Interlayer Magnetic Coupling and the Quantum Hall Effect in Multilayer Electron Systems

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We study the effect that electron-electron interaction has on the properties of a multilayer electron system. We consider the case corresponding to filling factor unity in each layer and find that, as a function of the sample parameters, the system has ferromagnetic, canted antiferromagnetic, or paramagnetic interlayer spin correlations. These three ground states are QHE phases because of the existence of a finite activation energy. In the ferromagnetic phase the gap is due to the intrawell exchange energy, whereas in the paramagnetic phase the gap appears due to spatial modulation of the interwell coherence. [S0031-9007(98)07800-4]

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The quantum Hall effect (QHE) is one of the most striking phenomena observed in two-dimensional electron gas (2DEG) systems [1]. The QHE occurs because the 2DEG becomes incompressible at certain filling factors. In the odd integer and fractional QHE, the energy gap source of the incompressibility is produced by interactions between electrons. In the even integer QHE the incompressibility is due to the quantization of the electron kinetic energy. Given the new physics which appears in 2DEG in the QHE regime, the question that arises is whether the quantum Hall phases are unique to 2DEG or can they occur in three-dimensional (3D) conductors [2]. In this direction, some studies in narrow gap 3D semiconductors in the strong magnetic field limit have shown some signatures of an incipient quantum Hall phase [3]. On the other hand, the progress in epitaxial growth has made it possible to fabricate semiconductor systems where 2DEG's with extra degrees of freedom exist:

(i) Wide parabolic quantum wells where a thick electron gas layer ($\sim$2000 Å) is formed. This system presents a clear QHE phase [4].

(ii) Double quantum well (DQW) systems with electrons confined to two parallel sheets separated by a distance comparable to that between electrons within a plane [5]. DQW systems present QHE at total integer filling factors, even in the absence of tunneling between the electron planes.

(iii) Superlattices, where an appreciable dispersion of the electronic spectrum in the direction perpendicular to the layers exits. Accurately quantized Hall plateaus have been observed in these multilayer systems [6,7] when a magnetic field is applied parallel to the superlattice axis. Studies of vertical transport in these supperlattices have shown [7] the existence of a chiral two-dimensional system that forms at the surface of the layered system [8]. The poor mobility of the bulk narrow gap semiconductors and the small number of new degrees of freedom of the parabolic and DQW's with respect the 2DEG, made the superlattices the best candidates for studying QHE phases in 3D conductors.

In this work we study the effect that the electron electron interaction has on the properties of the multilayer electron system. We consider the case corresponding to filling factor unity in each electron layer. Two points are raised in this paper: the magnetic order of the electron layers and the conditions for the occurrence of the QHE in this system. The main results we obtain are the following: (i) As a function of the sample parameters (Zeeman coupling, H, interlayer tunneling, t, and barrier thickness, d) we find that the system changes from a QHE state with interlayer ferromagnetic spin correlations to a new QHE state with canted antiferromagnetic interlayer correlations (see Fig. 1). This phase is similar to the canted phase [9] predicted to occur in DQW systems at total filling factor 2, and experimentally verified [10]. For larger values of the tunneling amplitude, we find that the system undergoes another phase transition towards a paramagnetic state. These transitions are second order phase transitions.

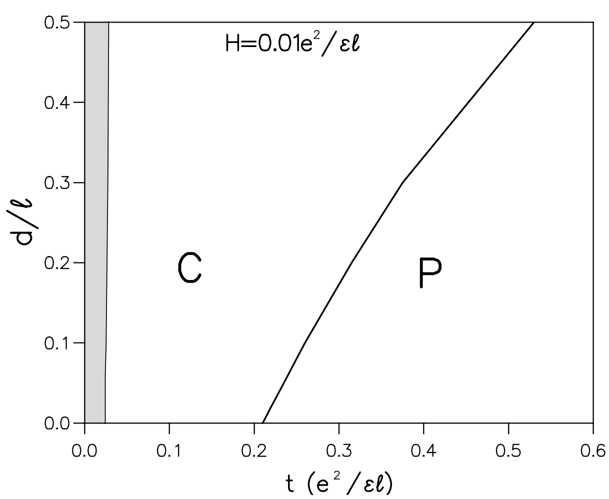

FIG. 1. Phase diagram for a multilayer system, with filling factor unity in each well. The Zeeman coupling is $H = 0.01e^2/\epsilon\ell$ and the electron layer thickness $b = 0.8\ell$. Three phases are present: a ferromagnetic phase (shadow region), a canted antiferromagnetic region (C), and a paramagnetic region (P).

