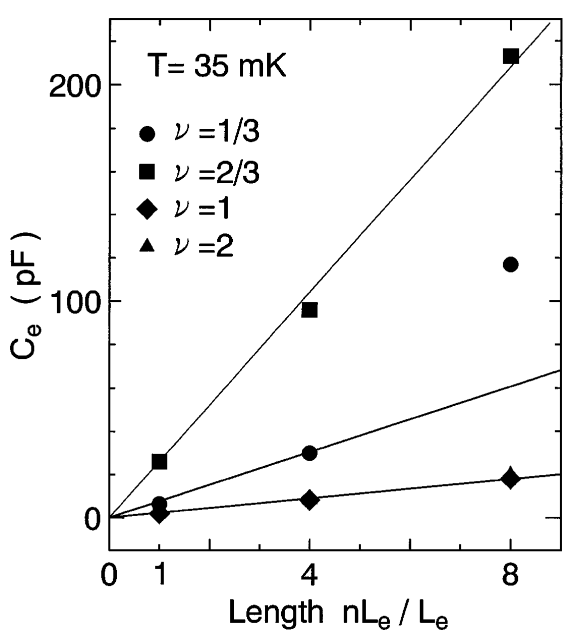

FIG. 3. Edge length dependence of the edge capacitance C_e at various ν. It is noted that C_e at $\nu = 1$ and 2 are almost the same.

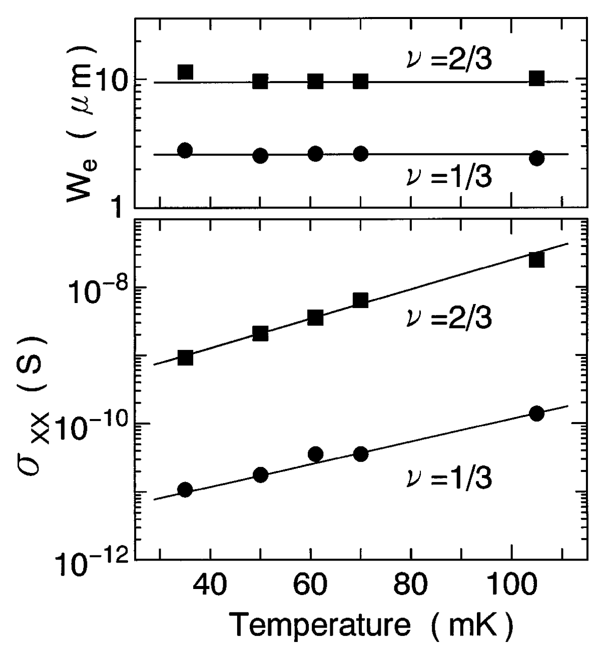

FIG. 4. Temperature dependence of the edge state width W_e and the bulk conductivity σ_{xx} in the FQHE regime ($\nu = \frac{1}{3}$ and $\frac{2}{3}$). Solid lines are provided as a guide to eyes.

model [16]. In this model, the screening effect of electron interactions near the sample boundary is taken into account. The spatial distribution of the lifted Landau level originating from the confinement potential becomes flat in the compressible region, where the electron concentration can vary. Since the edge state observed by the capacitance is considered to be the compressible region, the wide W_e can be explained qualitatively by this model. The reason why the wide W_e in the FQHE regime is also observed is not clear at present. One possible speculation is that the compressible-incompressible model is also valid in the FQHE regime. In the FQHE regime, the energy gap at $\nu = \frac{1}{3}$ and $\frac{2}{3}$ is formed by the many-body effect among electrons, while the energy gap in the IQHE region comes from by the Landau levels in the one-electron approximation picture. The incompressible region, where the carrier concentration is fixed due to the gap, is brought about in the bulk area. Near the sample edge, the carrier concentration (the filling factor) decreases due to the confinement potential, and the energy gap vanishes. The compressible region in which the carrier concentration is changeable is formed and the edge state spreads out.

It is observed that the evaluated W_e in the FQHE regime depends considerably on the modulation gate voltage in the ac capacitance measurement. As shown in Table I, the determined 1 mV modulation voltage is about 30% larger than that at 0.5 mV in the FQHE regime, though the W_e in the IQHE regime is hardly affected in this voltage range. An immediate explanation for the modulation voltage dependence would be that there is a significant electron heating by the modulation voltage; however, this is contradictory to the observation that the measured W_e does not change with bath temperatures between 35 and 110 mK, while the σ_{xx} changes considerably, indicating that the electron temperature is affected by the bath temperature. Another possible explanation is that the modulation voltage causes excitations of the composite fermions (CF) as follows: The concept of the CF has been proposed to explain the FQHE [25]. In this picture, the FQHE is regarded as the IQHE of the CF at about $\nu = \frac{1}{2}$, where the CF consists of an electron and two quantum fluxes $(2h/e)$. The CF experiences the effective magnetic field as $B_{\text{eff}} = B - B_{1/2}$, where $B_{1/2}$ is the magnetic field at $\nu = \frac{1}{2}$. The Landau level gap of the CF at $\nu = \frac{1}{3}$ (14.5 T) in this sample is estimated as 0.8 meV, if we assume an effective CF mass 10 times larger than that of the electron in GaAs [26] and $B_{\text{eff}} = 4.8$ T. This value is comparable to the modulation voltage and suggests that there could be a significant excitation across the Landau level gap.

The W_e at $\nu = \frac{2}{3}$ is much wider than that at $\nu = \frac{1}{3}$, as shown in Table I. This wide W_e may also be explained by the above-mentioned effect in the following way. Since the magnitude of B_{eff} at $\nu = \frac{2}{3}$ is half that at $\nu = \frac{1}{3}$, the Landau level gap at $\nu = \frac{2}{3}$ is also half that at $\nu = \frac{1}{3}$. The edge state at $\nu = \frac{2}{3}$ is much more influenced by the modulation voltage than that at $\nu = \frac{1}{3}$. The W_e at $\nu = \frac{2}{3}$ may broaden still more by the modulation voltage.

In summary, we have confirmed definitely the existence of the edge state in the FQHE regime from the magnetocapacitance measurement on the samples with various edge lengths. The W_e and σ_{xx} were determined from the frequency dependence of capacitance minima. The temperature dependence of W_e and σ_{xx} in the FQHE regime was measured. With increasing temperatures from 35 to 100 mK, the W_e is practically constant while the σ_{xx} increases an order of magnitude. Thus determined W_e is of the same order of W_e in the IQHE regime and is much wider than that expected from the one-electron approximation picture.

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