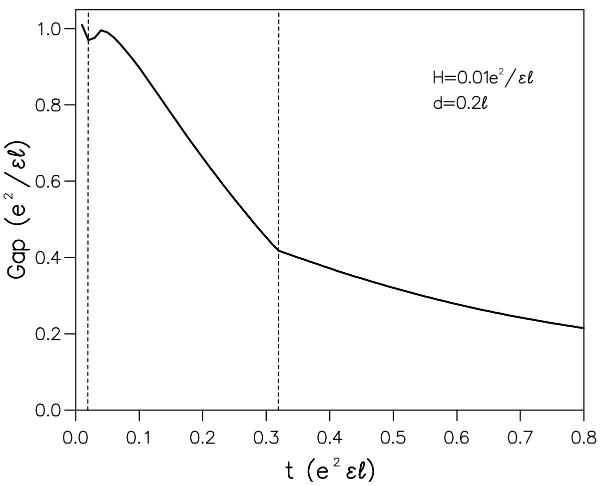

FIG. 2. Activation energy of a multilayer system as a function of the tunneling amplitude. The Zeeman coupling is $H = 0.01e^2/\epsilon\ell$, the barrier thickness $d = 0.2\ell$, and the electron layer thickness is $b = 0.8\ell$. The vertical lines indicate the values of t where the different phase transitions occur.

(ii) We also study the value of the activation energy as a function of the sample parameters. In Fig. 2 we plot this energy gap as a function of the tunneling amplitude for a multilayer system with $d = 0.2\ell$ and $H = 0.01e^2/\epsilon\ell$ (here ℓ is the magnetic length). We find that this gap is finite even for very large values of t, where the system is paramagnetic. This implies that the paramagnetic phase of the multilayer system is also a QHE phase. As we explain below, the energy gap in the paramagnetic phase appears because the system breaks spontaneously the translational symmetry along the multilayer axis by modulating the interwell coherence. From our results we conclude that in multilayer electron systems, with filling factor unity in each well, the QHE prevails in all phases due to the existence of a finite activation energy, even at $d, t \to \infty$.

We treat the electron electron interaction in the Hartree-Fock (HF) approximation. From previous works [5,9,10–12] in 2DEG and DQW systems, we expect the HF approximation to be a good approach for describing ML systems at filling factor unity in each well.

The calculations presented here employ realistic Coulomb interaction potentials and take into account interlayer tunneling and Zeeman coupling; therefore we expect our results to be qualitatively and quantitatively trustworthy. We take the multilayer vertical axis as the z direction and the electrons live in the x-y planes. The magnetic field, B, is applied in the z direction. Since B is very strong, we consider only states in the lowest energy Landau level of the lowest energy subband of each well. The Hamiltonian of the system is written as $\hat{H} = H_0 + V$, with

$$H_0 = -t \sum_{\langle i,j\rangle,k,\sigma} (C^\dagger_{i,\sigma,k} C_{j,\sigma,k} + \text{H.c.}) - H \sum_{i,\sigma,k} \sigma C^\dagger_{i,\sigma,k} C_{i,\sigma,k} \,, \quad (1)$$

where $C^\dagger_{i,\sigma.k}$ creates an electron in the lowest Landau level in layer i with spin σ ($\sigma = \pm 1$) in the direction of B, and with intra Landau level index k. The sum in the first term of Eq. (1) is over first neighbors layers. The many-body part of $\hat{H}$ takes the form

$$V = \frac{1}{2S} \sum_{\sigma,\sigma'} \sum_{i,j} \sum_{k,k',\mathbf{q}} V_{i,j}(q) e^{-q^2\ell^2/2} e^{iq_x(k-k')\ell^2} \times C^\dagger_{i,\sigma,k+q_y} C^\dagger_{j,\sigma',k'} C_{j,\sigma',k'+q_y} C_{i,\sigma,k} \,, \quad (2)$$

where S is the sample area and the interaction potential has the form $V_{i,j} = 2\pi e^2/\epsilon q F_{i,j}(q,b,d)$, with $F_{i,j}$ being finite layer thickness form factors [13], which depends on the thickness of the 2DEG in each layer b, and on the barrier thickness d.

The multilayer (ML) systems are doped in the barriers and in the absence of B, all layers have the same number of electrons. We consider that this is also the situation at large values of B, since in any other situation it would cost a large Hartree energy. At filling factor unity in each layer the intrawell correlation is not very important so we consider only solutions with translational symmetry in the plane (x,y) of the electron gases. Broken translational symmetries along the multilayer axis (z direction) are allowed, always with the condition of having filling factor unity in each layer. With these constraints, the HF expectation value of V takes the form

$$\langle V\rangle = -\frac{1}{2S} \sum_{i,j,\mathbf{q}} \sum_{\sigma,\sigma',k} V_{i,j}(q) e^{-q^2\ell^2/2} \langle C^\dagger_{i,\sigma,k} C_{j,\sigma',k}\rangle \langle C^\dagger_{j,\sigma',k-q_y} C_{i,\sigma,k-q_y}\rangle \,. \quad (3)$$

Here the sum in k is over all its possible values. By minimizing the energy $\langle\hat{H}\rangle = \langle H_0\rangle + \langle V\rangle$, we obtain the energy of the ground state of the system and its properties.

We have solved the Hamiltonian for different values of d, t, and H. For each layer i we calculate the expectation value of the total spin operator per electron $\langle \mathbf{S}_i\rangle$. For characterizing the ground state it is also necessary to quantify the interlayer coherence which is given by the following expectation value,

$$\Delta_{\sigma,\sigma'}(i) = \frac{1}{N_\phi} \sum_k \langle C^\dagger_{i,\sigma,k} C_{i+1,\sigma,k}\rangle \,. \quad (4)$$

Here $N_\phi = S/2\pi\ell^2$ is the the Landau level degeneracy. This quantity represents the coherence between wells.

Looking to the values of $\langle \mathbf{S}_i\rangle$ we find three different classes of ground states (see Figs. 1 and 3):

(1) *Ferromagnetic phase.*—All electron layers are fully spin polarized in the direction of the magnetic field, i.e., $\langle \mathbf{S}_i\rangle = (0,0,1/2)$. This phase occurs for small values of t or large values of H. In the ferromagnetic phase the intralayer coherence is more important than the interlayer coherence and all the expectation values of the operators $\Delta_{\sigma,\sigma'}(i)$ are zero, and there is not vertical kinetic energy contribution to the total energy. The ferromagnetic ground state is a QHE phase, and

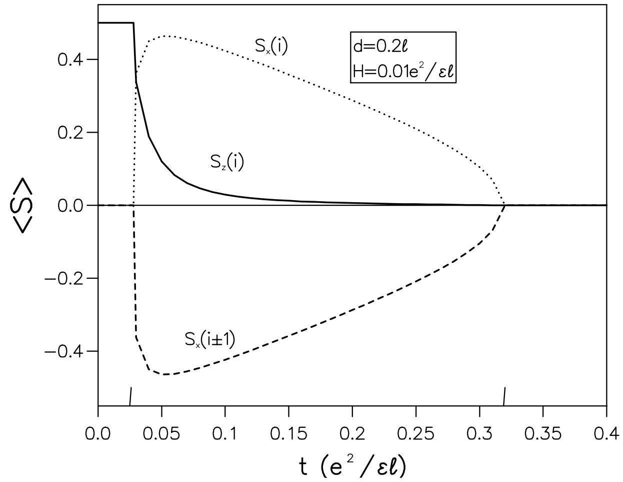

FIG. 3. Expectation values of the total spin operator per electron as a function of the tunneling amplitude. The Zeeman coupling is $H = 0.01e^2/\epsilon\ell$, the barrier thickness $d = 0.2\ell$, and the electron layer thickness is $d = 0.8\ell$. The vertical tick marks in the lower x axis indicate the values of t where the different phase transitions occur.

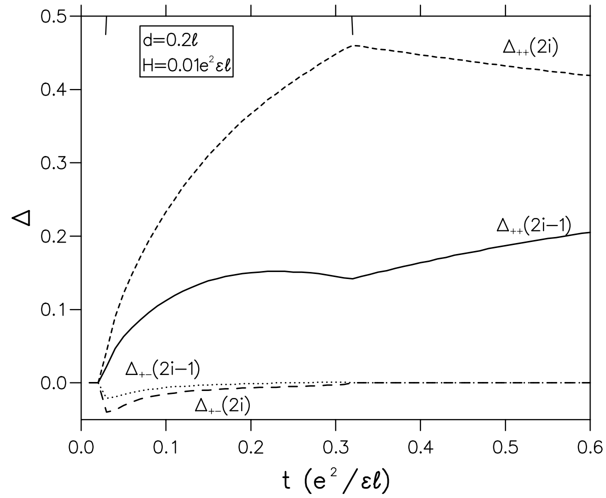

FIG. 4. Interlayer coherence parameters as a function of the tunneling amplitude. $\Delta_{++}(i)$ are real and we plot its real part. $\Delta_{+-}(i)$ are imaginary and we plot its imaginary part. The Zeeman coupling is $H = 0.01e^2/\epsilon\ell$, the barrier thickness $d = 0.2\ell$, and the electron layer thickness is $b = 0.8\ell$. The vertical dashed tick marks in the upper x axis indicate the values of t where the different phase transitions occur.

the activation gap (see Fig. 2) is the cost in energy of adding an electron to the system with the spin pointing antiparallel to the magnetic field. In this phase the ground state has the same translational symmetry as the Hamiltonian.

(2) *Canted antiferromagnetic phase.*—In this phase the total spin in each layer acquires a component perpendicular to the magnetic field, $\langle \mathbf{S}_i \rangle = (\mathbf{S}_{i,\perp}, S_{i,z})$, and the magnitude of $\langle \mathbf{S}_i \rangle$ is smaller than its maximum value, $1/2$. In this phase the sign of $\mathbf{S}_{i,\perp}$ alternates from layer to layer, i.e., $\mathbf{S}_{i,\perp} = -\mathbf{S}_{i\pm1,\perp}$, so that the translational symmetry of the Hamiltonian is spontaneously broken and the unit cell in the z direction consists of two electron layers which are labeled $2i - 1$ and $2i$. The z component of $\langle \mathbf{S}_i \rangle$ is finite and it has the same value in all layers, therefore in the canted phase there is an interlayer antiferromagnetic coupling of the transverse component of the total spin. By performing calculations in bigger size unit cells, we have checked that this phase is stable with respect to spiral ordering of the transverse component of $\langle \mathbf{S}_i \rangle$.

In the canted phase the interlayer coherence parameter is different from zero and verify the relations:

$$\Delta_{+,+}(i) = \Delta_{-,-}(i)\,,$$
$$\Delta_{+,-}(i) = -\Delta^*_{-,+}(i)\,, \tag{5}$$

but the interlayer coherence parameter depends on i. In Fig. 4 we plot $\Delta_{+,+}(2i)$, $\Delta_{+,+}(2i - 1)$, $\Delta_{+,-}(2i)$, and $\Delta_{+,-}(2i - 1)$, as a function of t. We see that

$$\Delta_{\sigma,\sigma'}(2i) \neq \Delta_{\sigma,\sigma'}(2i - 1)\,, \tag{6}$$

and this implies that there is a modulation of the interlayer coherence. Therefore in the canted phase the translational symmetry along the multilayer axis is broken not just by the antiferromagnetic ordering of the layers, but also by the modulation of the interlayer coherence. In the self-energy calculation, the modulation of the interlayer coherence acts as a spatial modulation of the hopping amplitude, and this modulation contributes to the opening of an energy gap at the Fermi level. The canted phase appears at intermediate values of the tunneling amplitude, see Figs. 1 and 3, and the reason for its existence is that in this phase the system can take advantage of the kinetic energy by creating interlayer antiferromagnetic spin correlations. The antiferromagnetic order is canted in order to minimize the loss of Zeeman energy. The canted ground state is a QHE phase. The transport activation energy is finite because the system is partially spin polarized in the direction of B, and because the interlayer coherence is spatially modulated.

Canted ground states corresponding to rotations of all the $\mathbf{S}_{i,\perp}$ are degenerated and therefore this phase should get a gapless collective mode associated with this degeneracy.

This phase can be considered as the ML generalization of the canted phase obtained in DQW systems [9]. The dependence on t of $\langle \mathbf{S}_i \rangle$ is very similar to that found in DQW systems. However, in the ML system there is also a modulation of the interlayer coherence which obviously cannot appear in DQW systems. In fact, the canted phase in infinite ML systems can be considered as the high magnetic field limit of the spin density wave ground state proposed by Celli and Mermin [14] to occur in 3D systems. The reason for this instability arises from the one-dimensional nature of the electron energies in a strong B.

(3) *Paramagnetic phase.*—In this phase the expectation value of the total spin operator is zero in all layers, $\langle \mathbf{S}_i \rangle = 0$. This phase occurs at large values of t, where the kinetic energy and interwell coherence energy is much bigger than the Zeeman and intrawell exchange

energy. In this phase $\Delta_{+,-}(i) = 0$, but the equal spin interwell coherence parameters are different from zero and verify $\Delta_{+,+}(2i - 1) = \Delta_{-,-}(2i - 1) \neq \Delta_{+,+}(i) = \Delta_{-,-}(2i)$. In this phase, the system breaks spontaneously the translational symmetry by modulating the interwell coherence along the vertical axis of the multilayer. In the paramagnetic ground state the unit cell consists of two electron layers. This modulation of the interwell coherence creates an energy gap at the Fermi energy, and the paramagnetic ground state is a QHE phase.

In these three magnetic phases there is a gap at the Fermi energy and electric resistance will be infinite for an electric field parallel to the ML axis. On the other hand, current flow perpendicular to the z direction will not be affected by the interlayer magnetic order and occurs without dissipation. The Hall conductivity is given by the classical expression $\sigma_{xy} = n^{3D}ec/B$, being n^{3D} the average 3D density $n^{3D} = 1/2\pi\ell^2 d$, so that $\sigma_{xy} = e^2/hd$. In this way the Hall conductance contributed by a layer of the ML is e^2/h.

The superlattices studied in Refs. [6,7] have thick barriers and they are in the ferromagnetic phase. For studying antiferromagnetic and paramagnetic phases, it is necessary ML's with thin barriers and large tunneling amplitudes. The ML's are usually doped in the barriers and in the case of thin barriers this produces a strong scattering of the electrons by impurities, which prevents many-body driven ground states. It is possible to circumvent this problem by working with superlattices superimposed on wide parabolic wells [15–17]. These systems are remotely doped, and it is possible to obtain ML's with thin barriers and high electron mobility. The ground states magnetic properties can be studied experimentally by using optically pumped nuclear magnetic resonance. This technique has been very useful for the study of the magnetic nature of 2DEG's [18]. Also it could be very useful in the application of a magnetic field, $B_{\parallel}$, parallel to the electron sheets. $B_{\parallel}$ changes the value of the Zeeman coupling, and as a function of its strength the ground state of the system could change. This phase transition could be identified by studying the activation energy as a function of $B_{\parallel}$ [19]. Strong enough $B_{\parallel}$ also destroys the interlayer coherence [20–22]. In the paramagnetic phase the activation energy is due to the spatial modulation of the interlayer coherence, and the application of a strong $B_{\parallel}$ would destroy the QHE.

In conclusion, we have studied the effect that the electron-electron interaction has on the properties of a multilayer electron system. We consider the case corresponding to filling factor unity in each layer. We have found that as a function of the sample parameters the system has ferromagnetic, canted antiferromagnetic, or paramagnetic interlayer spin correlations. We have obtained that these three ground states are QHE phases, because of the existence of a finite activation energy. In the ferromagnetic phase the gap is due to the intrawell exchange energy, whereas in the paramagnetic phase the gap appears due to the spatial modulation of the interwell coherence.

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